9/4/18 Class Notes

Review from last class

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ = The Identity Matrix

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ = A Reflection Matrix

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ = A Projection Matrix

$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ = A Rotation Matrix

$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ = A Dilation Matrix

* Look at multiplication on matrices and addition on vectors as operations on physical objects. Thus, order of operations matters.

Elimination and Back Substitution

b = bottles , K = Krabby Patties, p = pint

$$\begin{aligned} 3b &= 30 \\ b + 2k &= 20 \\ k + 4p &= 9 \\ k + p \times b &= ? \end{aligned} \implies \begin{aligned} 3b &= 30 \Rightarrow b = 10 \\ b + 2k &= 20 \Rightarrow 10 + 2k = 20 \Rightarrow 2k = 10 \Rightarrow k = 5 \\ k + 4p &= 9 \Rightarrow 5 + 4p = 9 \Rightarrow 4p = 4 \Rightarrow p = 1 \\ 5 + 1 \times 10 &= 15 \end{aligned}$$

$$\boxed{\begin{aligned} b + 7k + 3p &= 50 \\ b \quad + 2p &= 16 \\ 5b + k \quad &= 30 \end{aligned}}$$

b = ?

k = ?

p = ?

This is a Consistent system meaning a solution always exists.

$$\implies \begin{aligned} (b + 7k + 3p &= 50)\,2 \\ (b \quad + 2p &= 16)\,{-3} \\ 5b + k \quad &= 30 \end{aligned}$$

$$\boxed{\begin{aligned} b &= \tfrac{1}{5}\left(30 - \tfrac{290}{71}\right) \approx 5\text{ ish} \\ k &= \tfrac{290}{71} \approx 4\text{ ish} \\ p &= \tfrac{1}{2}(16 - b) \approx 5.5\text{ ish} \end{aligned}}$$

$+ \Rightarrow$

$$\begin{aligned} (-b + 14k &= 52)\,5 \\ + 5b + k &= 30 \\ \hline 71k &= 290 \\ k &= \tfrac{290}{71} \end{aligned}$$

* Here we eliminate b and p. (elimination)

$$\begin{aligned} 5b + \tfrac{290}{71} &= 30 \\ 5b &= 30 - \tfrac{290}{71} \\ b &= \tfrac{1}{5}\left(30 - \tfrac{290}{71}\right) \end{aligned}$$

* Here we substitute K back in. (back substitution)

$$\begin{aligned} b + 2p &= 16 \\ 2p &= 16 - b \\ p &= \tfrac{1}{2}(16 - b) \end{aligned}$$

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$$\left.\begin{aligned} b + 5k + 4p &= 39 \\ 2b + 10k + 8p &= 50 \end{aligned}\right\}$$

$b = ?$
$k = ?$
$p = ?$

The left-hand side of the second equation is double the left-hand side of the first equation. However, the right-hand side of the second equation is NOT double the right-hand side of the first equation. This means that this system is Inconsistent because there is no solution!

$$\begin{aligned} b + 5k + 4p &= 39 \\ 2b + 10k + 8p &= 78 \end{aligned} \Rightarrow \left.\begin{aligned} b + 5k + 4p &= 39 \\ 2(b + 5k + 4p &= 39) \end{aligned}\right\}$$

$b = ?$ $\quad b = 39 - 5k - 4p$
$k = ?$ $\quad$ K and p are Free Variables
$p = ?$

This is an underdetermined system because we don't have enough information to find a unique solution. However, it is still a consistent system because it has a solution. In fact, it has infinite many solutions!

$$\left.\begin{aligned} b + 3k &= 0 \\ 7b + 2k &= 0 \\ 5b - 4k &= 0 \end{aligned}\right\}$$

This is a Homogenous system meaning all three expressions equal 0. It is also an overdetermined system because there is only one unique solution.

$$\begin{aligned} b + 3k &= 0 \\ -19k &= 0 \\ -19k &= 0 \end{aligned} \Rightarrow k = 0 \qquad b + 0 = 0 \quad b = 0$$

$$\begin{aligned} b + 3k &= 3 \\ 7b + 2k &= 1 \\ 5b - 4k &= 5 \end{aligned}$$

$$\begin{aligned} b + 3k &= 3 \\ -19k &= -20 \\ -19k &= -10 \end{aligned}$$

These equations are inconsistent with one another so it is an inconsistent system.

	Consistent		Inconsistent	
Homogenous	$b+3k=0$ $7b+2k=0$ $5b-4k=0$	$b+5k+4p=0$ $2b+10k+8p=0$	Can't happen because 0 always works	
Non-homogenous	$b+3k=3$ $7b+2k=1$ $5b-4k=-5$	$b+5k+4p=39$ $2b+10k+8p=78$	$b+3k=5$ $7b+2k=1$ $5b-4k=5$	$b+5k+4p=39$ $2b+10k+8p=50$

$$\left(\begin{array}{cc|c} 1 & 3 & 3 \\ 7 & 2 & 1 \\ \underbrace{5 \;\; -4}_{\text{Coefficient Matrix}} & & -5 \end{array}\right)$$

Augmented Matrix