Directions:

Definition (Truth Set)**.** For any predicate $P(x)$ defined on a set $\mathcal{U}$ the ***truth set*** of $P(x)$, call it T_P, is a the set of all elements x in $\mathcal{U}$ for which $P(x)$ is true.

Figure 1 shows a Venn Diagram with truth set $T_P = \{b, c\}$ for a predicate $P(x)$ and truth set $T_Q = \{c, d\}$ for a predicate $Q(x)$. Both truth sets are subsets of the universal set $\mathcal{U} = \{a, b, c, d\}$.

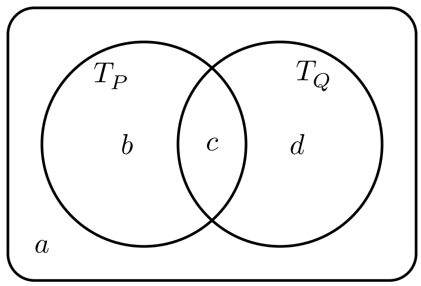

Figure 1: Venn Diagram of T_P and T_Q

For each of the statements below you need to answer these questions:

- Is the statement true for the diagram in figure 1?
- If it is not true, then which element or elements need to be removed to make the statement non-vacuously true?
- If a statement is true, which element or elements could you remove and still have it be non-vacuously true?
- Which pairs of statements are negations of one another?

To help with this you should first find the truth set for $\sim P(x)$, $T_{\sim P}$, and the truth set for $\sim Q(x)$, $T_{\sim Q}$.

Example:

For example $\forall x : P(x) \wedge Q(x)$ is not true unless we remove a, b, and d from figure 1. Its negation is $\exists x :\sim P(x) \vee \sim Q(x)$ which is true for this diagram and would still be true as long as we kept at least one of a, b, or d.

Statements:

1. $\forall x : P(x) \rightarrow Q(x)$
2. $\exists x : P(x) \rightarrow Q(x)$
3. $\forall x : P(x) \vee Q(x)$
4. $\exists x : P(x) \vee Q(x)$
5. $\exists x : P(x) \wedge \sim Q(x)$
6. $\exists x :\sim P(x) \wedge \sim Q(x)$
7. $\forall x : P(x) \wedge \sim Q(x)$
8. $\forall x :\sim P(x) \vee \sim Q(x)$
9. $\forall x :\sim P(x) \wedge \sim Q(x)$
10. $\exists x : P(x) \wedge Q(x)$