Introduction to Finite Automata

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2. New From Old
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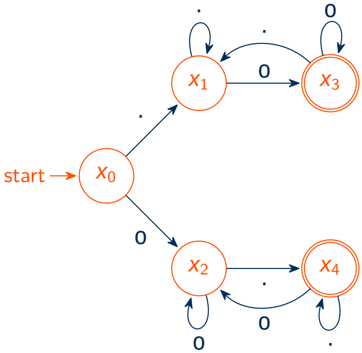
State Diagram and Finite Automaton



- States: $w = fx_0;x_1;x_2;x_3;x_4g$

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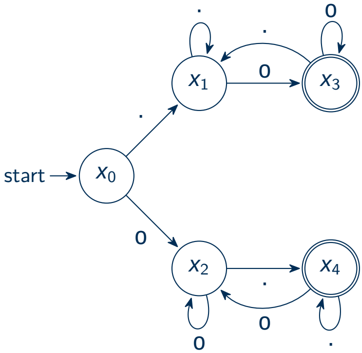

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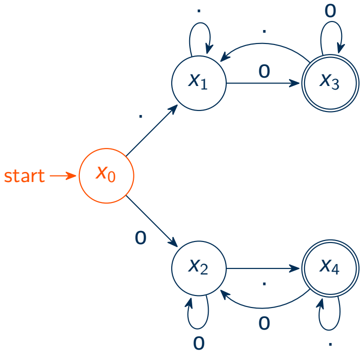

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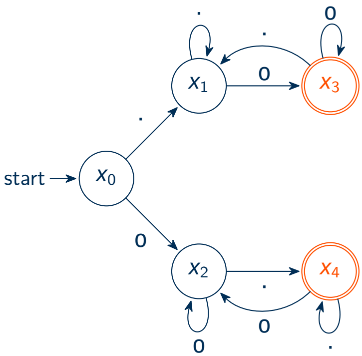

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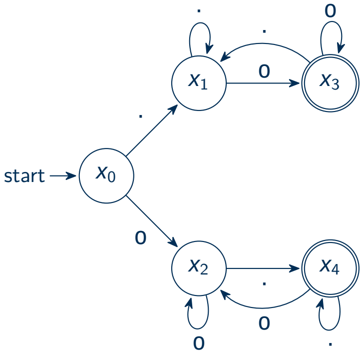

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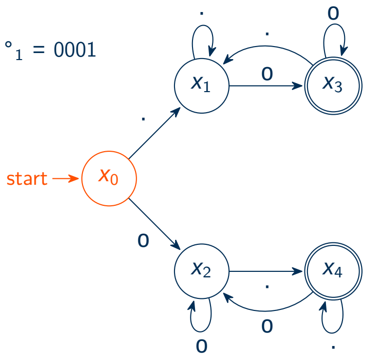

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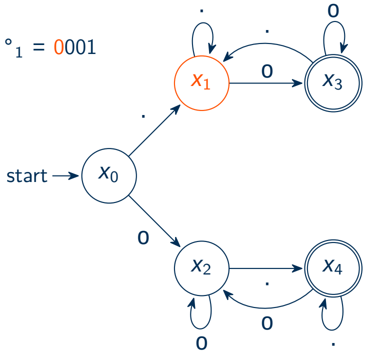

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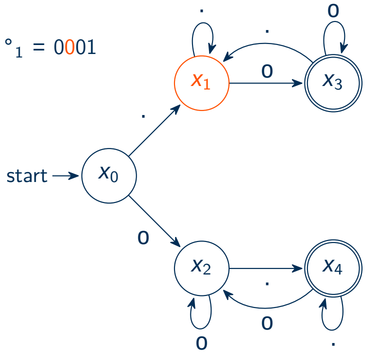

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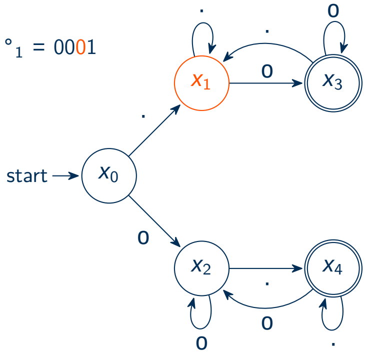

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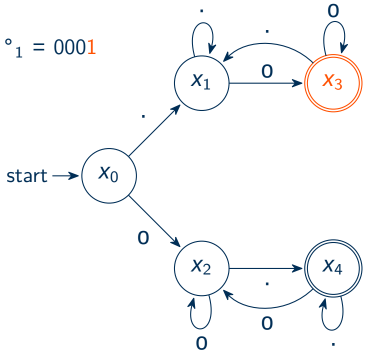

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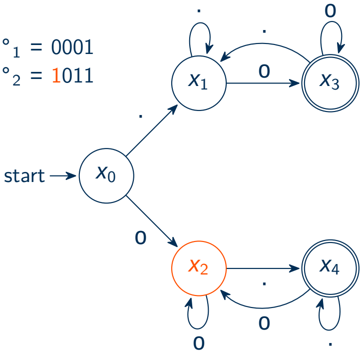
State Diagram and Finite Automaton

°$_1$ = 0001
°$_2$ = 1011



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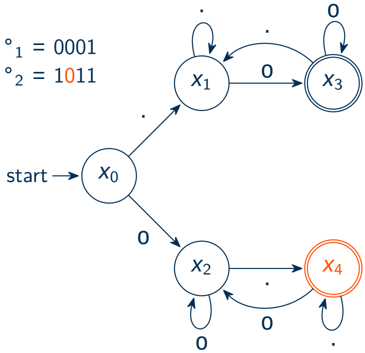

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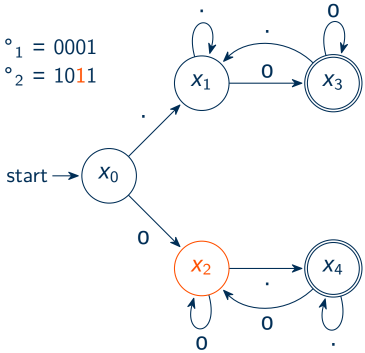

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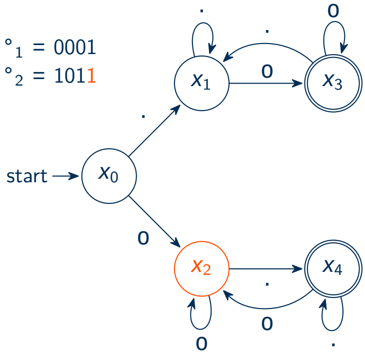

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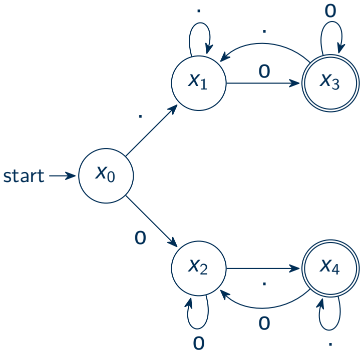
Formal Definition

Definition (Finite Automaton)

A *»j_§0 9«§nh9§nj* is a 5-tuple $(w; \; ; \; ; x_0; S)$, where

1. w is a finite set of *§9§0*,
2. is a finite *9eq\9@0§*,
3. $: w \quad ! \; w$ is the *§}9j _§_nj T«jH§_nj*,
4. $x_n \; 2 \; w$ is the *§9}§ n} _j_§_9e §9§0*, and
5. $S \quad w$ is the set of *9HH0q§ n} »j9e §9§0*.

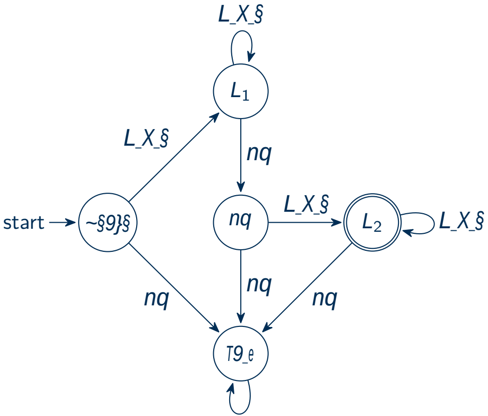
Transition Function



: w ! w

	0	1
x_0	x_1	x_2
x_1	x_1	x_3
x_2	x_4	x_2
x_3	x_1	x_3
x_4	x_4	x_2

Another Example



- w = f~§9}§; L_1; L_2; nq; T9_eg
- = nq [L_X_§
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- S = $f L_2 g$
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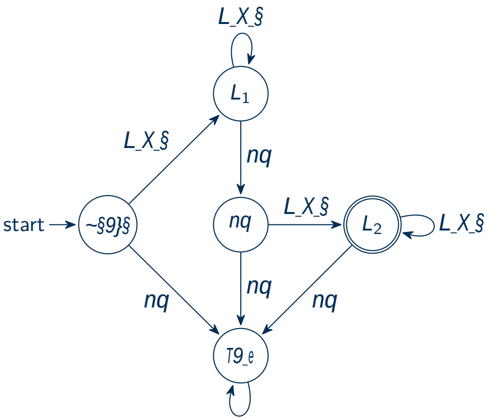

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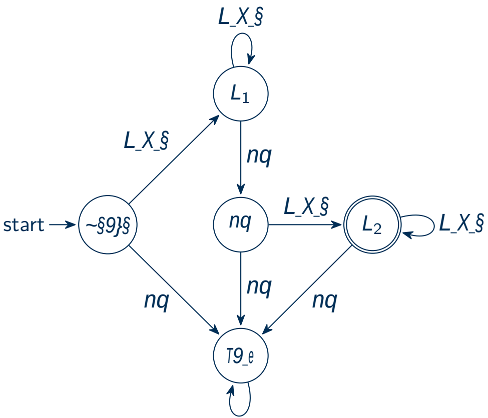

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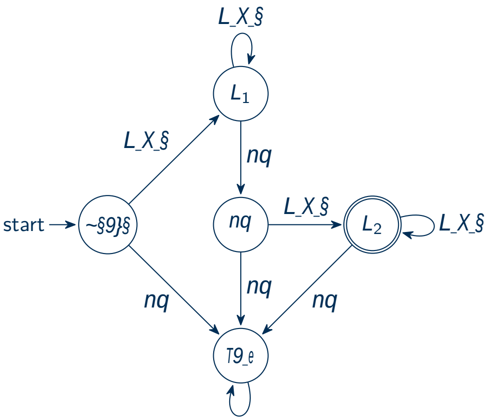

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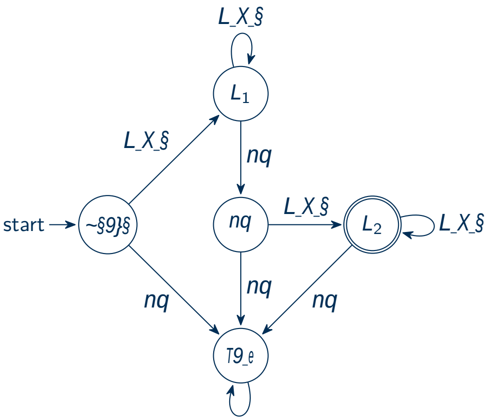

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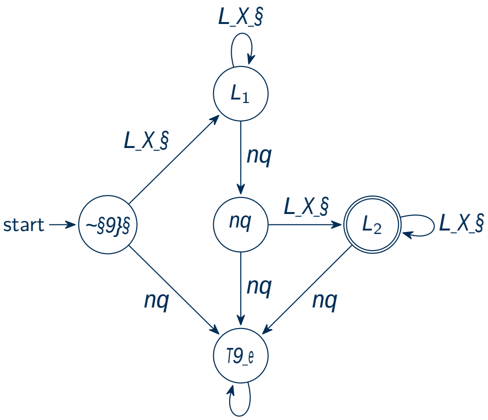

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The language *d* is called a *}0X«ɐ9} ɐ9jX«9X0* because is is recognized by a finite automaton.

Satisfying a Description

Problem

Gnj §}«H§ 9 »j_§0 9«§nh9§nj §\9§ 9HH0q§ nje´ °n}L _j = *f* · ; *og* 0jL_jX _j o·ou

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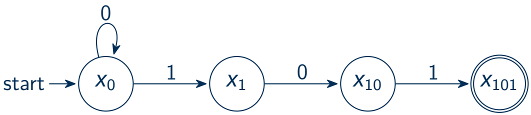

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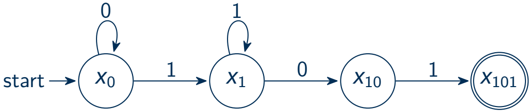

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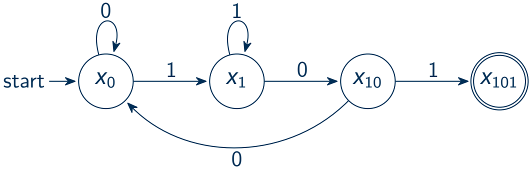

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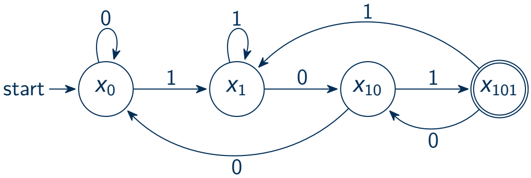

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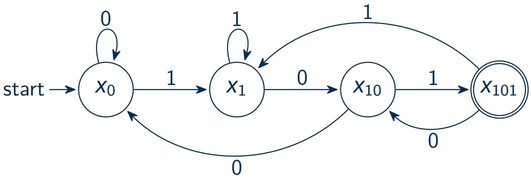

Check what happens to 101 independent of where we start.

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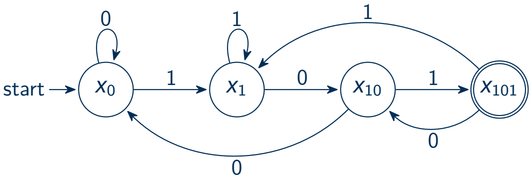

Check what happens to 010 independent of where we start.

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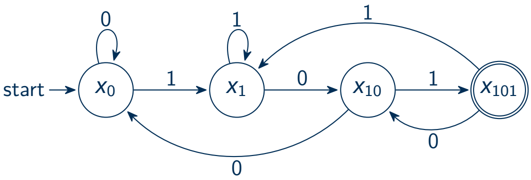

Note that this means we don't need to "remember" the whole string to check its ending.

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Creating New Automata

Definition

Let 8 and ? be regular languages. We define the regular operations «j_nj, HnjH9§0j9§_nj, and §9} as follows:

- ª j_nj: 8 [? = f²j² 2 8 n} ² 2 ?g
- GnjH9§0j9§_nj: 8 ? = f²´j² 2 8 9jL ´ 2 ?g
- ~§9}: 8 = f$²_1²_2$ $²_c$jc 0 9jL 8_ ²_ 2 8g

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- *8 = f0; 1g*

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- 8 = f"; 0; 1; 00; 01; ; 10; 11; 000; : : :g

Details on Union

Given to finite automata

$$g_1 = (w_1; \ _1; \ _1; x_1; S_1)\, 9jL \ g_2 = (w_2; \ _2; \ _2; x_2; S_2)$$

their union is constructed as follows:

- $w = w_1 \quad w_2$

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- $x_0 = (x_1; x_2)$

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- $x_0 = (x_1;x_2)$
- $S = f(}_1;}_2)j}_1\ 2\ S_1\ n}\ }_2\ 2\ S_2g$

Details on Union

Given to finite automata

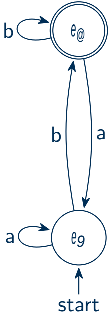
$$g_1 = (w_1; \ _1; \ _1; x_1; S_1)\ 9jL\ g_2 = (w_2; \ _2; \ _2; x_2; S_2)$$

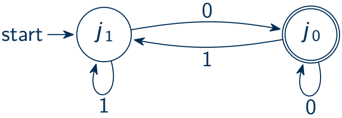
their union is constructed as follows:

- $w = w_1 \quad w_2$
- $\quad = \ _1 [\ _2$
- $8(\}_1; \}_2)\ 2\ w\ 89\ 2 \quad : \ ((\}_1; \}_2); 9) = (\ _1(\}_1; 9); \ _2(\}_2; 9))$
- $x_0 = (x_1; x_2)$
- $S = f(\}_1; \}_2)j\}_1\ 2\ S_1\ n\}\ \}_2\ 2\ S_2 g$

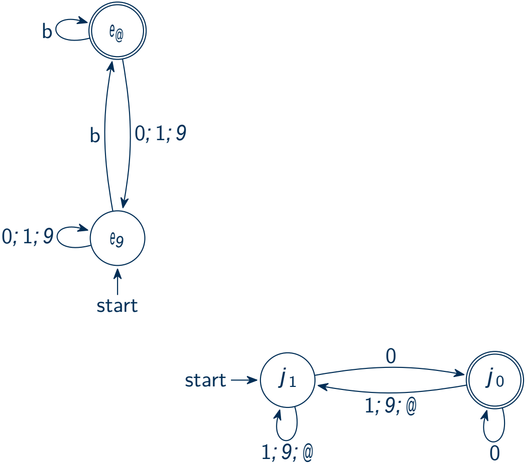
r i n§0 $\}_1$ 2 S_1 9jL $\}_2$ 2 S_2 °n«eL @0 _j§0} 0H§_njs

Union Example

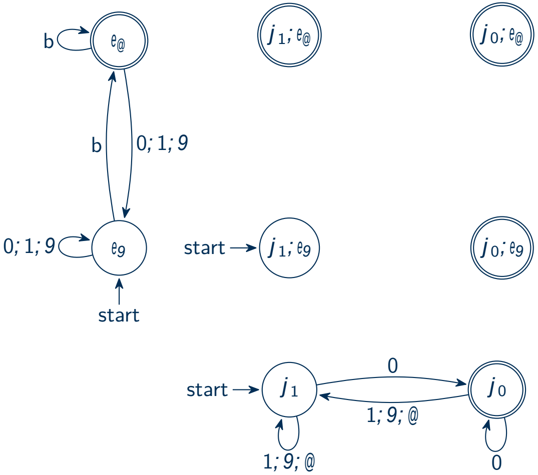




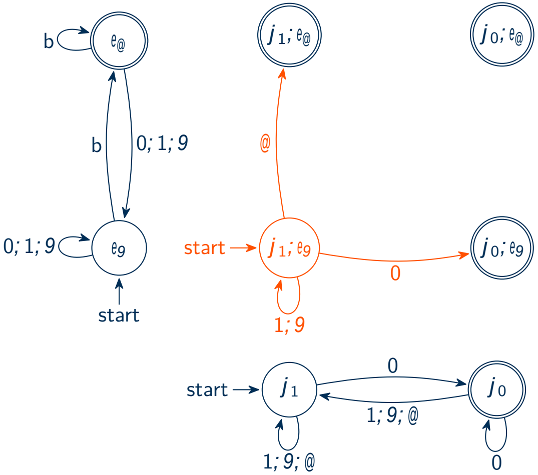
Union Example



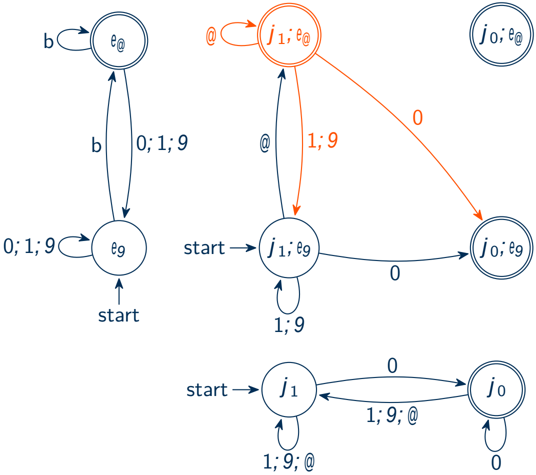
Union Example



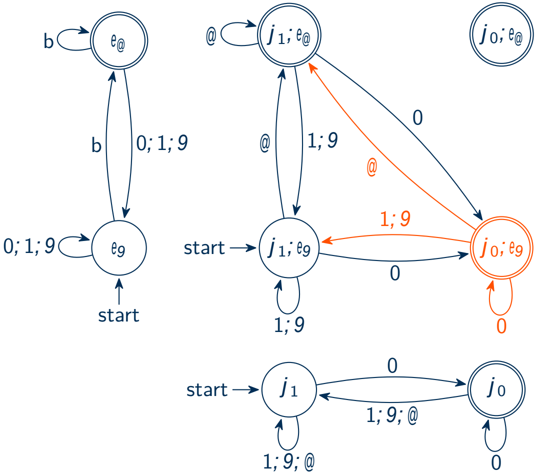
Union Example



Union Example



Union Example



Union Example

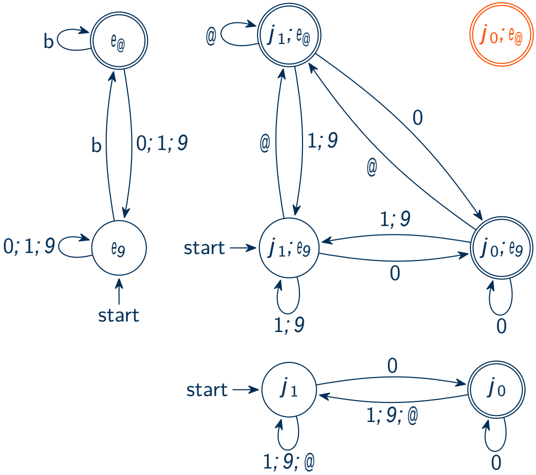
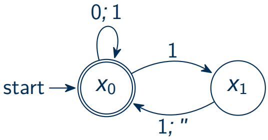


Table of Contents

1. Finite Automata
2. New From Old
3. Non-Deterministic vs. Deterministic
4. New From Old (again)
5. Next Class

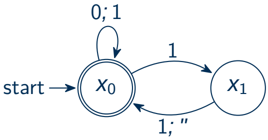
Non-Deterministic Automata



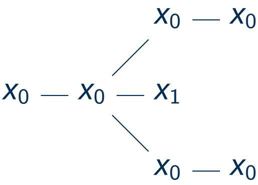
	0	1	"
x_0	fx_0g	$fx_0; x_1g$	;
x_1	;	fx_0g	fx_0g

° = 010

Non-Deterministic Automata

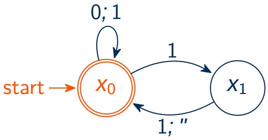


	0	1	"
x_0	fx_0g	$fx_0;x_1g$	;
x_1	;	fx_0g	fx_0g

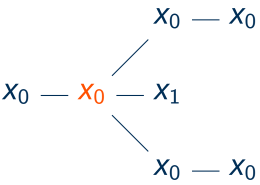


° = 010

Non-Deterministic Automata

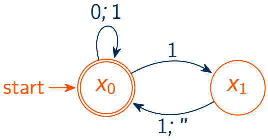


	0	1	″
x_0	$f x_0 g$	$f x_0 ; x_1 g$	;
x_1	;	$f x_0 g$	$f x_0 g$

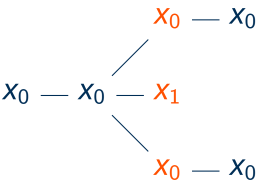


° = 010

Non-Deterministic Automata

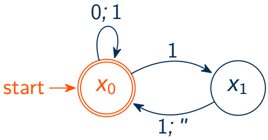


	0	1	"
x_0	$f x_0 g$	$f x_0; x_1 g$	;
x_1	;	$f x_0 g$	$f x_0 g$

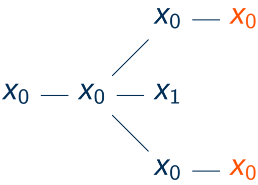


$$\circ = 010$$

Non-Deterministic Automata

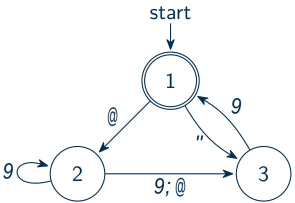


	0	1	″
x_0	$f x_0 g$	$f x_0 ; x_1 g$	;
x_1	;	$f x_0 g$	$f x_0 g$



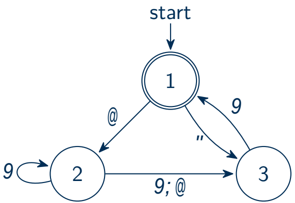
° = 010

Equivalence to Deterministic (Slide 1)



	9	@	"
1	;	{2}	{3}
2	{2,3}	{3}	;
3	{1}	;	;

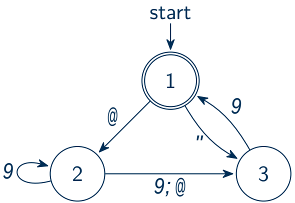
Equivalence to Deterministic (Slide 1)



	9	@	"
1	;	{2}	{3}
2	{2,3}	{3}	;
3	{1}	;	;

- w = f1; 2; 3g
- " = f9; @; "g
- ~§9}§ = f1g
- S = f1g

Equivalence to Deterministic (Slide 1)

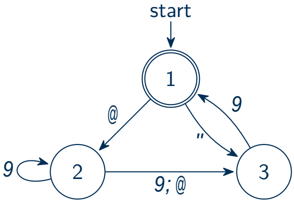


	9	@	"
1	;	{2}	{3}
2	{2,3}	{3}	;
3	{1}	;	;

- w^0 = $P(w)$ (Power set of w)

- w = f1; 2; 3g
- " = f9; @; "g
- ~§9}§ = f1g
- S = f1g

Equivalence to Deterministic (Slide 1)

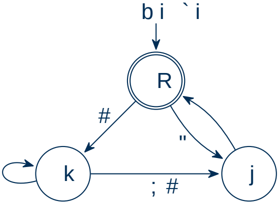


	9	@	''
1	;	{2}	{3}
2	{2,3}	{3}	;
3	{1}	;	;

- w^0 = P (w) (Power set of w)
- ^{0} = *f9; @g*

- *w* = *f*1; 2; 3*g*
- '' = *f9; @; ''g*
- *~§9}§* = *f*1*g*
- *S* = *f*1*g*

1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua



		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;

Z = f R; k; jg

" = f ; #; "g

ai `i= f Rg

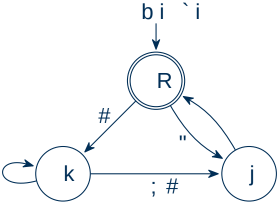
6 = f Rg

Z0 = P (Z) USQr2` bZ2Vi Q7

0 = f ; #g

1(f Rg) = f R; jg

1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua



	;	#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;

Z = f R, k, jg

" = f ; #, "g

ai `= f Rg

6 = f Rg

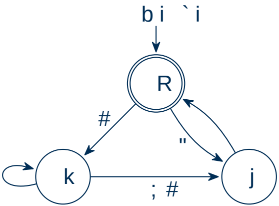
Z0 = P (Z) USQr2` bZ2V Q7

0 = f ; #g

1(f Rg) = f R, jg

ai `= f R, jg

1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua



		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;

Z = f R, k, jg

" = f ; #, "g

ai `= f Rg

6 = f Rg

Z⁰ = P(Z) USQr2` bZ2Vi Q7

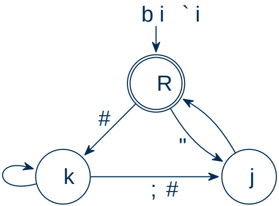
⁰ = f ; #g

1(f Rg) = f R, jg

ai `= f R, jg

6⁰ = ff Rg; f R, kg; f R, jg; f R, k, jgg

1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua



Z = f R; k; jg

" = f ; #; "g

ai `i= f Rg

6 = f Rg

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;

Z0 = P (Z) USQr2` bZ2i Q 7 Z

0 = f ; #g

1(f Rg) = f R; jg

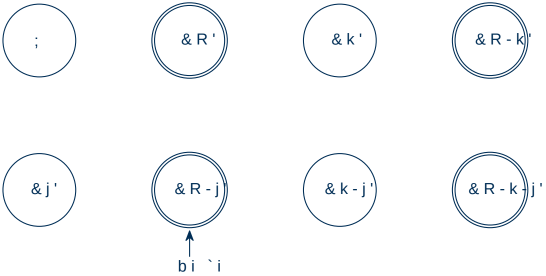
ai `i= f R; jg

60 = ff Rg; f R; kg; f R; jg; f R; k; jgg

0 =?

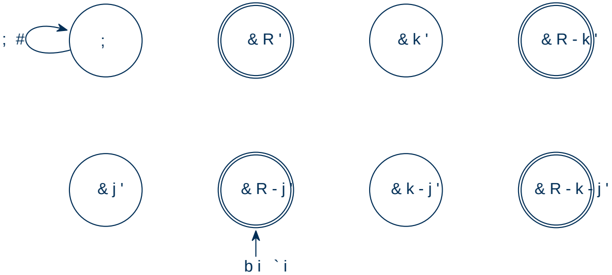
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



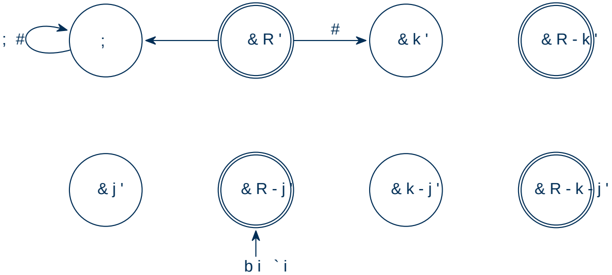
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



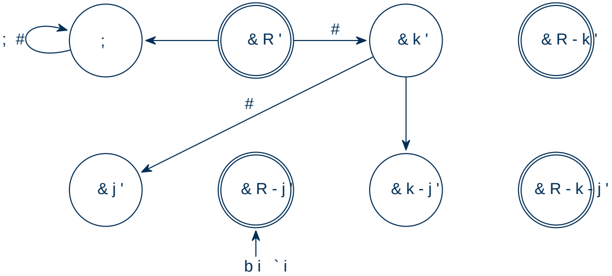
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



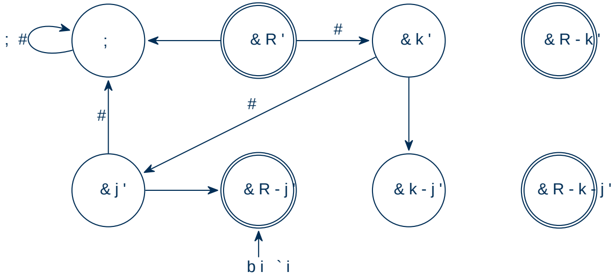
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ual

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



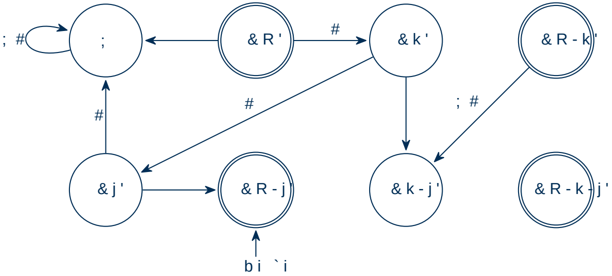
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



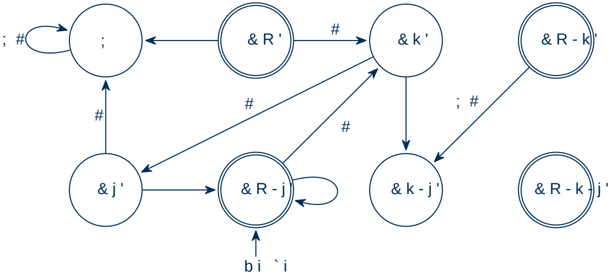
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



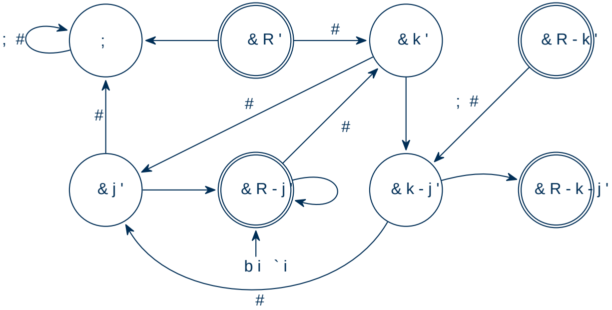
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



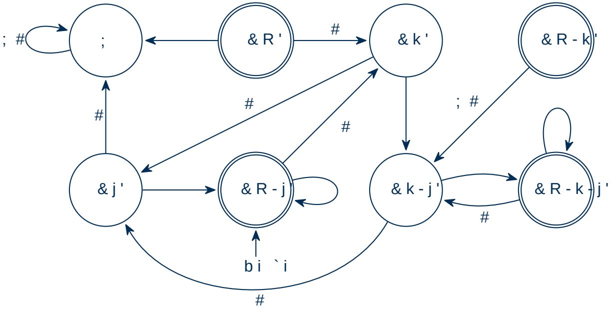
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



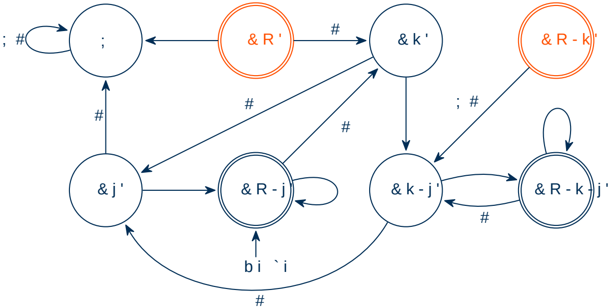
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

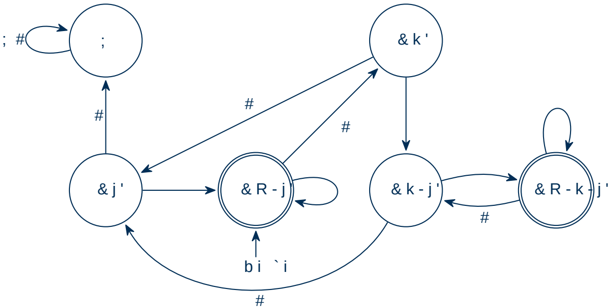
		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;



q2 + M 2HBKBM i2 bi i2b i? i `2 QMHv

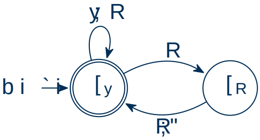
1[mBp H2M+2 iQ .2i2`KBMBbiB+ Ua

		#	"
R	;	&k'	&j'
k	&k-j'	&j'	;
j	&R'	;	;

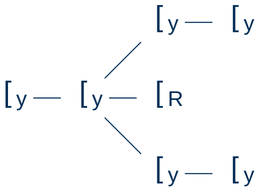


q2 + M 2HBKBM i2 bi i2b i? i `2 QMHv

LQM@.2i2`KBMMBbiB+ miQK i



	y	R	"
[y	f [yg	f [y; [Rg	;
[R	;	f [yg	f [yg



r = yRy

h #H2 Q7 *QMi2Mib

R 6BMBi2 miQK i

K L2r 6`QK PH/

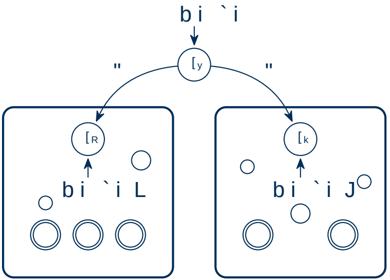
j LQM@.2i2`KBMBbiB+ pbX .2i2`KBMBbiB+

9 L2r 6`QK PH/ U ; BMV

8 L2ti *H bb

lMBQM _2pB[b'Bi2/,

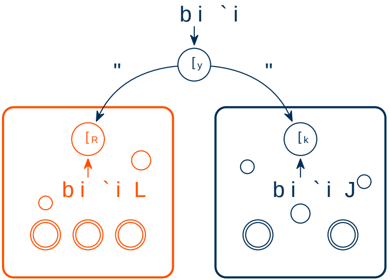
bbmK2 i? i i?2 `2;mH `BH `M2;Tm̀2;b2 2Mi2/ #v i?2 }M
L M/ i?2 `2;mH ` H BMb;m2 T;`22b2Mi2/ #v i?2 }MJ Bi2
i?2M i?2 MB+QM #2 `2T`2b2Mi2/ b #2HQrX



([;) = R([;) [2 ZR
k([;) [2 Zk
f [R; [kg [= [y^ = "
; [= [y^ 6= "

lMBQM _2pB b'Bi2/,

bbmK2 i? i i?2 `2;mH` BH `M2;`Tm2;b2 2Mi2/ #v i?2 }M
L M/ i?2 `2;mH ` H BM;`2 T`2;2b2Mi2/ #v i?2 }MB i2
i?2M i?2 MBB+QM #2 `2T`2b2Mi2/ b #2HQrX

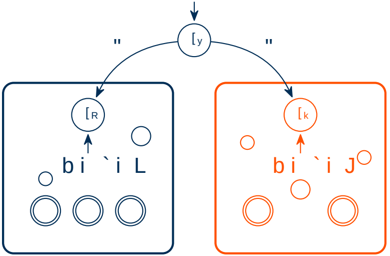


8
([;) = R([;) [2 ZR
k([;) [2 Zk
f [R; [k g [= [y^ = "
; [= [y^ 6= "

lMBQM _2pB[bB'Bi2/,

bbmK2 i? i i?2 `2;mH`B bH `M2;Tm`2b2 2Mi2/ #v i?2 }M
L M/ i?2 `2;mH` H`BM;mb;2 T`2 2b2Mi2/ #v i?2 }MB i2
i?2M i?2 W `B+QM #2 `2T`2b2Mi2/ b #2HQrX

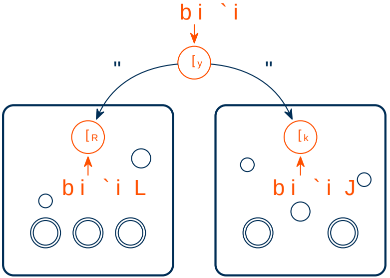
bi `i



([;) = 8 R([;) [2 ZR
k([;) [2 Zk
f [R [k g [= [y^ = "
; [= [y^ 6= "

lMBQM _2pBbBi2/,

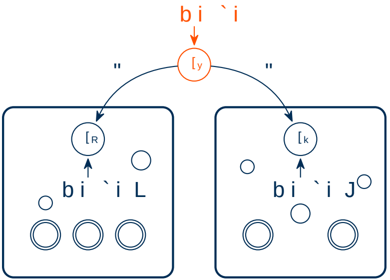
bbmK2 i? i i?2 `2;mH` BH `M2;m 2b2 Mi2/ #v i?2 }M
L M/ i?2 `2;mH` H BMb;m2T`2 2b2Mi2/ #v i?2 }MBi2
i?2M i?2 mMBQM #2 `2T`2b2Mi2/ b #2HQrX



([;) = (R([;) [2 ZR
k([;) [2 Zk
f [R; [k g [= [y^ = "
; [= [y^ 6 "

lMBQM _2pBb'Bi2/,

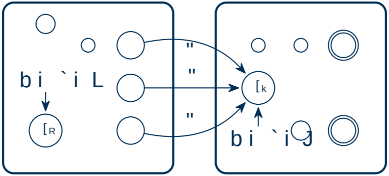
bbmK2 i? i i?2 `2;mH `BH `M2;m 2;b2Mi2/ #v i?2 }M
L M/ i?2 `2;mH ` H BMb;m2 T`2 2b2Mi2/ #v i?2 }M BMi2
i?2M i?2 MBB+QM #2 `2T`2b2Mi2/ b #2HQr X



([;) = 8 R([;) [2 ZR
k([;) [2 Zk
f [R [k g [= [y^ = "
; [= [y^ 6 "

*QM+ i2M iB'QM,

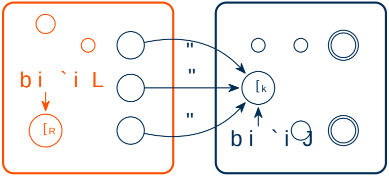
bbmK2 i? i i?2 `2;mH `BHQ`M2;m`2;b2 Mi2/ #v i?2 }M
L M/ i?2 `2;mH ` ?HBMb;m2 T`22b2Mi2/ #v i?2 }MBi2
i?2M i?2 *QM+ i2M+iBQM#M2 `2T`2b2Mi2/ b #2HQrX



8
([;) = R([;) [2 ZR^ [626R
R([;) [2 6R^ 6 "
R([;) [f [kg [2 6R^ = "
k([;) [2 Zk

*QM+ i2M iB'QM,

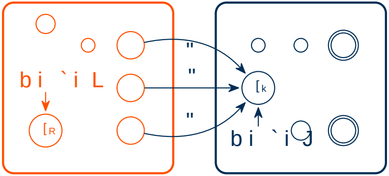
bbmK2 i? i i?2 `2;mH `BH `M2;Tm̀2;b2 2Mi2/ #v i?2 }M
L M/ i?2 `2;mH ` H BMb;m̀2 T;`2 2 b2Mi2/ #v i?2 }M B i2
i?2M i?2QBM+ i2M+iBMQ#M2 `2T`2b2Mi2/ b #2HQrX



([;) = 8 ... R([;) [2 ZR^ [626R
R([;) [2 6R^ 6 "
R([;) [f [kg [2 6R^ = "
k([;) [2 Zk

*QM+ i2M iB'QM,

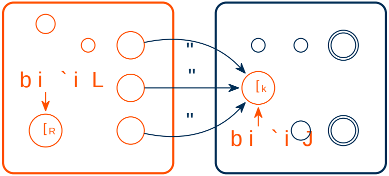
bbmK2 i? i i?2 `2;mH `BH `M2;Tm`2;b2 2Mi2/ #v i?2 }M
L M/ i?2 `2;mH ` H BMb;`m2 T`22 b2Mi2/ #v i?2 }M2 Bi2
i?2M i?2 QB`M+ i2M+iBMQ#M2 `2T`2b2Mi2/ b #2HQrX



8
([;) = R([;) [2 ZR^ [626R
R([;) [2 6R^ 6 "
R([;) [f [kg [2 6R^ = "
k([;) [2 Zk

*QM+ i2M iB'QM,

bbmK2 i? i i?2 `2;mH `B H `M2;Tm`2;b2 2Mi2/ #v i?2 }M
L M/ i?2 `2;mH ` H BMb;`m2 T;`2 2b2Mi2/ #v i?2 }MBi2
i?2M i?2 *QM+ i2M+iBMQ#M2 `2T`2b2Mi2/ b #2HQrX



([;) = 8 > < > :

R([;) [2 ZR^ [626R

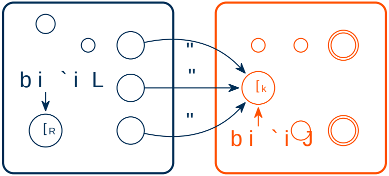
R([;) [2 6R^ 6 "

R([;) [f [kg [2 6R^ = "

k([;) [2 Zk

*QM+ i2M iB'QM,

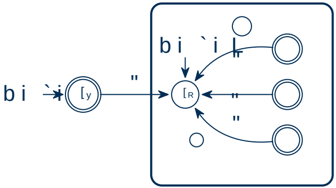
bbmK2 i? i i?2 `2;mH `Biv `2;mH `Biv b2Mi2/ #v i?2 }M
L M/ i?2 `2;mH ` H BMb;m2 T`2b2Mi2/ #v i?2 }MBi2
i?2M i?2 +QM+ i2M+iBQM `2T`2b2Mi2/ b #2HQrX



([;) = 8 R([;) [2 ZR^ [626R
R([;) [2 6R^ 6 "
R([;) [f [kg [2 6R^ = "
k([;) [2 Zk

ai `,

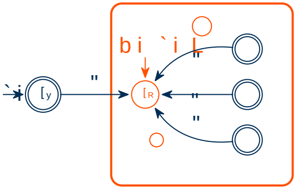
bbmK2 i? i i?2 `2;mH `BH `M2;Tm`2;b2 Mi2/ #v i?2 }M L i?2M i?2 mbM`-vQT2` iQ` TTHB2M/ #Q `2T`2b2Mi #2HQrX



([;) = 8 R([;) [2 Z R^ 626 R
R([;) [2 6 R^ 6 "
R([;) [f [Rg [2 6 R^ = "
f [Rg [= [y^ = "
; [= [y^ 6 "

ai `,

b b m K 2 i ? i i ? 2 ` 2 ; m H `B H `M 2 ;Tm 2 ;b 2 M i 2 / # v i ? 2 }M
L i ? 2 M i ? 2 m b M ` `-vQ T 2 ` i Q ` T T H+B M/ # Q ` 2 T ` 2 b 2 M i
2 H Q r X

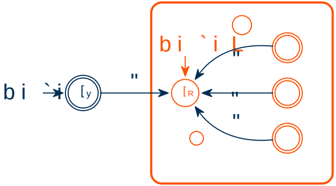


([;) =

	R([;)	[2 ZR^ 626R
	R([;)	[2 6R^ 6 "
	R([;) [f [R g	[2 6R^ = "
f [R g		[= [y^ = "
;		[= [y^ 6 "

ai `,

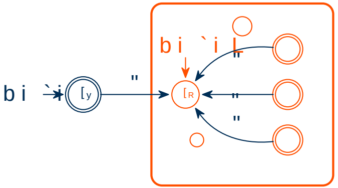
bbmK2 i? i i?2 `2;mH `BH `M2;Tm2;b2 2Mi2/ #v i?2 }M L i?2M i?2 mbM``-vQT2` iQ` TTHB2M/ #Q `2T`2b2Mi #2HQrX



([;) = 8 R([;) [2 ZR^ 626R
R([;) [2 6R^ 6 "
R([;) [f [Rg [2 6R^ = "
f [Rg [= [y^ = "
; [= [y^ 6 "

ai `,

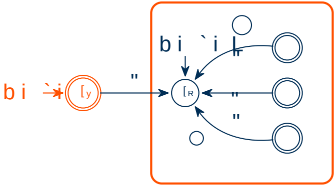
b b m K 2 i ? i i ? 2 ` 2 ; m H `B H `M 2 ;T m 2 ; b 2 M i 2 / # v i ? 2 } M
L i ? 2 M i ? 2 m b M ` `-v Q T 2 ` i Q ` T T H B 2 / # Q ` 2 T ` 2 b 2 M i
2 H Q r X



([;) = 8 ([;) [2 Z R ^ 6 2 6 R
([;) [2 6 R ^ 6 "
([;) [f [R g [2 6 R ^ = "
f [R g [= [y ^ = "
; [= [y ^ 6 "

ai `,

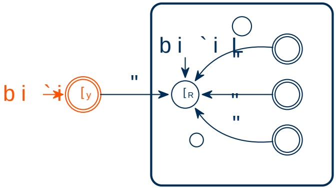
bbmK2 i? i i?2 `2;mH `BH `M2;Tm`2;b2 Mi2/ #v i?2 }M
L i?2M i?2 mbM`-vQT2` iQ` TTHB M/ #Q `2T`2b2Mi
#2HQrX



([;) = 8 (R([;) [2 ZR^ 626R ; R([;) [2 6R^ 6 " ; R([;) [f [Rg [2 6R^ = " ; f [Rg [= [y^ = " ; ; [= [y^ 6 "

ai `,

bbmK2 i? i i?2 `2;mH `BH `M2;Tm`2;b2 Mi2/ #v i?2 }M
L i?2M i?2 mbM``-vQT2` iQ` TTH+B M2/ #Q `2T`2b2Mi
#2HQrX



([;) =

8
R([;) [2 Z R^ 626R
R([;) [2 6R^ 6 "
R([;) [f [Rg [2 6R^ = "
f [Rg [= [y^ = "
; [= [y^ 6 "

h #H2 Q7 *QMi2Mib

R 6BMBi2 miQK i

k L2r 6`QK PH/

j LQM@.2i2`KBMBbiB+ pbX .2i2`KBMBbiB+

9 L2r 6`QK PH/ U ; BMV

8 L2ti *H bb

L2ti *H bb

_2;mH ` 1tT`2bbBQMb

L2ti *H bb

_2;mH ` 1tT`2bbBQMb

:2M2` HBx2/ LQM/2i2`KBMBbiB+ 6BMBi2 mi

L2ti *H bb

_2;mH ` 1tT`2bbBQMb

:2M2` HBx2/ LQM/2i2`KBMBbiB+ 6BMBi2 mi

SmKTBM; G2KK

Next Class

- Regular Expressions
- Generalized Nondeterministic Finite Automata
- Pumping Lemma
- Nonregular Languages

Introduction to Finite Automata

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