Introduction to Finite Automata

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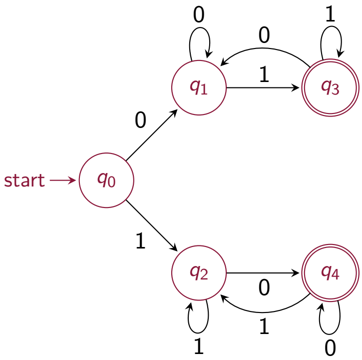
State Diagram and Finite Automaton



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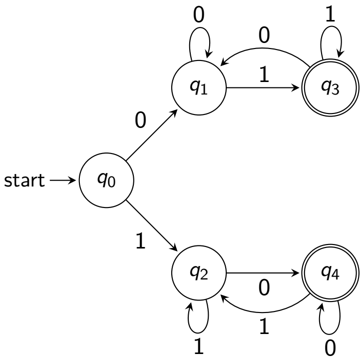
State Diagram and Finite Automaton



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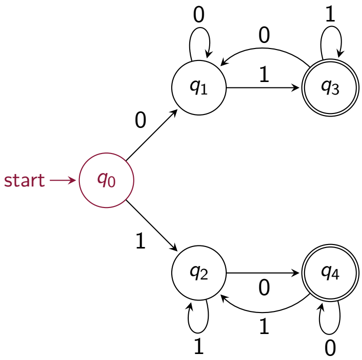

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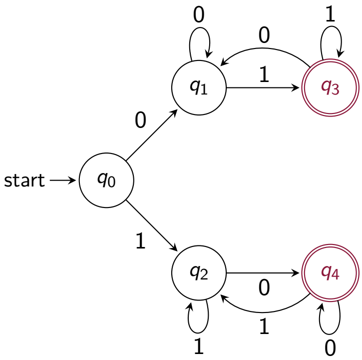

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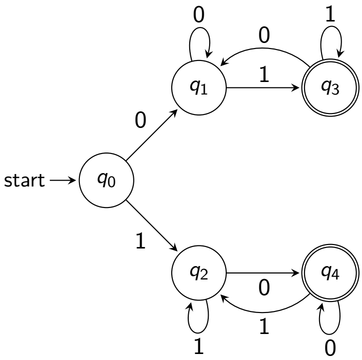

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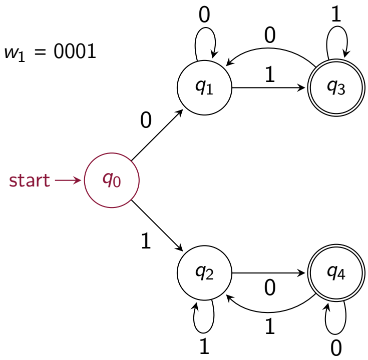

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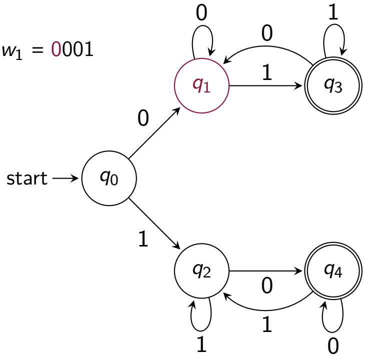

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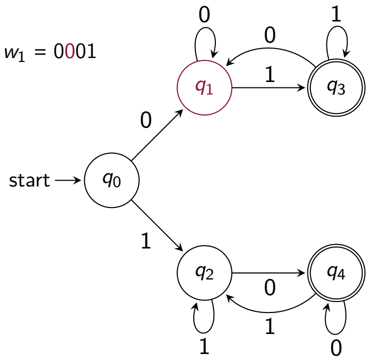

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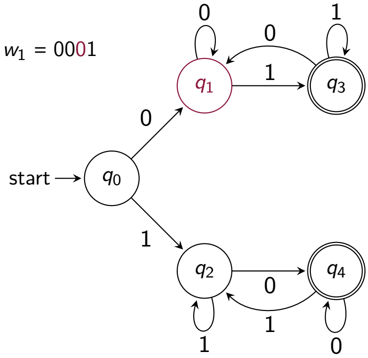

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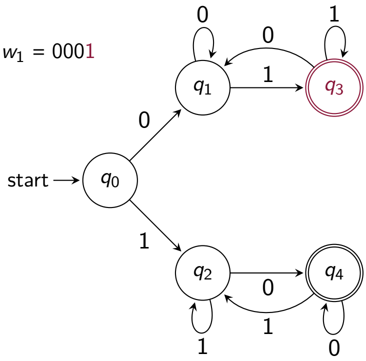

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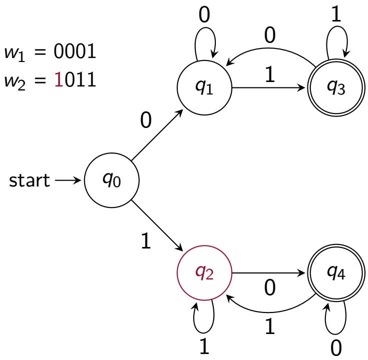

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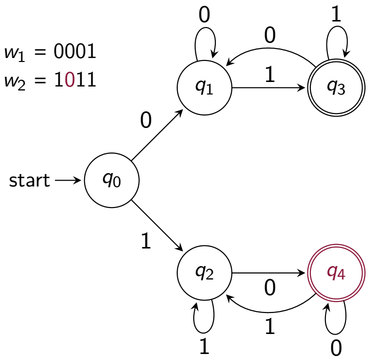

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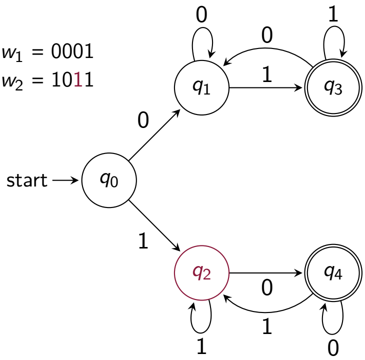
State Diagram and Finite Automaton

$w_1 = 0001$
$w_2 = 1011$



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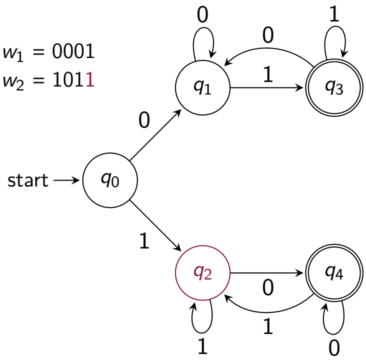

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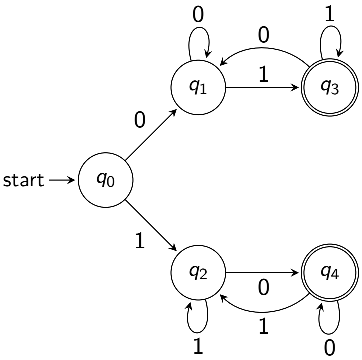
Formal Definition

Definition (Finite Automaton)

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of ***states***,
2. Σ is a finite ***alphabet***,
3. $\delta : Q \times \Sigma \rightarrow Q$ is the ***transition function***,
4. $q_o \in Q$ is the ***start or initial state***, and
5. $F \subseteq Q$ is the set of ***accept or final states***.

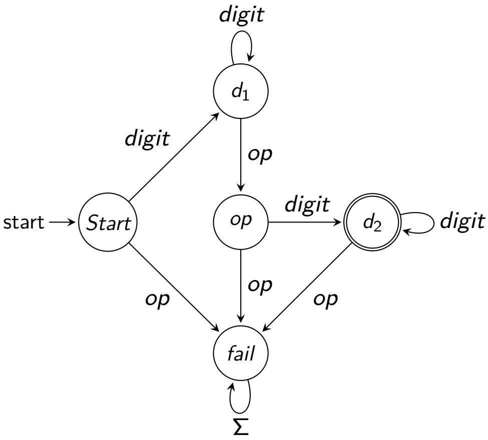
Transition Function



$$\delta : Q \times \Sigma \to Q$$

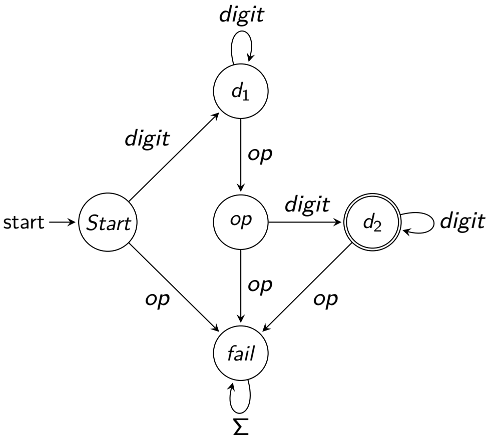
δ	0	1
q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_4	q_2
q_3	q_1	q_3
q_4	q_4	q_2

Another Example



- $Q = \{Start, d_1, d_2, op, fail\}$
- $\Sigma = op \cup digit$
  - $op = \{+, -, \times\}$
  - $digit = \{0, 1, 2, \ldots, 9\}$
- $F = \{d_2\}$
- $L = ?$

Another Example



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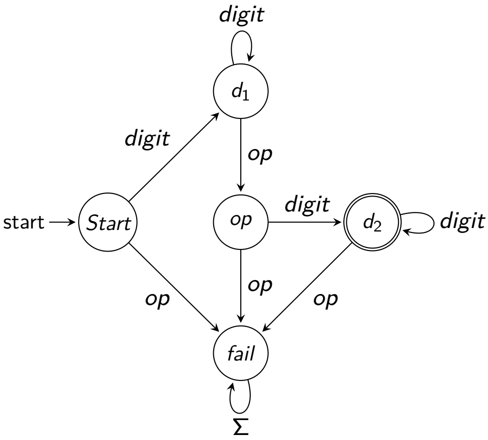
- $w = 25 + 362$

Another Example



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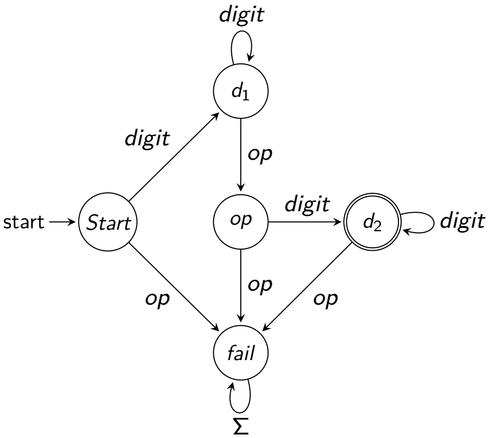
- $w = 25 + 362$
- $w = \times 35 - 67$

Another Example



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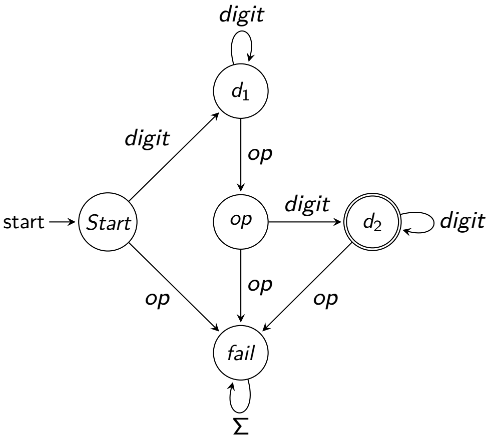
- $w = 25 + 362$
- $w = \times 35 - 67$
- $w = 265 - 456 \times 18$

Another Example



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- L=*Combine two integers*
- $w = 25 + 362$
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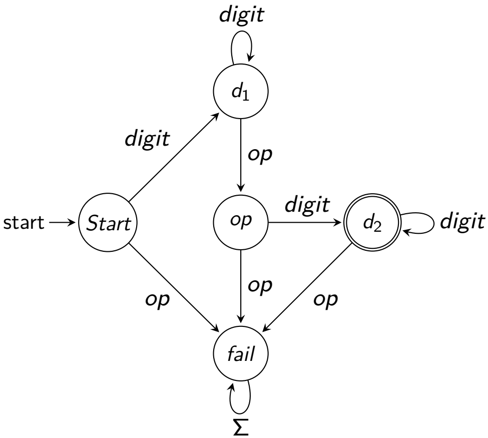

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- L=*Combine two integers*
- $w = 25 + 362$
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The language L is called a ***regular language*** because is is recognized by a finite automaton.

Satisfying a Description

Problem

Construct a finite automaton that accepts only words in $\Sigma = \{0, 1\}$ ending in 101.

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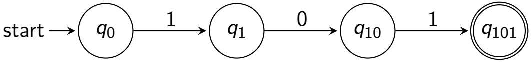

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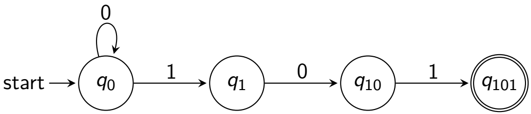

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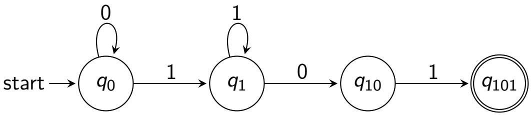

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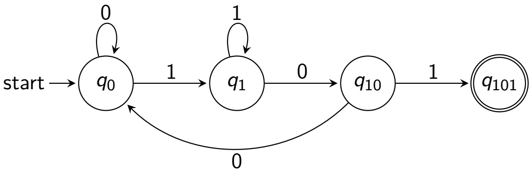

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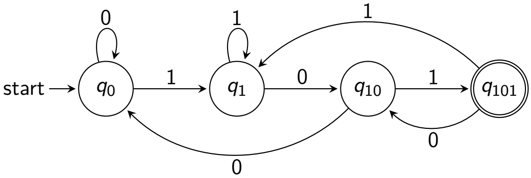

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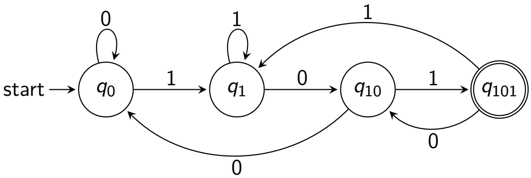

Check what happens to 101 independent of where we start.

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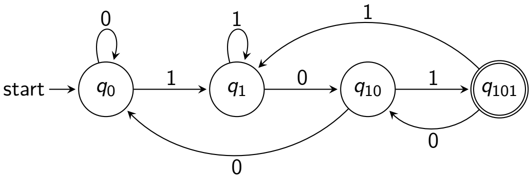

Check what happens to 010 independent of where we start.

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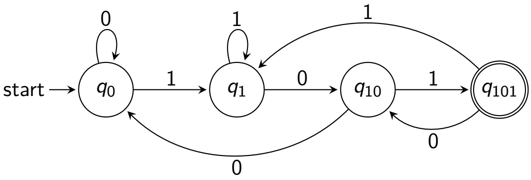

Note that this means we don't need to "remember" the whole string to check its ending.

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Creating New Automata

Definition

Let A and B be regular languages. We define the regular operations ***union***, ***concatenation***, and ***star*** as follows:

- ***Union***: $A \cup B = \{x | x \in A \text{ or } x \in B\}$
- ***Concatenation***: $A \circ B = \{xy | x \in A \text{ and } y \in B\}$
- ***Star***: $A^* = \{x_1 x_2 \cdots x_k | k \geq 0 \text{ and } \forall i\, x_i \in A\}$

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- $A^* = \{\varepsilon, 0, 1, 00, 01, , 10, 11, 000, \ldots\}$

Details on Union

Given to finite automata

$$M_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1) \textit{ and } M_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$$

their union is constructed as follows:

- $Q = Q_1 \times Q_2$

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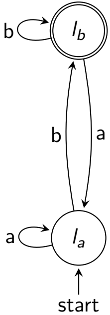
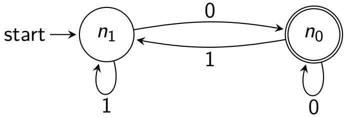
- $q_0 = (q_1, q_2)$
- $F = \{(r_1, r_2) | r_1 \in F_1 \; or \; r_2 \in F_2\}$

(Note $r_1 \in F_1 \; and \; r_2 \in F_2$ would be intersection)

Union Example





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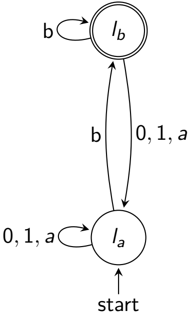
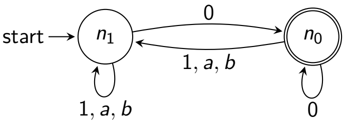
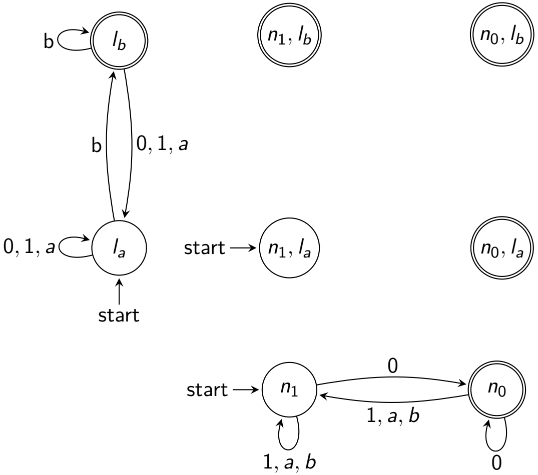
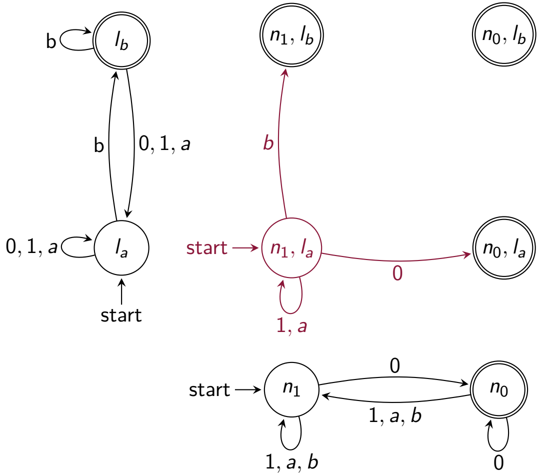




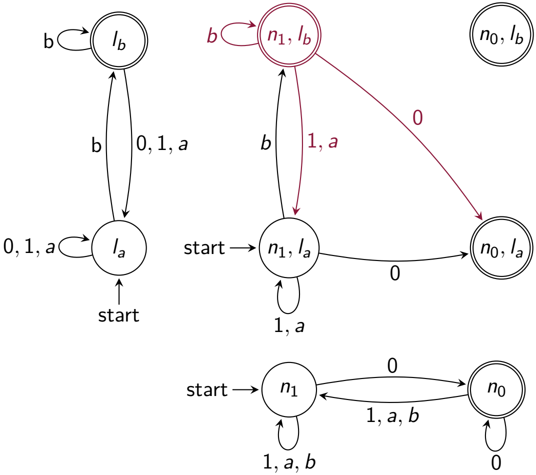
Union Example



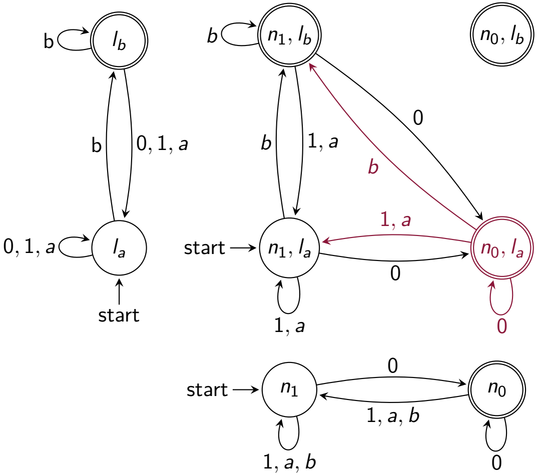
Union Example



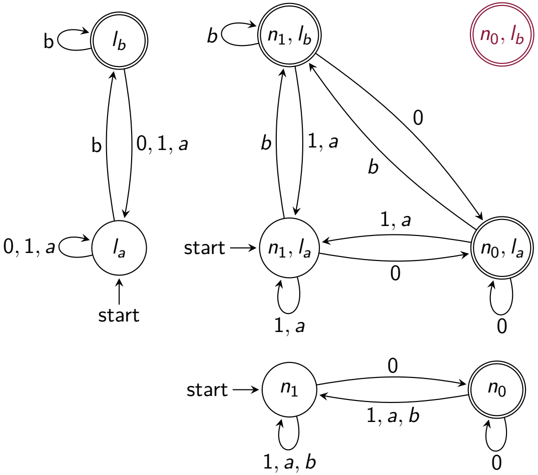
Union Example



Union Example



Union Example



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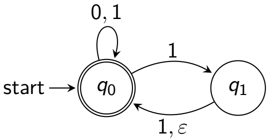
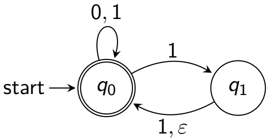
Non-Deterministic Automata



δ	0	1	ε
q_0	$\{q_0\}$	$\{q_0, q_1\}$	$\varnothing$
q_1	$\varnothing$	$\{q_0\}$	$\{q_0\}$

$$w = 010$$

Non-Deterministic Automata



δ	0	1	ε
q_0	$\{q_0\}$	$\{q_0, q_1\}$	$\varnothing$
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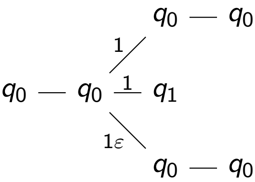
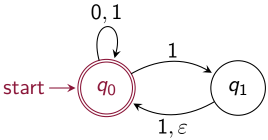
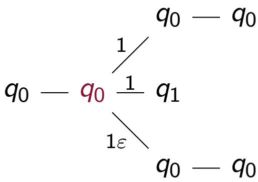


$w = 010$

Non-Deterministic Automata



δ	0	1	ε
q_0	$\{q_0\}$	$\{q_0, q_1\}$	$\varnothing$
q_1	$\varnothing$	$\{q_0\}$	$\{q_0\}$



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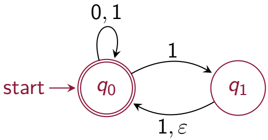

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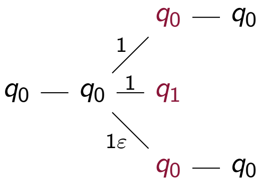

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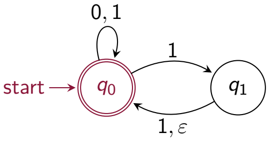

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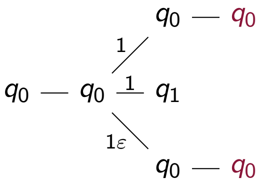
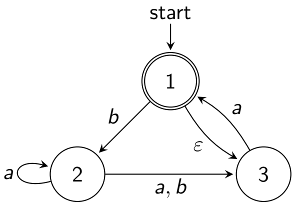


$w = 010$

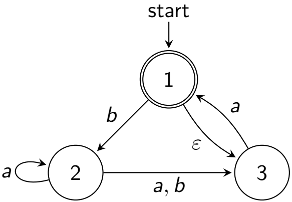
Equivalence to Deterministic (Slide 1)



δ	a	b	ε
1	$\varnothing$	$\{2\}$	$\{3\}$
2	$\{2,3\}$	$\{3\}$	$\varnothing$
3	$\{1\}$	$\varnothing$	$\varnothing$

- $Q' = \mathscr{P}(Q)$ (Power set of Q)
- $\Sigma' = \{a, b\}$
- $E(\{1\}) = \{1, 3\}$
- $Start = \{1, 3\}$
- $F' = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$
- $\delta' = ?$

Equivalence to Deterministic (Slide 1)



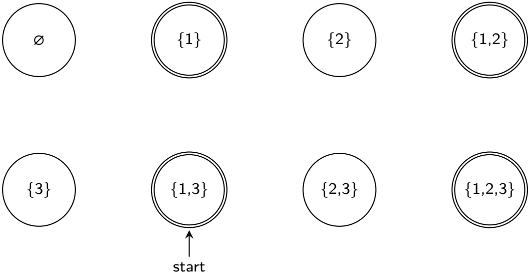
δ	a	b	ε
1	$\varnothing$	$\{2\}$	$\{3\}$
2	$\{2,3\}$	$\{3\}$	$\varnothing$
3	$\{1\}$	$\varnothing$	$\varnothing$

- $Q = \{1, 2, 3\}$
- $\Sigma_\varepsilon = \{a, b, \varepsilon\}$
- $Start = \{1\}$
- $F = \{1\}$

- $Q' = \mathscr{P}(Q)$ (Power set of Q)
- $\Sigma' = \{a, b\}$
- $E(\{1\}) = \{1, 3\}$
- $Start = \{1, 3\}$
- $F' = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$
- $\delta' = ?$

Equivalence to Deterministic (Slide 2)

δ	a	b	ε
1	$\varnothing$	$\{2\}$	$\{3\}$
2	$\{2,3\}$	$\{3\}$	$\varnothing$
3	$\{1\}$	$\varnothing$	$\varnothing$



Equivalence to Deterministic (Slide 2)

δ	a	b	ε
1	$\varnothing$	$\{2\}$	$\{3\}$
2	$\{2,3\}$	$\{3\}$	$\varnothing$
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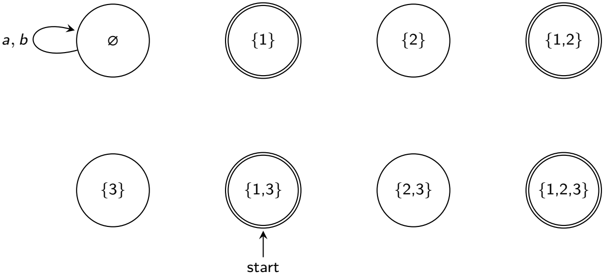

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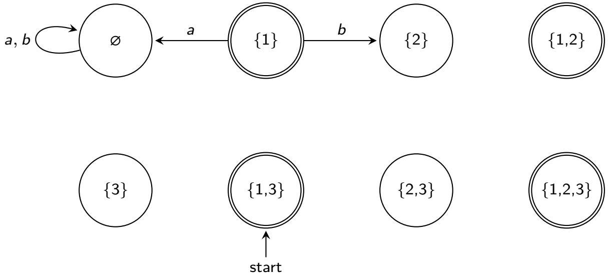

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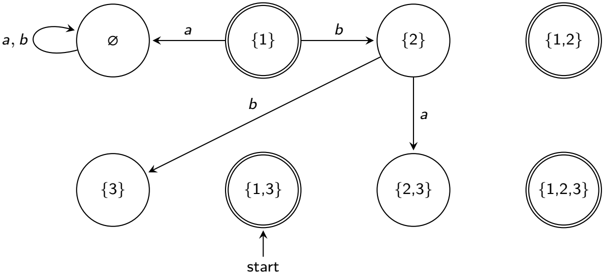

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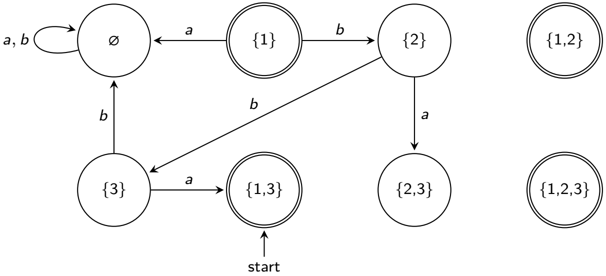

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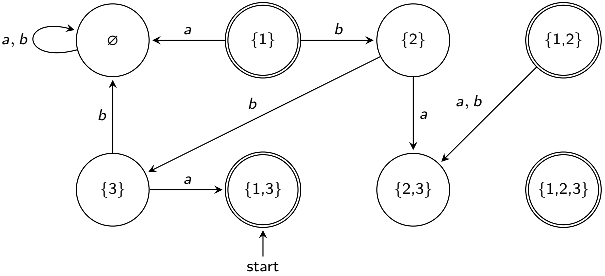

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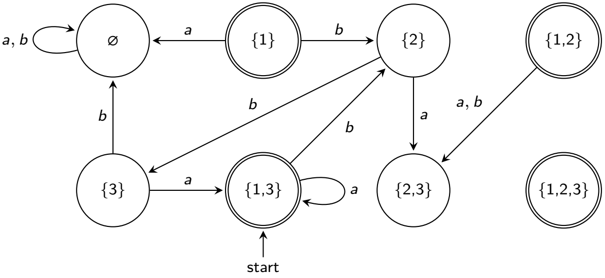

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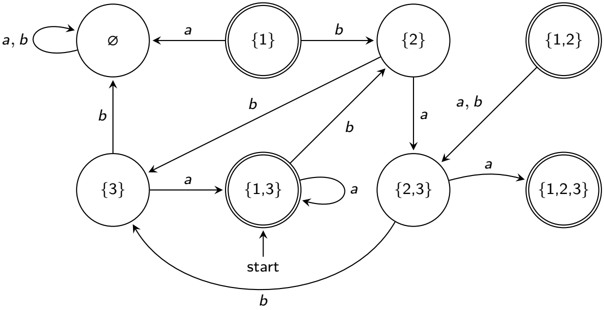

Equivalence to Deterministic (Slide 2)

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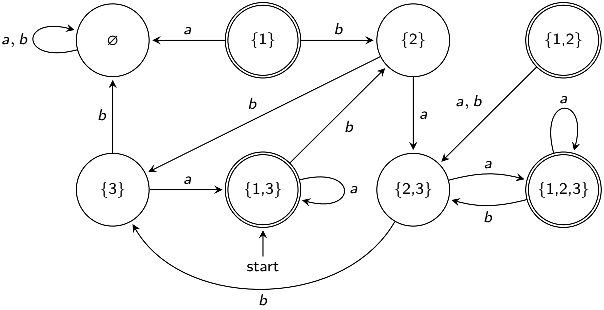

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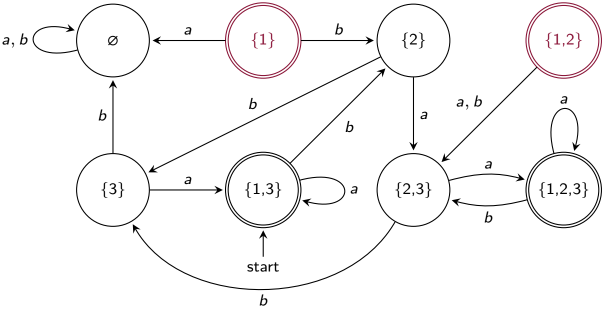

We can eliminate states that are only "sources."

Equivalence to Deterministic (Slide 2)

δ	a	b	ε
1	$\varnothing$	$\{2\}$	$\{3\}$
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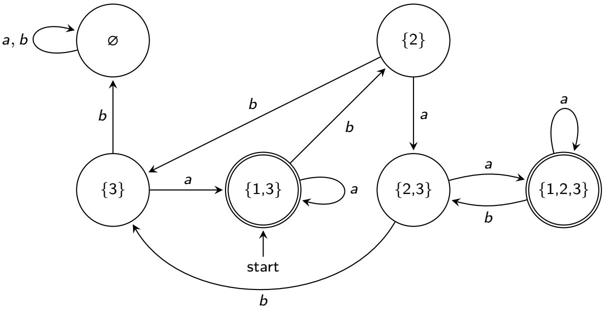
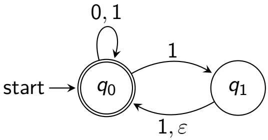


We can eliminate states that are only "sources."

Non-Deterministic Automata



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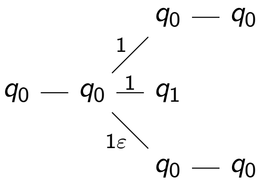

$w = 010$

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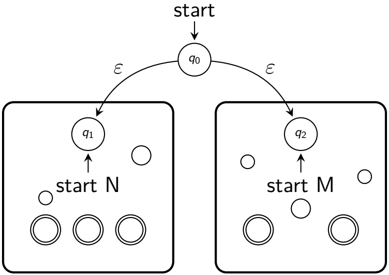
Union Revisited: $A \cup B$

Assume that the regular language A is represented by the finite automaton N and the regular language B is represented by the finite automaton M, then their **union** can be represented as below.



$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \\ \delta_2(q,a) & q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \wedge a = \varepsilon \\ \varnothing & q = q_0 \wedge a \neq \varepsilon \end{cases}$$

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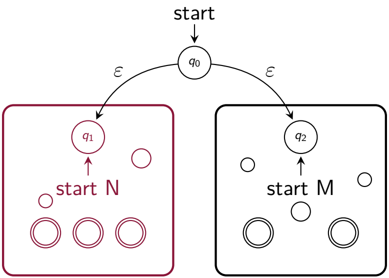

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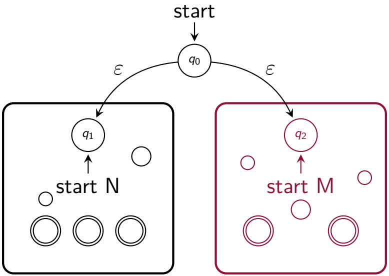

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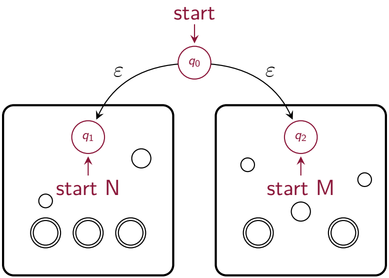

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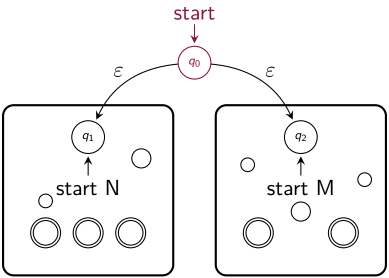

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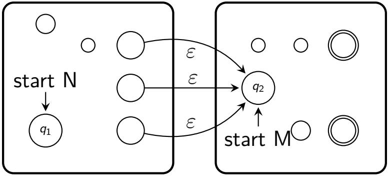
Concatenation: $A \circ B$

Assume that the regular language A is represented by the finite automaton N and the regular language B is represented by the finite automaton M, then their **concatenation** can be represented as below.



$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \wedge q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \wedge a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_2\} & q \in F_1 \wedge a = \varepsilon \\ \delta_2(q,a) & q \in Q_2 \end{cases}$$

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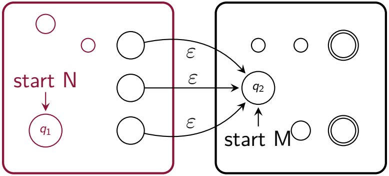

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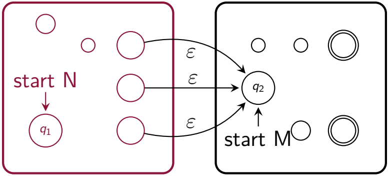

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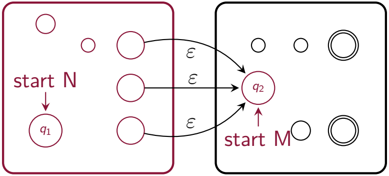

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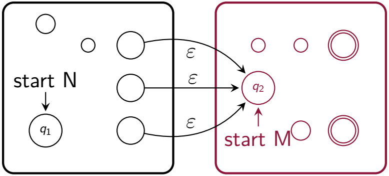

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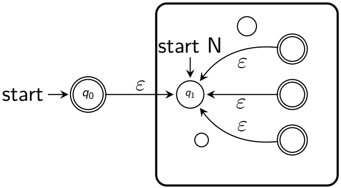
Star: A^*

Assume that the regular language A is represented by the finite automaton N then the unary **star, $*$,** operator applied to A can be represented as below.



$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \wedge a \notin F_1 \\ \delta_1(q,a) & q \in F_1 \wedge a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \wedge a = \varepsilon \\ \{q_1\} & q = q_0 \wedge a = \varepsilon \\ \varnothing & q = q_0 \wedge a \neq \varepsilon \end{cases}$$

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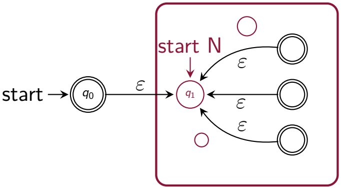

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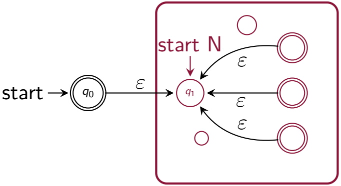

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \wedge a \notin F_1 \\ \delta_1(q,a) & q \in F_1 \wedge a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \wedge a = \varepsilon \\ \{q_1\} & q = q_0 \wedge a = \varepsilon \\ \emptyset & q = q_0 \wedge a \neq \varepsilon \end{cases}$$

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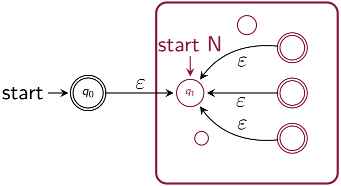

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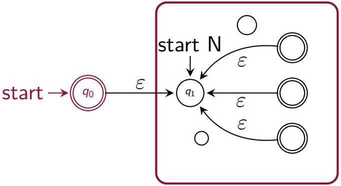

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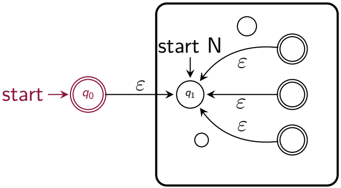

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Introduction to Finite Automata

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