

Instructions

Below are 11 practice exam problems which you must turn in when you come in to take the exam; these must be written up neatly or typed on separate paper and in accordance with the guidelines in your syllabus. Your grade will be based on you completing all the questions and on the quality of your work. In addition there is a list of challenge questions; two of these will be on the exam and will make up 20% of the exam grade. Finally, there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam.

Practice Exam Problems:

1. Write the first four terms of the sequence defined below.

$$b_j = \frac{5-j}{5+j}$$

2. Write an explicit formula for the sequence below.

$$\left(1 - \frac{1}{2}\right), \left(\frac{1}{2} - \frac{1}{3}\right), \left(\frac{1}{3} - \frac{1}{4}\right), \left(\frac{1}{4} - \frac{1}{5}\right), \left(\frac{1}{5} - \frac{1}{6}\right), \dots$$

3. Show that the value of the finite sum is 121.

$$\sum_{k=0}^{10} 2k + 1$$

4. Show that the value of the finite product is $1/1024$.

$$\prod_{i=1}^4 \left(\frac{1}{2}\right)^i$$

5. Write the following using summation notation:

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2.$$

6. Use

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

to show that the sum $S = 3 + 4 + 5 + 6 + \dots + 1000$ is equal to 500497.

7. Rewrite the summation by separating off the final term.

$$\sum_{i=1}^{n+1} \frac{1}{i^2 + 1}$$

8. Make a change of variable (i.e. re-index) in the summation using $j = i - 1$:

$$\sum_{i=1}^{n+1} \frac{(i-1)^2}{i \cdot n}.$$

9. Write out the next five terms of the recursive sequence defined by

$$s_0 = 1, \quad s_1 = 1, \quad s_n = \frac{1}{s_{n-1} + s_{n-2}}.$$

10. Use iteration to show that the explicit formula for the recursive relation $h_k = 2^k - h_{k-1}$, where $h_0 = 1$, is

$$h_k = (-1)^k \left(\frac{1 - (-2)^{k+1}}{3} \right).$$

11. Verify that $F_k = 3F_{k-3} + 2F_{k-4}$ for all integers $n \geq 4$, where $F_n = F_{n-1} + F_{n-2}$ is the Fibonacci Sequence.

Extra: Assume the sequence satisfies the given recurrence relation and initial conditions and find an explicit formula for the sequence.

$$s_0 = 0, \quad s_1 = 1, \quad s_k = -4s_{k-1} - 4s_{k-2}, \quad \text{for } k > 1$$

Extra: Let b_0, b_1, b_2, \dots be a sequence defined by the explicit formula

$$b_n = C \cdot 3^n + D \cdot (-2)^n \quad \text{for all } n \geq 0,$$

where C and D are real numbers. Show that for any choice of C and D ,

$$b_k = b_{k-1} + 6b_{k-2} \quad \text{for } k \geq 2.$$

Challenge Problems:

(listed by section and problem number) You don't need to write these up. Two of these will be on the exam and will count for 20% of the exam grade.

§ 5.1: 80,87

§ 5.6: 17,23

§ 5.7: 22, 52

Additional Practice Problems:

(listed by section and problem number)

§ 5.1: 1,3,5,11,12,14,20,23,37,46,51,55

§ 5.7: 1ab,5,10,19,24

§ 5.6: 1,3,5,11,17ac,26

Extra - 5.8: 1,5,8,11,13,22