Vocabulary:

- **Unit 1:** Matrix, Vector, Augmented Matrix, System of Equations, Vector Equation, Matrix Equation, Pivot Position, Consistent System, Inconsistent System, Homogeneous, Non-Homogeneous, Overdetermined, Underdetermined, Dot Product
- **Unit 2:** Vector Space, Subspace, Linearly Independent, Linearly Dependent, Span of a set of Vectors, Column Space, Row Space, Null space, Basis
- **Unit 3:** Linear Transformations, Matrix of a Transformation, One-to-One, Onto, Kernel, Zero Divisor, Transpose, Inverse Matrix, Null Space, Projection, Orthogonal Vectors, Orthogonal Complement, Projection of a vector onto a subspace, Orthonormal Basis, Change of Basis Matrix (also called change of coordinate matrix)
- **Unit 4:** Determinants, Cofactors, Minor Matrix, Eigenvector, Eigenvalue, Characteristic Polynomial, Similar Matrices, Diagonalizable

Basic Skills: Be able to ...

- **Unit 1: Systems of Equations**
  1. ...find general solutions to systems of equations.
  2. ...change systems from equations to matrices to vectors.
  3. ...row reduce matrices.
  4. ...identify a system as homogeneous, non-homogeneous, overdetermined, underdetermined, consistent, or inconsistent.
  5. ...find the dot product of vectors and project one onto another.
  6. ...identify when two vectors are orthogonal.
- **Unit 2: Vectors Spaces**
  1. ...determine if a vector is in the span of a set of vectors, and if a set of vectors is a basis.
  2. ...find the row, column, and null space of a matrix.
  3. ...determine if a set is a vector space.
  4. ...find a basis for the span of a set of vectors.
- **Unit 3: Transformations and Projections**
  1. ...find the domain, codomain, range, and kernel of a linear transformation.
  2. ...determine if a transformation is one-to-one or onto.
  3. ...find change of basis matrices.
  4. ...project a vector onto a subspace.
  5. ...apply Gram-Schmidt orthogonalization process.
- **Unit 4: Determinants and Eigen-Stuff**
  1. ...find determinants, eigenvectors, and eigenvalues.
  2. ...diagonalize a matrix.
  3. ...explain the geometric significance of the eigenvectors/values.

Your final exam will consist of 10 questions based on the basic skills, like those below, and 5 vocabulary questions. The exam is on Thursday December 13^{th} at 11 am in the usual room. You can do this practice exam for extra credit, but you don't have to, it is due when you come in for the final exam.

1. Use Gauss-Jordan elimination to solve the system of equations:

$$\begin{aligned} 2x - 6y + 5z &= 0 \\ x - 3y + 2z &= 0 \end{aligned}$$

2. Which of these adjectives apply to the system of equations homogeneous, non-homogeneous, overdetermined, underdetermined, consistent, or inconsistent? (You should not need to do any calculations.)

$$\begin{aligned} 2x + 3y - 9z &= 0 \\ 10x + 15y - 45z &= 1 \end{aligned}$$

3. Let $V = span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ and assume that none of the vectors are zero or a multiple of one another. Given

$$3\vec{v}_1 - \vec{v}_2 + 7\vec{v}_3 = \vec{0}$$

find a basis for V.

4. Find the row, column, and null spaces for the matrix.

$$B = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 7 & 3 \\ 1 & -2 & -2 \end{bmatrix}$$

5. Define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T(\vec{v}) = \langle x - 2y, 3x - y - z \rangle$$

Find the matrix for the transformation, then find the domain, codomain, range, and kernel of the transformation.

6. Find the change of basis matrix $Rep_{\mathcal{E}\mathcal{B}}$ from the standard basis for $\mathbb{R}^2$ to

$$\mathcal{B} = \{\langle 2, 3\rangle, \langle 3, 5\rangle\}$$

7. Use Gram-Schmidt to change the basis to an orthogonal basis:

$$\mathcal{C} = \{\vec{v} = \langle 1,\ 0,\ 1\rangle, \vec{w} = \langle 2,\ 1,\ 0\rangle, \vec{u} = \langle 0,\ 11,\ 2\rangle\}.$$

If you start by keeping the first vector and projecting the second one onto that you should get

$$\widehat{\mathcal{C}} = \{\vec{v} = \langle 1,\ 0,\ 1\rangle, \widehat{w} = \langle 1,\ 1,\ -1\rangle, \widehat{u} = \langle -4,\ 8,\ 4\rangle\}.$$

8. Diagonalize the matrix:

$$A = \begin{bmatrix} 1 & -1 & 7 \\ 0 & 2 & -9 \\ 0 & 0 & -1 \end{bmatrix}$$

You should get something like $A = PDP^{-1}$ with

$$P = \begin{bmatrix} 1 & 1 & -2 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$