MAT272

10/18/18 Class Notes

$$\begin{pmatrix} 4t + 3r \\ -4s - r \\ 6t + 7s \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow$$

Write as a
(1) System of equations
(2) A vector equation
(3) A matrix equation
(4) An augmented matrix

Useful for word problems

(1)
$$\begin{aligned} 7t \quad +3r &= 0 \\ -4s - r &= 0 \\ 6t + 7s \quad &= 0 \end{aligned}$$

Useful for bases

(2)
$$\begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix} t + \begin{pmatrix} 0 \\ -4 \\ 7 \end{pmatrix} s + \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Useful for linear transformations

(3)
$$\begin{pmatrix} 7 & 0 & 3 \\ 0 & -4 & -1 \\ 6 & 7 & 0 \end{pmatrix} \begin{pmatrix} t \\ s \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(4)
$$\left(\begin{array}{ccc|c} 7 & 0 & 3 & 0 \\ 0 & -4 & -1 & 0 \\ 6 & 7 & 0 & 0 \end{array}\right) \xrightarrow{R_1 - R_2} \left(\begin{array}{ccc|c} 1 & -7 & 3 & 0 \\ 0 & -4 & -1 & 0 \\ 6 & 7 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 - 6 \cdot R_1} \left(\begin{array}{ccc|c} 1 & -7 & 3 & 0 \\ 0 & -4 & -1 & 0 \\ 0 & 49 & -18 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 + 12 \cdot R_2} \left(\begin{array}{ccc|c} 1 & -7 & 3 & 0 \\ 0 & -4 & -1 & 0 \\ 0 & 1 & -30 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 \sim R_2} \left(\begin{array}{ccc|c} 1 & -7 & 3 & 0 \\ 0 & 1 & -30 & 0 \\ 0 & -4 & -1 & 0 \end{array}\right)$$

$$\xrightarrow{4R_3 + R_2} \left(\begin{array}{ccc|c} 1 & -7 & 3 & 0 \\ 0 & 1 & -30 & 0 \\ 0 & 0 & -121 & 0 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{121} \cdot R_3} \left(\begin{array}{ccc|c} 1 & -7 & 3 & 0 \\ 0 & 1 & -30 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 + 30 \cdot R_1} \left(\begin{array}{ccc|c} 1 & -7 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{R_1 + 7 \cdot R_2 - 3 \cdot R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

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A quick note from the exam:

$$\begin{array}{c} \text{10 columns} \\ \left[\begin{array}{cccccc|c} 1 & 0 & * & \cdots & \cdots & * & 0 \\ 0 & 1 & * & \cdots & \cdots & * & 0 \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \end{array}\right] \end{array}$$

This is a 3x10 augmented matrix

- The Column Space is 2-dimensional
- The Row Space is 2-dimensional
- The Column Space lives in 3 dimensions
- The Row Space lives in 10 dimensions

> Unit 3's Homework Exercises are not from the textbook. They are on Dr. Rocca's website under "Homework Exercises on Linear Transformations and Projections".

Definitions

Function: $f: X \to Y$

The set of inputs is called the domain.

The set of possible outputs is called the codomain.

The range (or image) is the set of all $y \in Y$ such that there is actually an $x \in X$ with $y = f(x)$

Linear Functions

or Linear Transformations ← Most common term used.

or homomorphisms

$T: V \longrightarrow W$

$\rightarrow T(\vec{v}) = \vec{w}$

$\rightarrow T(a\vec{v}_1 + b\vec{v}_2) = aT(\vec{v}_1) + bT(\vec{v}_2)$

$\hookrightarrow T(\vec{0}) = \vec{0}$

$\hookrightarrow T(-\vec{v}_1) = -T(\vec{v}_1)$

An example of a linear transformation of a function:

$\frac{d}{dx} f(x)$ = derivative of f

Sum Rule: $\frac{d}{dx}(f+g)(x) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

Constant Multiple Rule: $\frac{d}{dx}(cf)(x) = c\frac{d}{dx}f(x)$

A linear transformation: $\frac{d}{dx}(af+bg)(x) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$

Finding the derivative of a function is a linear transformation.

$\frac{d}{dx}: C \longrightarrow C$ where C = the set of all everywhere infinitely differentiable functions

$\frac{d}{dx}f(x) = 0 \iff f(x)$ is a constant

These functions are in the kernel (or Null space) of the transformation. The Kernel (or Null space) of a linear transformation is the set of all vectors whose image is the zero vector.

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A linear transformation is <u>one-to-one</u> if every output is associated with a unique input.
$\frac{d}{dx}$ is not one-to-one. Ex: $\frac{d}{dx} e^{kx}+7 = ke^{kx}$
$\frac{d}{dx} e^{kx}-13 = ke^{kx}$.

A linear transformation is <u>onto</u> if every element of the codomain is in the range.
$\frac{d}{dx}$ is onto.