

Writing Up a Math Problem



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The Problem

Projectile Motion

The height of a ball thrown in the air is given by the equation

$$s(t) = -4.9t^2 + 20t + 2 \text{ meters,}$$

where t measures time in seconds. Using this find the time at which the ball reaches its maximum height and the time at which it lands on the ground.

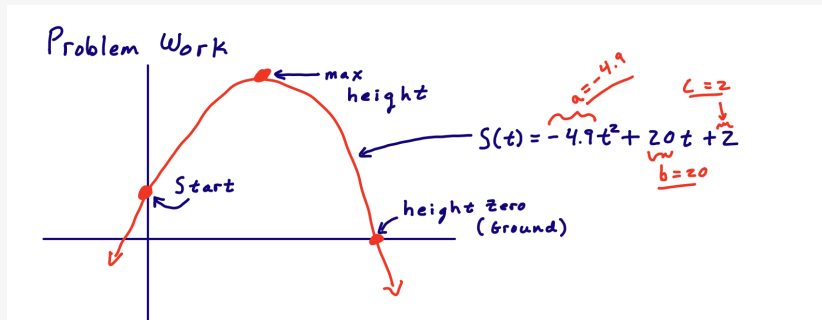


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Picture



Algebra

$$\text{Vertex at } t = -b/2a = \frac{-20}{2(-4.9)} = \frac{-20}{-9.8} \approx 2.041 \text{ seconds}$$

$$\text{Roots at } t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{400 + 39.2}}{-9.8}$$

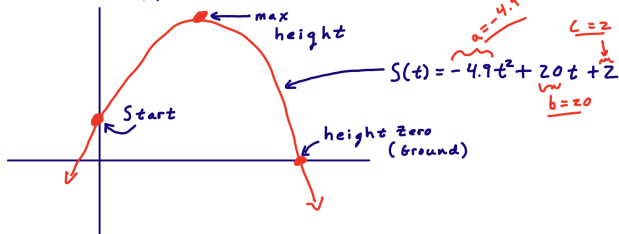
$$= \frac{20 \pm \sqrt{439.2}}{9.8}$$

$$\approx 4.179 \text{ or } -0.098$$



All Together

Problem Work



$$\text{Vertex at } t = -b/2a = \frac{-20}{2(-4.9)} = \frac{-20}{-9.8} \approx 2.041 \text{ seconds}$$

$$\begin{aligned} \text{Roots at } t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{400 + 39.2}}{-9.8} \\ &= \frac{20 \pm \sqrt{439.2}}{9.8} \\ &\approx 4.179 \text{ or } -0.098 \end{aligned}$$



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Flip the Question

Answer

The ball whose height is given by

$$s(t) = -4.9t^2 + 20t + 2 \text{ meters}$$

reaches its maximum height at $t \approx 2.041$ seconds and lands on the ground after $t \approx 4.179$ seconds.

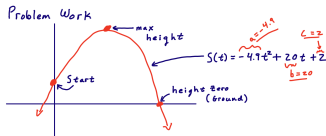


Add Details

Answer

The path of the ball is a parabola, the maximum height is the vertex at

$$t = \frac{-b}{2a} = \frac{-20}{-9.8} \approx 2.041 \text{ sec.}$$



The ball lands on the ground when the height is 0 meters; the roots of the parabola. Using the quadratic equation we get

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{400 + 39.2}}{-9.8} \approx 4.179 \text{ or } -0.098.$$

Only the positive value makes sense, so $t \approx 4.179 \text{ sec.}$



Final Solution All Together

Answer

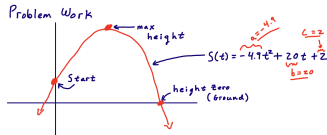
The ball whose height is given by

$$s(t) = -4.9t^2 + 20t + 2 \text{ meters}$$

reaches its maximum height at $t \approx 2.041$ seconds and lands on the ground after $t \approx 4.179$ seconds.

The path of the ball is a parabola, the maximum height is the vertex at

$$t = \frac{-b}{2a} = \frac{-20}{-9.8} \approx 2.041 \text{ sec.}$$



The ball lands on the ground when the height is 0 meters; the roots of the parabola. Using the quadratic equation we get

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{400 + 39.2}}{-9.8} \approx 4.179 \text{ or } -0.098.$$

Only the positive value makes sense so $t \approx 4.179$ sec.

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The Problem

Horsepower Exercise

The horsepower (hp) that a shaft can safely transmit varies jointly with its speed (in revolutions per minute (rpm)) and the cube of the diameter. If the shaft of a certain material 3 inches in diameter can transmit 45 hp at 100 rpm, what must the diameter be in order to transmit 60 hp at 150 rpm?



The Scrap Work

$$\begin{aligned}
 \text{h.p.} &= K \cdot \text{rpm} \cdot d^3 \quad \left. \vphantom{\text{h.p.}} \right\} \text{From text} \\
 K &= \frac{\text{hp}}{\text{rpm} d^3} = \frac{45}{100 \cdot 3^3} = \frac{1}{60} \\
 60 &= \frac{1}{2} \cdot \text{rpm} \cdot d^3 \rightarrow d^3 = \frac{120}{5} = 24 \\
 &\quad d = \sqrt[3]{24} \approx 2.88
 \end{aligned}$$



The Write-Up

Horsepower Solution

Horse power transmitted by a shaft varies jointly with speed and the cube of its diameter

$$\text{hp} = k \cdot \text{rpm} \cdot (d^3).$$

A 3 inch diameter shaft transmitting 45 hp at 100 rpm gives

$$k = \frac{\text{hp}}{\text{rpm} \cdot (d^3)} = \frac{45}{100 \cdot (3^3)} = \frac{1}{60}.$$

Therefore, to generate 60 hp at 150 rpm we need a shaft with diameter

$$\begin{aligned} d &= \sqrt[3]{\frac{\text{hp}}{k \cdot \text{rpm}}} = \sqrt[3]{\frac{60}{(1/60) \cdot 150}} \\ &= \sqrt[3]{24} \\ &= 2\sqrt[3]{3} \text{ inch} \\ &\approx 2.88 \text{ inch.} \end{aligned}$$

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Take 1

0.4226



Take 2

42.26%



Take 3

Probability is 42.26%



Take 4

The probability is 42.26%.



Take 5

The probability of being dealt one pair is 42.26%.



Take 6

The probability of being dealt one pair when playing poker, assuming there are no wilds, is 42.26%.



Question

Assume that you are playing a game of poker with no wild cards. What is the probability of being dealt a hand with a one pair and three other non-matching cards, like $\{5,5,7,Q,J\}$?



Writing Up a Math Problem

