Exploring Groups v 0.3

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1 Directions

The goal of this packet is to understand ***groups*** in a concrete physical and visual way beyond the definition:

Definition 1 (Groups)**.** A set G together with a binary operation $*$ is a ***group*** if for all $a, b, c \in G$:

1. $a * b \in G$, (closure)
2. $a * (b * c) = (a * b) * c$, (associativity)
3. there exists e_G for all a such that $a * e_G = e_G * a = a$, (identity element) and
4. for all a there exists a' such that $a * a' = a' * a = e_G$ (inverse elements).

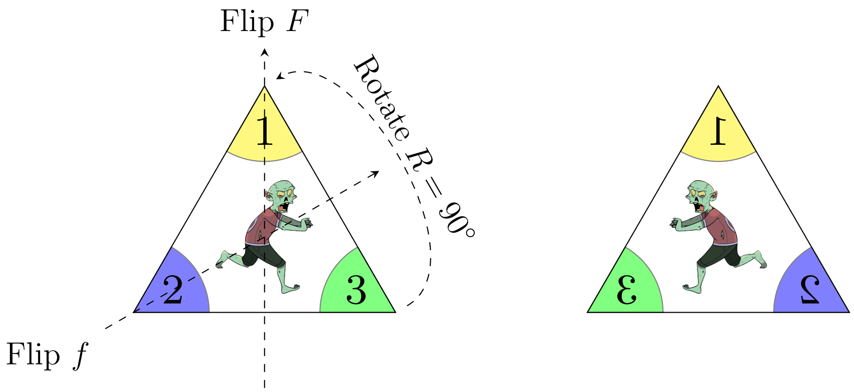
If in addition $a * b = b * a$ we say the group is an ***abelian group***.

Here are the directions ...

2 Moving Triangles

Spinning Triangles

In this section we are looking at symmetries of an equilateral triangle. The transformations are a horizontal flip F, diagonal flip f, or a 120° rotation R. For each picture list the permutation or combination of permutations of the corners and then list the ways we could carry out the transformations with just flips or with just the diagonal flip and the rotation.

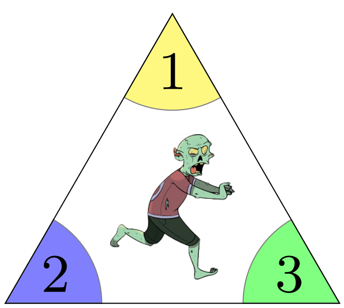


Perm.: (23)

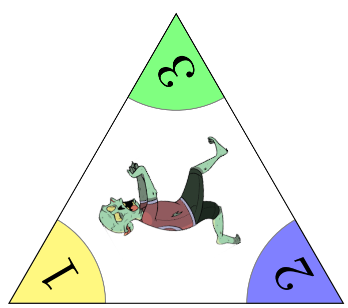
Flips: F

Flip'n Roate: $R \circ f$

Figure 1: Flip'n Triangles



Perm.:
Flips:
Flip'n Roate:



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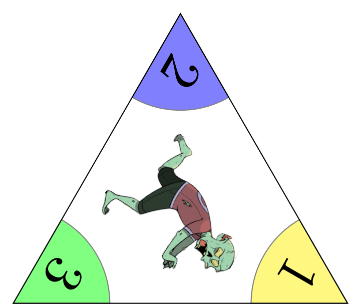

Perm.:
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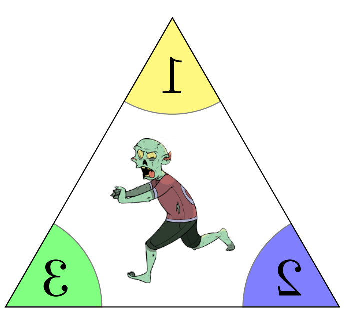
Flip'n Roate:



Perm.:
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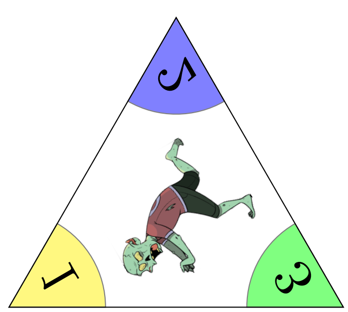
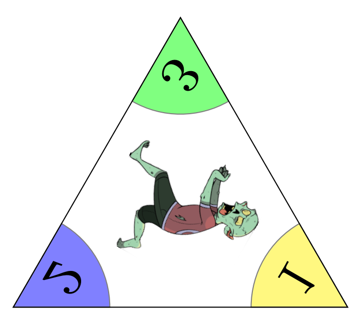
Flip'n Roate:



Perm.:
Flips:
Flip'n Roate:



Perm.:
Flips:
Flip'n Roate:

Reflections:

1. What permutation represents f?

2. What permutation represents F?

3. What permutation represents R?

4. What combination of f and F gives R?

5. What combination of f and R gives F?

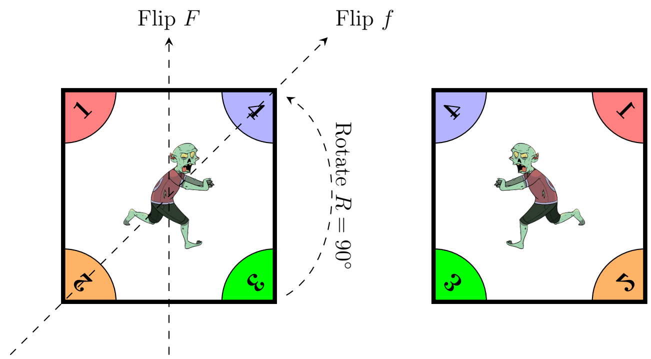
6. With three objects how many ways can we permute them?

7. Based on your previous answer, can you associate all the possible permutations with motions of the triangle?

3 Moving Squares

Flip'n Squares!

In this section we are looking at symmetries of the square. The transformations are a horizontal flip F, diagonal flip f, or a $90°$ rotation R. For each picture list the permutation or combination of permutations of the corners and then list the ways we could carry out the transformations with just flips or with just the diagonal flip and the rotation.



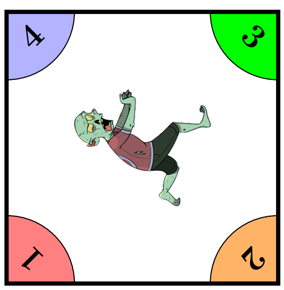
Perm.: (14)(23)

Flips: F

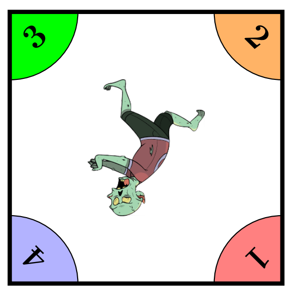
Flip'n Roate: $R \circ f$

Figure 2: Flip'n Square Demo

Perm.:
Flips:
Flip'n Roate:



Perm.:
Flips:
Flip'n Roate:



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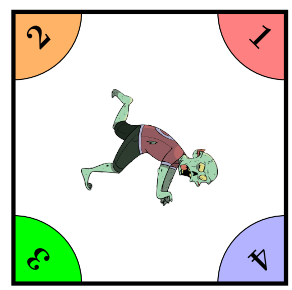

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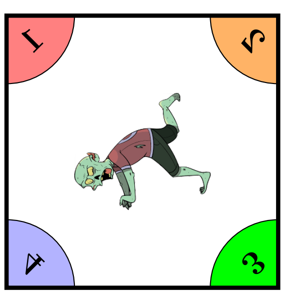

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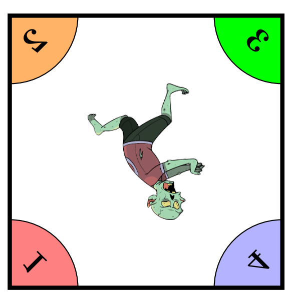

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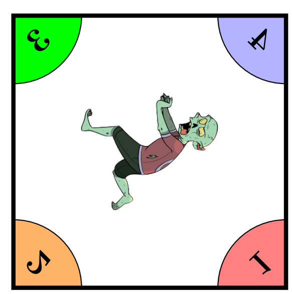


Perm.:
Flips:
Flip'n Roate:

Reflections:

1. What permutation represents f?

2. What permutation represents F?

3. What permutation represents R?

4. What combination of f and F gives R?

5. What combination of f and R gives F?

6. Would it ever be possible to move the square and get the permutation (123)?

7. With four objects how many ways can we permute them?

8. Based on your previous answer, why could you never associate all the possible permutations with motions of the square?

4 More on Permutations

Swapping Chips:

Given four objects in arranged in a row, suppose that we want to shuffle the object in position 1 to position 2, position 2 to position 4, position 4 to position 3, and position 3 to position 1, this could look like figure 3.

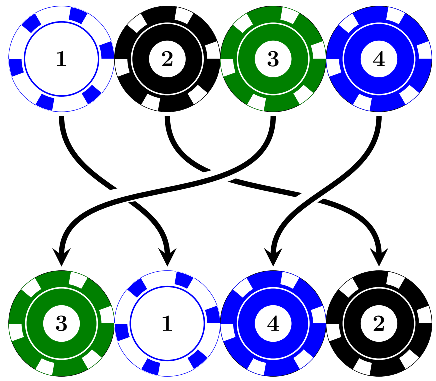


Figure 3: Permutation $\sigma = (1243)$ Take 1

Alternatively, we could swap positions 2 and 3, then positions 1 and 2 and 3 and 4. This gives the same final result but in two (or three) steps instead of one, as we see in figure 4.

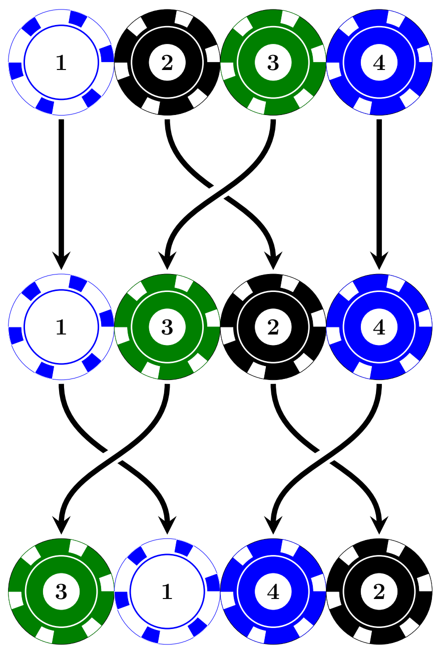
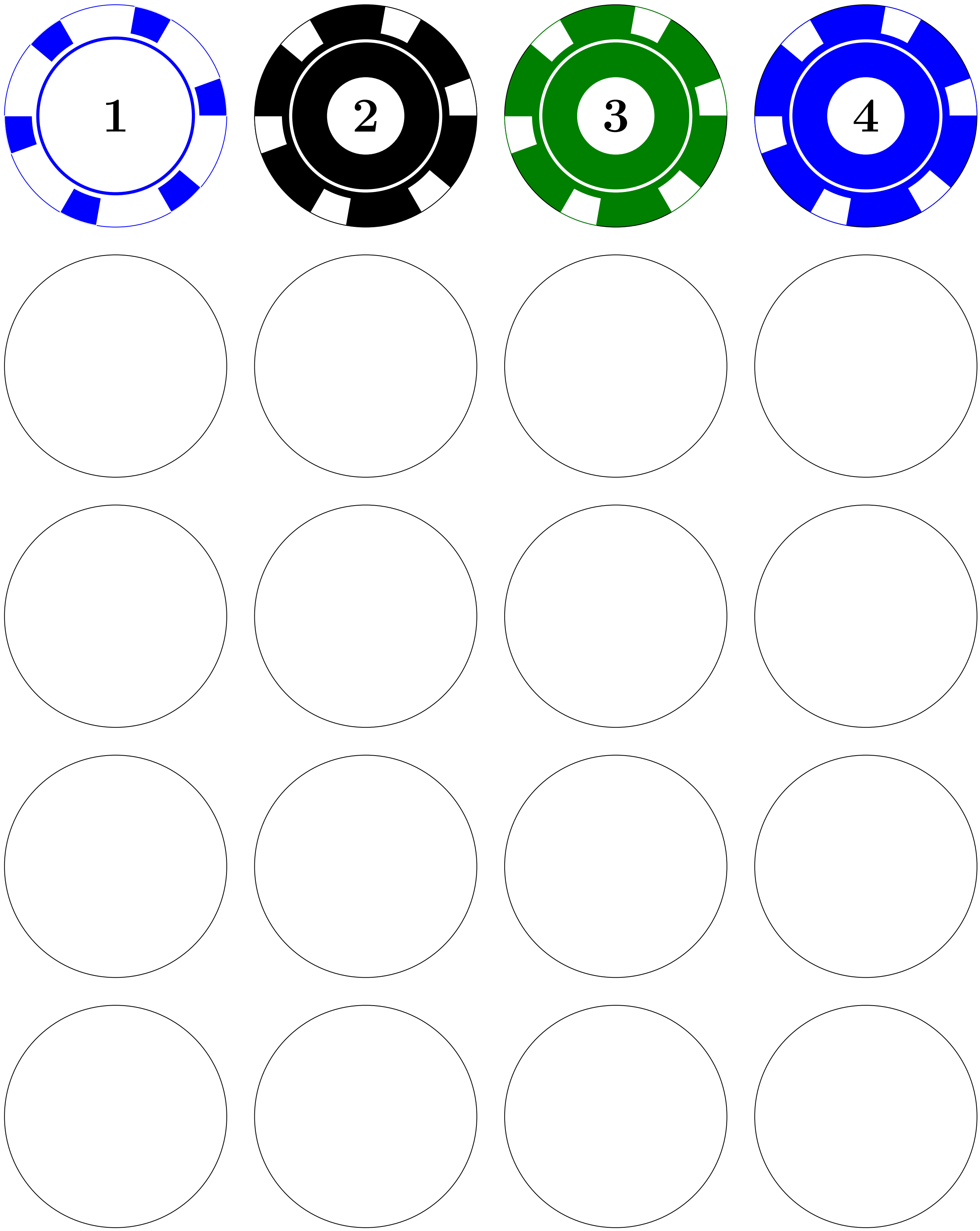
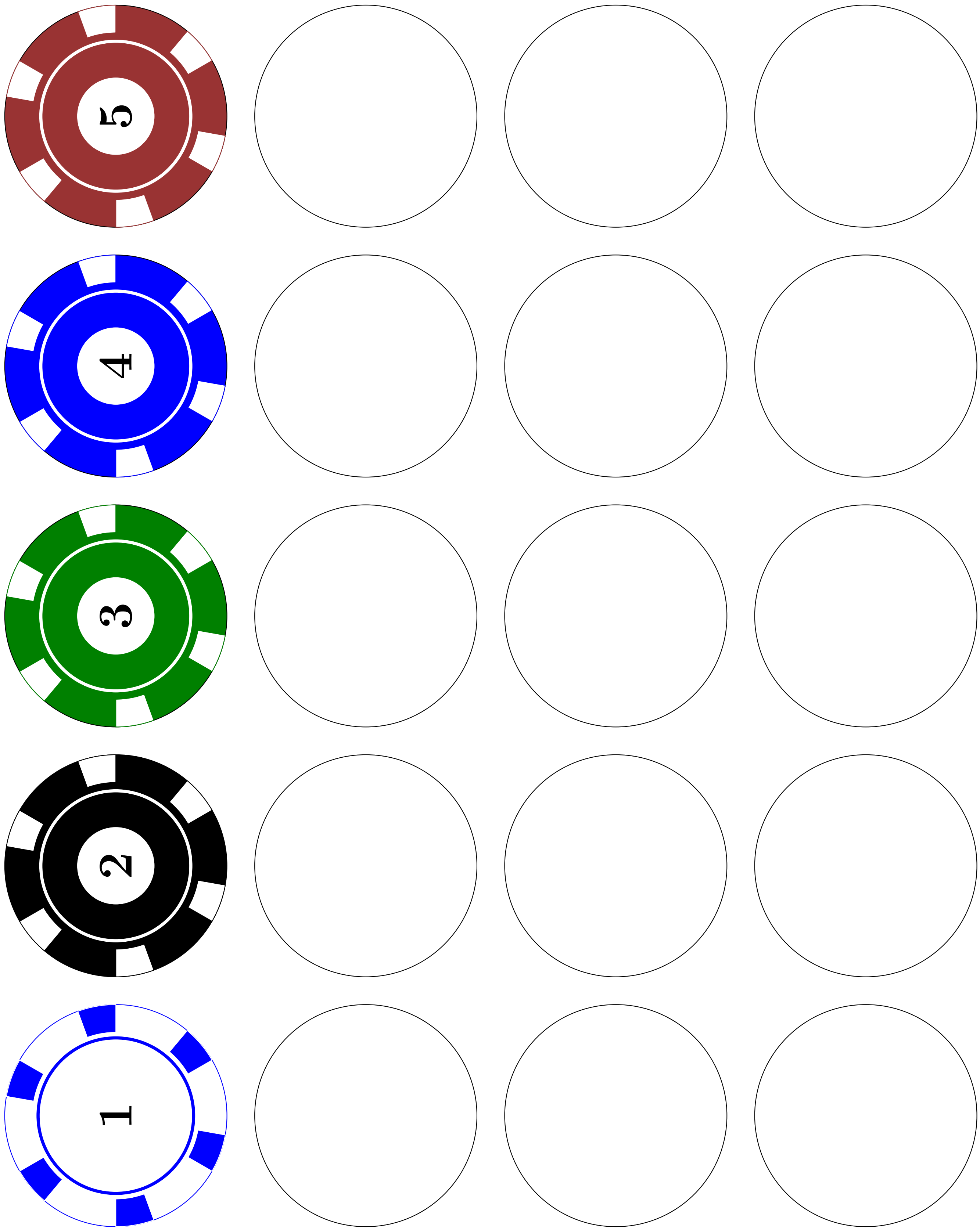


Figure 4: Permutation $\sigma = (1243) = (34)(12)(23)$ Take 2

On the following page, place four chips (white, black, green, and blue) in random order at the bottom of the page. Then, using another set of chips figure out which sequence of permutations will take you from the top to the bottom. What result do you get if you go from top to bottom in "one step" ? What if you only swap two chips at a time? What if you only swap adjacent chips? Finally, repeat this with five chips on the page after that.


1
2
3
4


5
4
3
2
1

Reflections:

1. When you went from top to bottom in "one step," did any of the permutations share numbers or were they disjoint?

2. Were you always able to get from top to bottom by only swapping two chips at a time?

3. Were you always able to get from top to bottom by only swapping adjacent chips?

4. If you carried out the permutation (23), then (34), then 12 on 7, what do you end up with?

5. If you carried out the permutation (34), then (12), then 23 on 7, what do you end up with?

6. How do your answers to the previous two questions compare to each other? What does this tell you about composing permutations?