

Instructions

Below are 17 practice exam problems which you must turn in the class before the exam. These must be written up neatly or typed on separate paper in accordance with the guidelines in your syllabus. Your grade will be based on you completing all the questions and on the quality of your work. In addition there is a list of challenge questions; two of these will be on the exam and will make up 20% of the exam grade. Finally, there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam and have answers in the text.

Please recall that in lieu of a final exam, the fourth exam will include questions from the previous exams. Specifically,

- 60% of the exam will be exercise type problems from chapters 9 and 10,
- 20% of the exam will be challenge problems from chapters 9 and 10, and
- 20% of the exam will be exercise type problems from the previous exams.

Practice Exam Problems:

1. Suppose that in a certain state all license plates consist of four letters followed by three digits (see figure 1 for an example).
 - (a) How do we know there are 456,976,000 different license plates?
 - (b) How do we know there are 1,757,600 plates that begin with A and end with 0?
 - (c) How do we know there are 258,336,000 plates on which all the letters and digits are distinct?
2. Use suppose figure 3a is a map between cities A , B and C and use it to answer the following; assume we never visit a city twice:
 - (a) How do we know there are 14 ways we can travel from *City A* to *City B*?
 - (b) How do we know there are 12 ways we can travel from *City A* to *City B* with a stop in *City C*?
3. Suppose that a student council consists of 15 members, 8 men and 7 women.
 - (a) Why are there 1960 ways to form a six person committee with 3 men and 3 women?
 - (b) Why are there 4977 ways to form a six person committee with at least 1 women?
4. Why are there 151,200 distinguishable ways to arrange the letters of the word

HULLABALOO?

5. If n is a positive integer how many solutions are there to the equation

$$x_1 + x_2 + x_3 = n$$

if we assume $x_i \geq 0 \forall i$? What if we assume $x_i \geq 1 \forall i$? (Note that your answers will be in terms of n .)

6. How many cards must you select from a standard deck of 52 cards in order to guarantee that two of them are the **same suit**? How many must you select to guarantee that you get two with the **same rank**? There is a picture of a deck of cards in figure 2 if you need it.
7. Show that in any set of thirteen integers chosen from 2 through 40 there must be at least two with a common divisor greater than 1.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

8. Use the **Binomial Theorem** to prove that for all integers $n \geq 0$

$$3^n = \binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n}.$$

(Hint: $3=1+2$)

9. Find the coefficient of $x^3y^2z^5$ in the expansion of $(x + y + z)^{10}$.
10. A **bridge** is an edge whose removal disconnects a graph. Are there any bridges in graph 3a? What about graph 3b? Justify your answers.
11. Does the graph 3a have an **Euler Circuit**? Justify your answer.
12. Describe a **Hamilton Circuit** in graph 3a.
13. Write the **adjacency matrix** for graph 3c.
14. Sketch a graph with **adjacency matrix**

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

15. Find the matrix product

$$\begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

16. Which graph(s) in figure 3 are **trees** and which are **binary trees**? Explain your conclusion.
17. Can you draw a **connected graph** with 9 edges and 9 vertices? What about a **tree** with 9 edges and 9 vertices? If so, sketch them. If not, explain why?

Challenge Questions:

(listed by section and problem number) You don't need to write these up. You will need to do two of these on the exam and they will count for 20% of the exam grade. There will be one from chapter 9 and one from chapter 10.

§ 9.4: 37

§ 9.6: 20

§ 10.2: 22

§ 9.5: 23

§ 10.1: 43

§ 10.4: 27

Additional Practice Problems:

(listed by section and problem number)

§ 9.2: 1, 4, 6, 9, 11ab, 14abd, 16, 19, 21, 34;

§ 9.3: 3, 6, 11, 14;

§ 9.4: 1, 3, 9, 10, 14, 20, 24;

§ 9.5: 3, 6, 8, 15, 19;

§ 9.6: 3, 5, 10, 11;

§ 9.7: 1, 3, 5, 10, 19, 23, 25, 29, 31, 37;

§ 10.1: 1,4,6a,8a,9a,12,14,19,29;

§ 10.2: 2a,3a,4ac,5a,9ad,20;

§ 10.4: 7a,8,9,10,11,12,13

Reference Figures

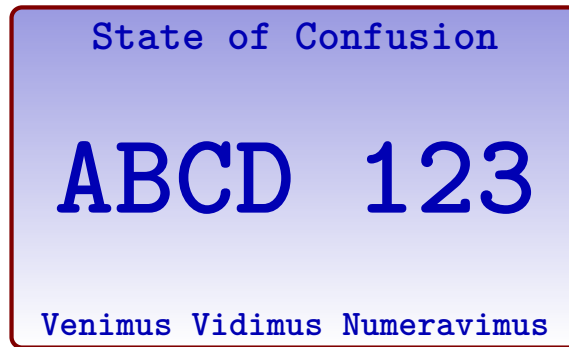


Figure 1: License Plate

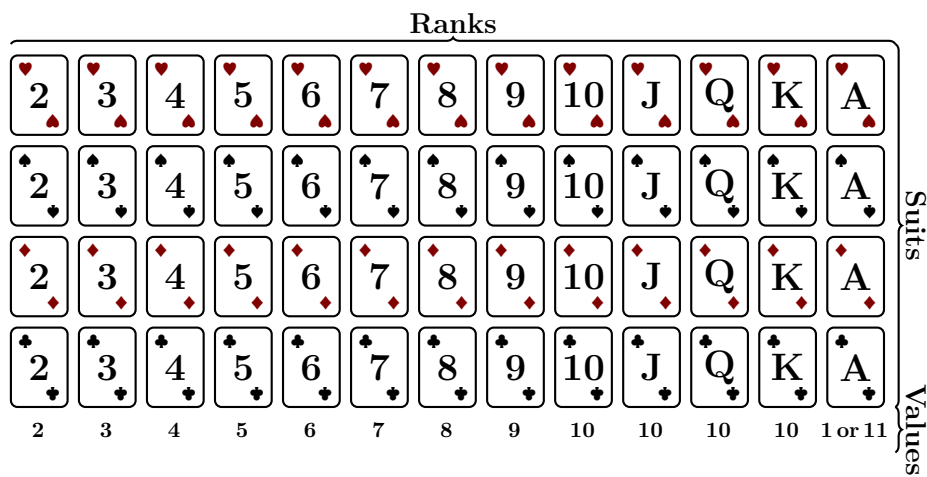


Figure 2: Deck of Cards

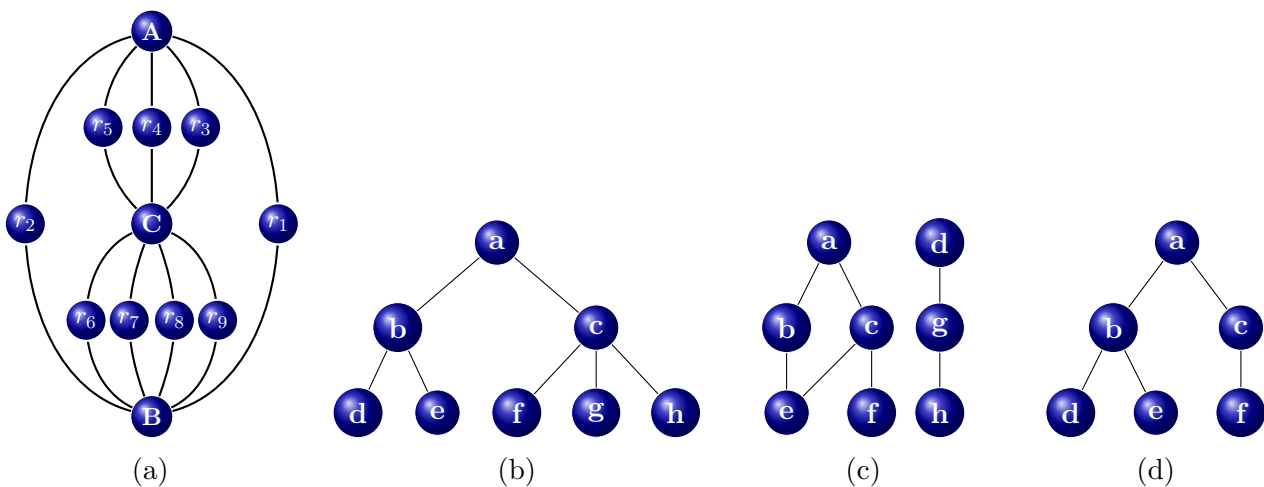


Figure 3: Reference Graphs