

# Discrete Math Review

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# Table of Contents

- 1 Sets, Relations, and Functions
- 2 Graph Theory
- 3 Theorems and Proofs
- 4 Next Class



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1 Sets, Relations, and Functions

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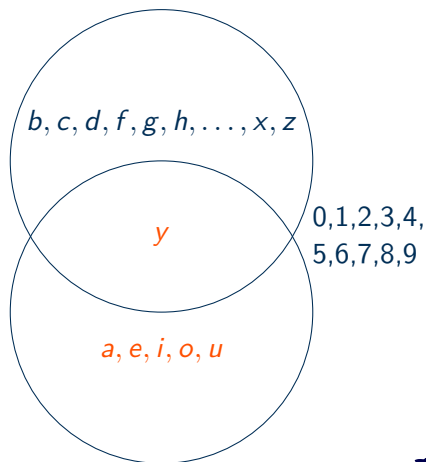


# Sets

- Sets:

$A = \{a, e, i, o, u, y\}$  and

$B = \{b, c, d, f, g, h, \dots, z\}$

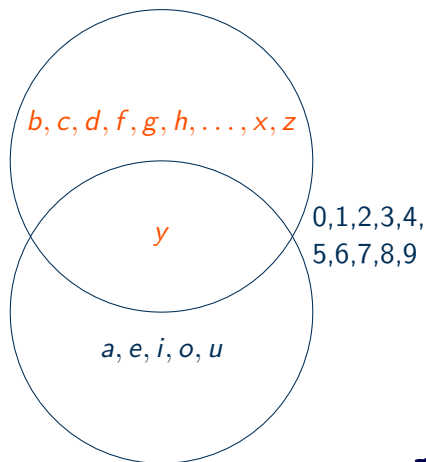


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# Sets

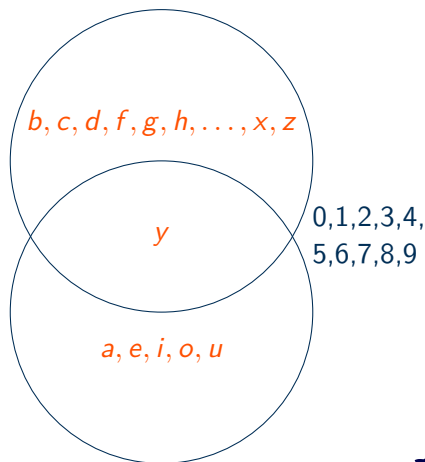
- Sets:

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- Union:

$A \cup B = \{a, b, c, d, e, \dots, z\}$



# Sets

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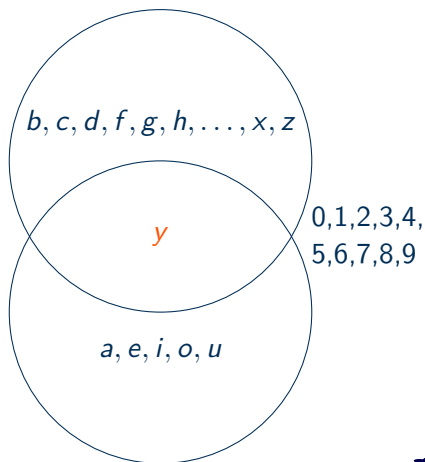
$B = \{b, c, d, f, g, h, \dots, z\}$

- Union:

$A \cup B = \{a, b, c, d, e, \dots, z\}$

- Intersection:

$A \cap B = \{y\}$



# Sets

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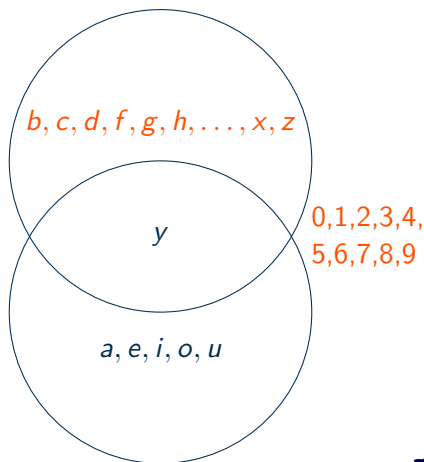
$A \cup B = \{a, b, c, d, e, \dots, z\}$

- Intersection:

$A \cap B = \{y\}$

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$A^c = (B \setminus \{y\}) \cup \{0, 1, \dots, 9\}$





# Sets

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$A = \{a, e, i, o, u, y\}$  and

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- Union:

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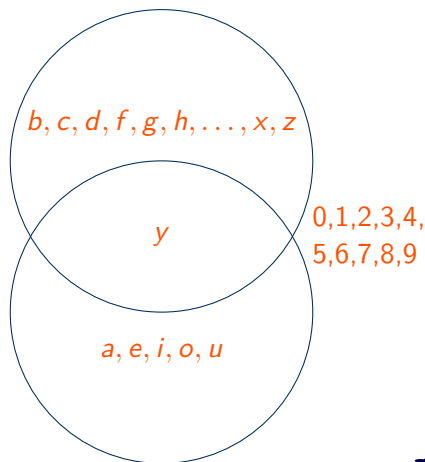
$A \cap B = \{y\}$

- Complement:

$A^c = (B \setminus \{y\}) \cup \{0, 1, \dots, 9\}$

- Universal Set:

$\mathcal{U} = A \cup B \cup \{0, 1, \dots, 9\}$



# New Sets from Old

- $A = \{a, b, c\}$  and  $B = \{0, 1, 2\}$



# New Sets from Old

- $A = \{a, b, c\}$  and  $B = \{0, 1, 2\}$
- Cartesian Product:

$$A \times B = \{(a, 0), (a, 1), (a, 2), (b, 0), (b, 1), (b, 2), (c, 0), (c, 1), (c, 2)\}$$



# New Sets from Old

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- Cartesian Product:

$$A \times B = \{(a, 0), (a, 1), (a, 2), (b, 0), (b, 1), (b, 2), (c, 0), (c, 1), (c, 2)\}$$

- Power Set:

$$\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

$$|\mathcal{P}(A)| = 2^{|A|}$$



# Cardinality of a Power Set: $|\mathcal{P}(S)| = 2^{|S|}$

- $A = \{0, 1\}$  and  $B = \{0, 1, 2\}$



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# Cardinality of a Power Set: $|\mathcal{P}(S)| = 2^{|S|}$

- $A = \{0, 1\}$  and  $B = \{0, 1, 2\}$
- $\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$
- $\mathcal{P}(B) = ?$

$$\begin{aligned}\mathcal{P}(B) &= \mathcal{P}(A) \cup \left( \bigcup_{s \in \mathcal{P}(A)} \{s \cup \{2\}\} \right) \\ &= \{\emptyset, \{0\}, \{1\}, \{0, 1\}\} \cup \{\{2\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\} \\ &= \{\emptyset, \{0\}, \{1\}, \{0, 1\}, \{2\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}\end{aligned}$$



# Cardinality of a Power Set: $|\mathcal{P}(S)| = 2^{|S|}$

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- $|\mathcal{P}(B)| = ?$

$$\begin{aligned}
 |\mathcal{P}(B)| &= |\mathcal{P}(A)| + \left| \bigcup_{s \in \mathcal{P}(A)} \{s \cup \{2\}\} \right| \\
 &= |\mathcal{P}(A)| + \sum_{s \in \mathcal{P}(A)} |\{s \cup \{2\}\}| \\
 &= |\mathcal{P}(A)| + |\mathcal{P}(A)| \\
 &= 2 \cdot |\mathcal{P}(A)|
 \end{aligned}$$



# Cardinality of a Power Set: $|\mathcal{P}(S)| = 2^{|S|}$

- $A = \{0, 1\}$  and  $B = \{0, 1, 2\}$
- $\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$
- $\mathcal{P}(B) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$
- $|\mathcal{P}(B)| = 2 \cdot |\mathcal{P}(A)| = 2 \cdot 2^{|A|} = 2^{|A|+1} = 2^{|B|}$



# Relations

## Definition (Relation)

A **relation** between two sets is a subset of their Cartesian product.



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Given:

$$A \times B = \{(a, 0), (a, 1), (a, 2), (b, 0), (b, 1), (b, 2), (c, 0), (c, 1), (c, 2)\}$$



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A sample relation might be:

$$\mathcal{R} = \{(a, 0), (a, 1), (a, 2), (b, 1), (b, 2), (c, 2)\}$$



# Relations

## Definition (Relation)

A **relation** between two sets is a subset of their Cartesian product.

Given:

$$A \times A = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$$



# Relations

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A **relation** between two sets is a subset of their Cartesian product.

Given:

$$A \times A = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$$

A sample relation might be:

$$\mathcal{O} = \{(a, b), (a, c), (b, c)\}$$



# Equivalence Relation

## Definition (Equivalence Relation)

A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.





# Equivalence Relation

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Given the relation on  $A$ :

$$\mathcal{O} = \{(a, b), (a, c), (b, c)\}$$



# Equivalence Relation

## Definition (Equivalence Relation)

A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.

Given the relation on  $A$ :

$$\mathcal{O} = \{(a, b), (a, c), (b, c)\}$$

Since  $a$  does not relate to its self ( $a \not\sim a$ ) this is not **reflexive**.



# Equivalence Relation

## Definition (Equivalence Relation)

A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.

Given the relation on  $A$ :

$$\mathcal{O} = \{(a, b), (a, c), (b, c)\}$$

Since  $a$  relates to  $b$  ( $a \sim b$ ) but  $b$  does not relate to  $a$  ( $b \not\sim a$ ) this is not **symmetric**.



# Equivalence Relation

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A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.

Given the relation on  $A$ :

$$\mathcal{O} = \{(a, b), (a, c), (b, c)\}$$

Since  $a \sim b$  and  $b \sim c$  and  $a \sim c$  this is **transitive**.



# Equivalence Relation

## Definition (Equivalence Relation)

A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.

Given the relation on  $A$ :

$$\mathcal{C} = \{(a, a), (a, b), (b, a), (b, b), (c, c)\}$$



# Equivalence Relation

## Definition (Equivalence Relation)

A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.

Given the relation on  $A$ :

$$\mathcal{C} = \{(a, a), (a, b), (b, a), (b, b), (c, c)\}$$

Since  $a \sim a$ ,  $b \sim b$ , and  $c \sim c$  this is **reflexive**.



# Equivalence Relation

## Definition (Equivalence Relation)

A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.

Given the relation on  $A$ :

$$\mathcal{C} = \{(a, a), (a, b), (b, a), (b, b), (c, c)\}$$

Since  $a \sim b$  and  $b \sim a$  this is **symmetric**.



# Equivalence Relation

## Definition (Equivalence Relation)

A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.

Given the relation on  $A$ :

$$\mathcal{C} = \{(a, a), (a, b), (b, a), (b, b), (c, c)\}$$

Since  $a \sim b$ ,  $b \sim a$  and  $a \sim a$  (also,  $b \sim a$ ,  $a \sim b$ , and  $b \sim b$ ) this is **transitive**.





# Equivalence Relation

## Definition (Equivalence Relation)

A relation between a set and its self is an **equivalence relation** if and only if it is **reflexive**, **symmetric**, and **transitive**.

Given the relation on  $A$ :

$$\mathcal{C} = \{(a, a), (a, b), (b, a), (b, b), (c, c)\}$$

This relation is an **equivalence relation**.



# Function

## Definition (Function)

A **function** is a relation between two sets, the first called the **domain** and the second the **co-domain**, such that for all  $x$  in the domain there exists a unique  $y$  in the co-domain such that  $(x, y)$  is in the relation.



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Given:

$$A \times B = \{(a, 0), (a, 1), (a, 2), (b, 0), (b, 1), (b, 2), (c, 0), (c, 1), (c, 2)\}$$

The relation:

$$\mathcal{R} = \{(a, 0), (a, 1), (a, 2), (b, 1), (b, 2), (c, 2)\}$$

is not a function



# Function

## Definition (Function)

A **function** is a relation between two sets, the first called the **domain** and the second the **co-domain**, such that for all  $x$  in the domain there exists a unique  $y$  in the co-domain such that  $(x, y)$  is in the relation.

Given:

$$A \times B = \{(a, 0), (a, 1), (a, 2), (b, 0), (b, 1), (b, 2), (c, 0), (c, 1), (c, 2)\}$$

But, the relation:

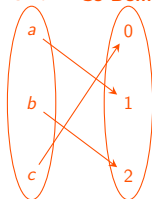
$$S = \{(a, 1), (b, 2), (c, 0)\}$$

is a function



# Visualizing Functions

Domain Co-Domain

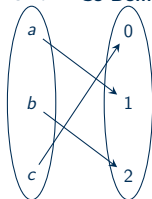


1 - 1 & onto



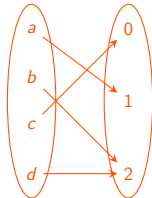
# Visualizing Functions

Domain Co-Domain



1 - 1 & onto

Domain Co-Domain

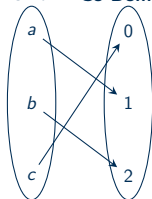


onto



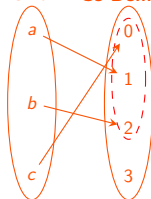
# Visualizing Functions

Domain Co-Domain



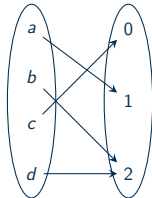
$1-1$  & onto

Domain Co-Domain



$1-1$

Domain Co-Domain

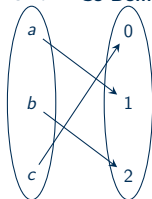


onto



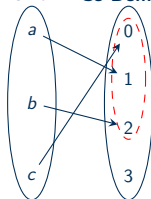
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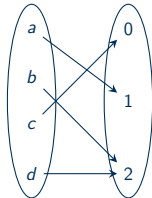
$1-1$  & onto

Domain Co-Domain



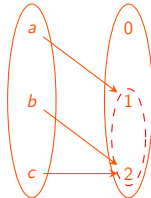
$1-1$

Domain Co-Domain



onto

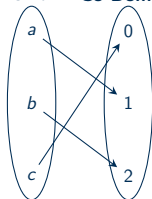
Domain Co-Domain





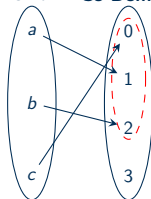
# Visualizing Functions

Domain Co-Domain



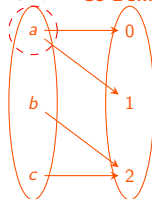
1 - 1 & onto

Domain Co-Domain



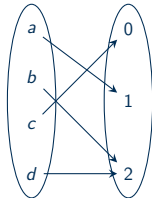
1 - 1

Domain Co-Domain



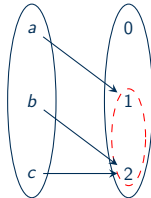
Not a function

Domain Co-Domain



onto

Domain Co-Domain

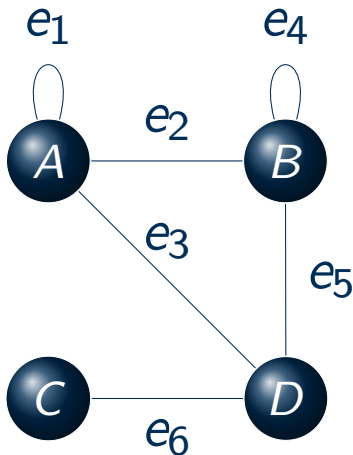


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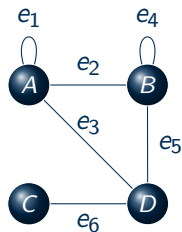
- 1 Sets, Relations, and Functions
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# Graphs



# Graphs

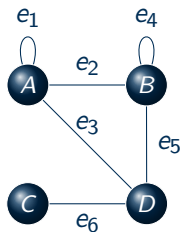


- Vertex Set:

$$V = \{A, B, C, D\}$$



# Graphs



- Vertex Set:

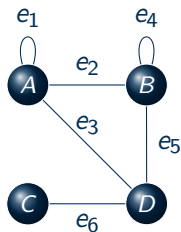
$$V = \{A, B, C, D\}$$

- Edge Set:

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$



# Graphs



- Vertex Set:

$$V = \{A, B, C, D\}$$

- Edge Set:

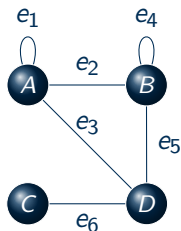
$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

- Edge Set:

$$E = \{(A, A), (A, B), (A, D), (B, B), (B, D), (C, D)\}$$



# Graphs



- Vertex Set:

$$V = \{A, B, C, D\}$$

- Edge Set:

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

- Edge Set:

$$E = \{(A, A), (A, B), (A, D), (B, B), (B, D), (C, D)\}$$

- Graph:

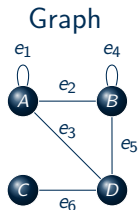
$$G = (V, E)$$

$$= (\{A, B, C, D\},$$

$$\{(A, A), (A, B), (A, D), (B, B), (B, D), (C, D)\})$$



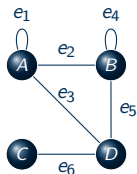
# Types of Graphs



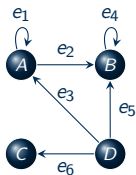


# Types of Graphs

## Graph

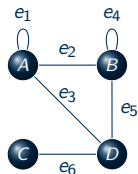


## Directed Graph

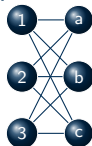


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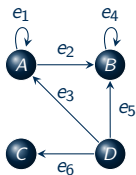
## Graph



## Bipartite Graph

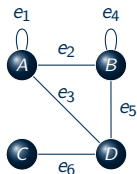


## Directed Graph

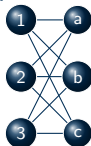


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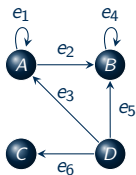
## Graph



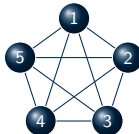
## Bipartite Graph



## Directed Graph

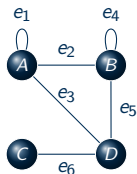


## Complete Graph

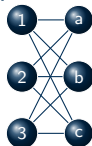


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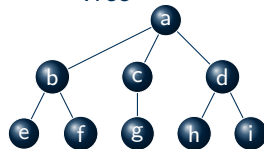
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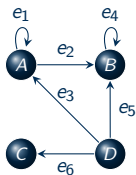
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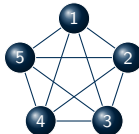
## Tree



## Directed Graph

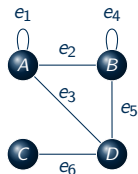


## Complete Graph

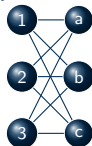


# Types of Graphs

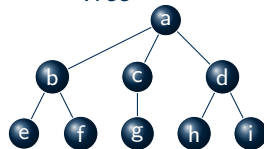
## Graph



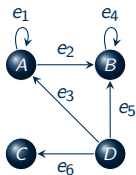
## Bipartite Graph



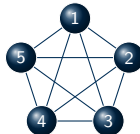
## Tree



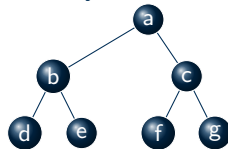
## Directed Graph



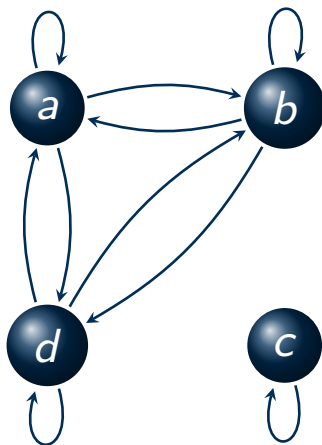
## Complete Graph



## Binary Tree

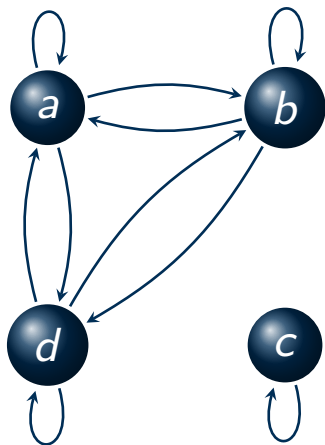


# Relations as Graphs



# Relations as Graphs

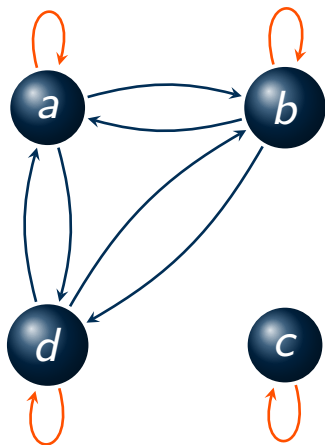
Equivalence Relation?



# Relations as Graphs

Equivalence Relation?

- Reflexive ✓

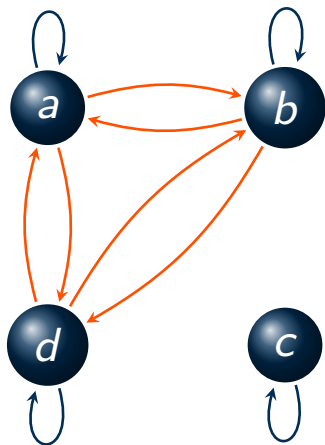




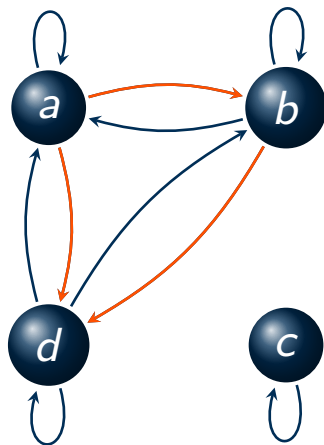
# Relations as Graphs

Equivalence Relation?

- Reflexive ✓
- Symmetric ✓



# Relations as Graphs

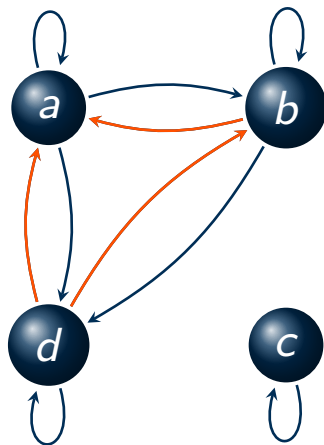


Equivalence Relation?

- Reflexive ✓
- Symmetric ✓
- Transitive ✓



# Relations as Graphs

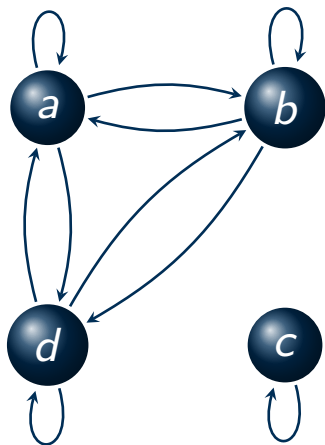


Equivalence Relation?

- Reflexive ✓
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- Transitive ✓



# Relations as Graphs



## Equivalence Relation✓

- Reflexive ✓
- Symmetric ✓
- Transitive ✓
- Equivalence Classes

$$A = \{a, b, d\} \text{ \& } C = \{c\}$$



# Table of Contents

- 1 Sets, Relations, and Functions
- 2 Graph Theory
- 3 Theorems and Proofs**
- 4 Next Class



# Direct Proof

## Theorem (De Morgan's Law)

*Given two sets  $A$  and  $B$ , the complement of their union is equal to the intersection of their complements:*

$$(A \cup B)^c = A^c \cap B^c.$$



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$$(A \cup B)^c = A^c \cap B^c$$

as desired.



# By Cases

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*Given any integer  $n$ , either  $n^2$  or  $n^2 - 1$  is divisible by four.*



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(Case2) Now, if  $n$  is an odd integer then we write  $n = 2k + 1$  for some unique  $k$ . Thus,

$$n^2 - 1 = 4k^2 + 4k + 1 - 1 = 4(k^2 + k)$$

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Therefore, for any integer  $n$  we have shown that either  $n^2$  or  $n^2 - 1$  is divisible by four.





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*If  $n^2$  is even, then  $n$  is even.*



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which is odd. Therefore, if  $n$  is odd, then  $n^2$  is odd and so if  $n^2$  is even, then  $n$  is even.



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# Induction

## Theorem

*Any tree with  $n$  vertices has  $n - 1$  edges.*



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(Base Case) When there is only one vertex there are no edges since trees do not contain loops and there is not a second vertex to connect to.

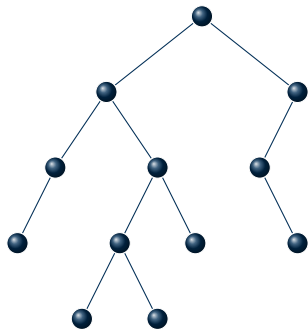


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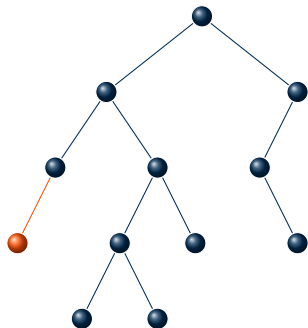
(Induction Step) Assume that the theorem is true for some  $k \geq 2$  and consider a tree with  $k + 1 \geq 3$  vertices.



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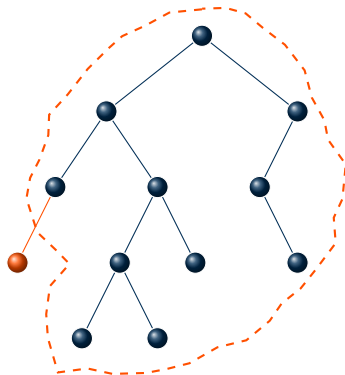
(Induction Step) Assume that the theorem is true for some  $k \geq 2$  and consider a tree with  $k + 1 \geq 3$  vertices. Since there are at least two vertices the tree must contain at least one leaf.



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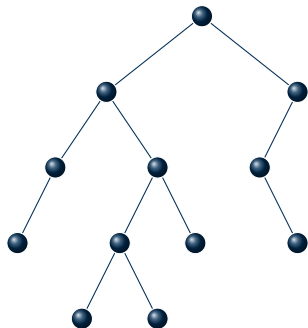
(Induction Step) Assume that the theorem is true for some  $k \geq 2$  and consider a tree with  $k + 1 \geq 3$  vertices. Since there are at least two vertices the tree must contain at least one leaf. Removing the leaf removes one vertex and one edge; we now have a subtree with  $k$  vertices and, by induction,  $k - 1$  edges.



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(Induction Step) Assume that the theorem is true for some  $k \geq 2$  and consider a tree with  $k + 1 \geq 3$  vertices. Since there are at least two vertices the tree must contain at least one leaf. Removing the leaf removes one vertex and one edge; we now have a subtree with  $k$  vertices and, by induction,  $k - 1$  edges. Thus the original tree has  $k + 1$  vertices and  $k$  edges.



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# Next Class

- Deterministic and Non-Deterministic Finite State Automata



# Discrete Math Review

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