Sept 20th Class Notes

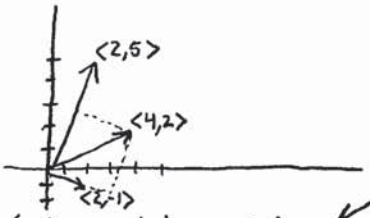
A system of equations $\begin{cases} 2x+2y=4 \\ 5x-y=2 \end{cases}$ ↔ Rewritten as a matrix

Rewritten as a matrix equation

Matrix Equation $\left\{ \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \right.$

Row x Vector

$\left(\begin{array}{cc|c} 2 & 2 & 4 \\ 5 & -1 & 2 \end{array}\right\}$ An augmented matrix



Rewritten as a vector equation

$$\begin{pmatrix} 2 \\ 5 \end{pmatrix} x + \begin{pmatrix} 2 \\ -1 \end{pmatrix} y = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

Dot Product ≡ Inner Product ≡ Scalar Product

$\vec{v} = \langle 1,0,7 \rangle$

$\vec{u} = \langle 0,2,-3 \rangle$

$\vec{v}\vec{u} = (1\cdot 0)+(0\cdot 2)+(7\cdot -3) = -21$

If rewritten as matrices ⟶ $\begin{pmatrix} \rightarrow \\ 1\ 0\ 7 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}\downarrow = (1\cdot 0+0\cdot 2+7\cdot -3) = -21$

Practice

$\vec{w} = \langle 2,3,9,0 \rangle$ $\qquad$ $\vec{x} = \langle 3,-5 \rangle$

$\vec{z} = \langle 1,-2,0,-7 \rangle$ $\qquad$ $\vec{y} = \langle -10,-6 \rangle$

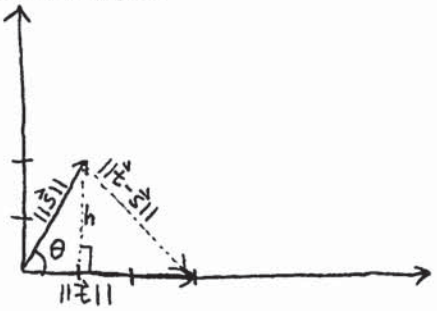
$\vec{w}\cdot\vec{z} = (2\cdot 1)+(3\cdot -2)+(9\cdot 0)+(0\cdot -7) = -4$ $\qquad$ $\vec{x}\cdot\vec{y} = (3\cdot -10)+(5\cdot -6) = 0$

Significance of the Dot Product

$\vec{s} = \langle 1,2 \rangle$

$\vec{t} = \langle 3,0 \rangle$

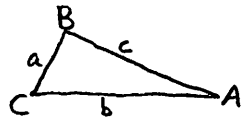
$\vec{s}\cdot\vec{t} = (1\cdot 3)+(2\cdot 0) = 3$



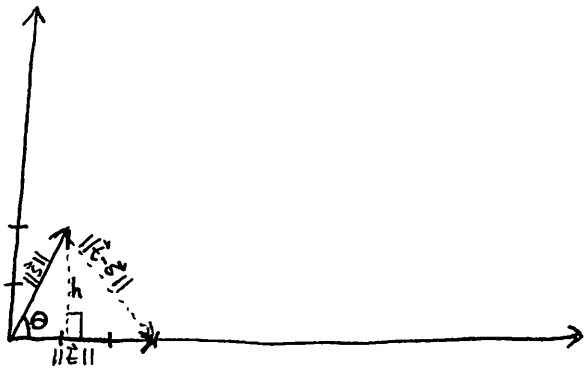
$\vec{s}\cdot\vec{t} = \underbrace{\|\vec{s}\|\cdot\|\vec{t}\|\cdot\cos\theta}$

The magnitude of $\vec{s}$ times the magnitude of $\vec{t}$ times $\cos\theta$

Sept 20th Class Notes Cont...



Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$



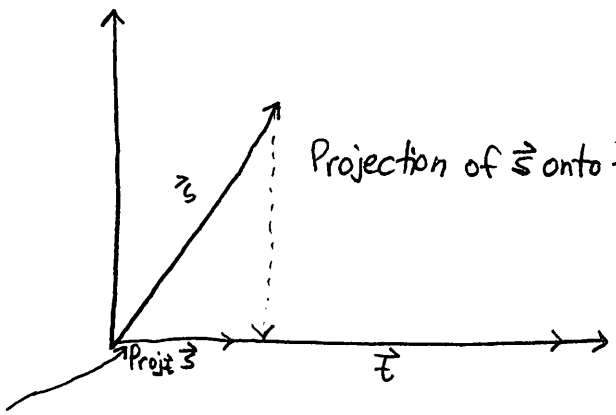
$$\|\vec{t}-\vec{s}\|^2 = \|\vec{s}\|^2 + \|\vec{t}\|^2 - 2\|\vec{s}\|\|\vec{t}\|\cos(\theta)$$

$$\|\vec{s}\|\|\vec{t}\|\cos(\theta) = \frac{\|\vec{t}-\vec{s}\|^2 - \|\vec{s}\|^2 - \|\vec{t}\|^2}{-2} = \vec{s}\cdot\vec{t}$$

$$\vec{s}\cdot\vec{t} = \|\vec{s}\|\|\vec{t}\|\cos(\theta)$$

$$\theta = \arccos\left(\frac{\vec{s}\cdot\vec{t}}{\|\vec{s}\|\|\vec{t}\|}\right)$$

$$\vec{s}\cdot\vec{s} = \|\vec{s}\|\|\vec{s}\|\cos(0) = \|\vec{s}\|^2$$



Projection of $\vec{s}$ onto $\vec{t}$ = $\text{Proj}_{\vec{t}}\,\vec{s}$

Vector moving in same direction as $\vec{t}$ with length 1 $\left(\frac{1}{\|\vec{t}\|}\right)$

$$\cos(\theta) = \frac{\|\text{Proj}_{\vec{t}}\,\vec{s}\|}{\|\vec{s}\|} = \frac{\vec{s}\cdot\vec{t}}{\|\vec{s}\|\|\vec{t}\|}$$

$$\|\text{Proj}_{\vec{t}}\,\vec{s}\| = \frac{\vec{s}\cdot\vec{t}}{\|\vec{t}\|}$$

$$\text{Proj}_{\vec{t}}\,\vec{s} = \frac{\vec{s}\cdot\vec{t}}{\|\vec{t}\|^2}\cdot\vec{t} = \frac{\vec{s}\cdot\vec{t}}{\vec{t}\cdot\vec{t}}\cdot\vec{t}$$

$\vec{s} = \langle 2, -3, 7\rangle$

$\vec{v} = \langle x, y, z\rangle$

These vectors are perpendicular if $\vec{s}\cdot\vec{v} = 0 \rightarrow 2x - 3y + 7z = 0$