Use these predicates to help describe figure 1.

- $C(x)$: x is a consonant
- $V(x)$: x is a vowel
- $Z(x)$: x is a zombie
- $S(x)$: x is a stick figure
- $N(x)$: x is a brain
- $R(x)$: x is ***red***
- $B(x)$: x is ***blue***
- $G(x)$: x is ***green***
- $P(x)$: x is a pentagon (5-sides)
- $H(x)$: x is a heptagon (7-sides)
- $W(x, y)$: x is in the same row as y
- $L(x, y)$: x is in the same column as y

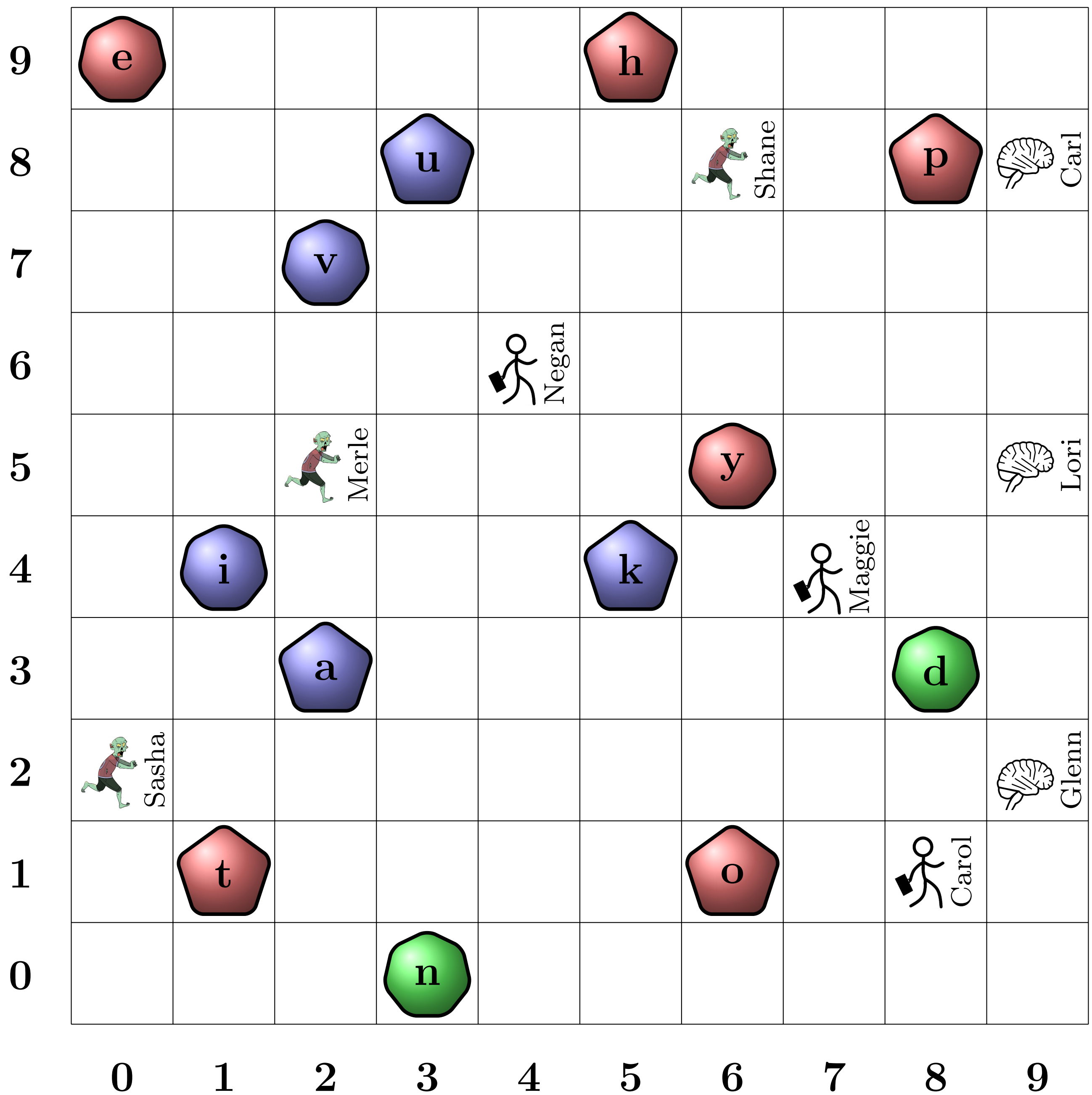

Figure 1: Tarski's World

Truth Sets

Given a predicate the ***truth set*** for that predicate is the collection of all elements where it is true. For example for $V(x) : x$ is a vowel, the truth set is

$$T_V = \{a, e, i, o, u, y\}$$

and for $P(x) : x$ is a pentagon the truth set is

$$T_P = \{t, a, u, h, k, o, p\}.$$

Figure 2 shows how these two sets are related, in particular we can see that

$$V(x) \wedge P(x) \textit{ and } V(x) \wedge \sim P(x) \textit{ and } \sim V(x) \wedge P(x)$$

are all true if we pick the right value of x.

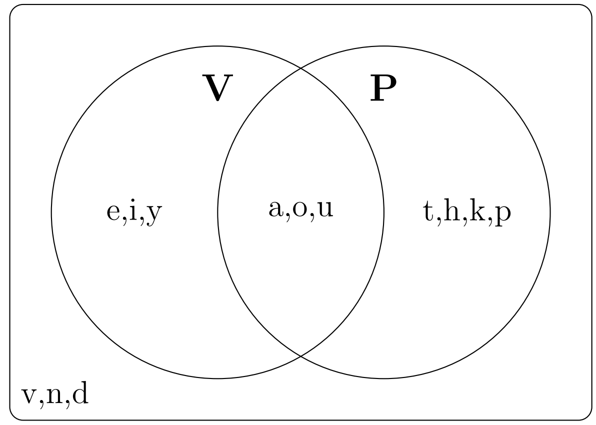

Figure 2: Venn Diagram of T_V and T_P

We can also write truth sets for the predicates like $W(x, y) : x$ is in the same row as y:

$$T_W = \{(e, h), (h, e), (u, p), (p, u), (i, k), (k, i), (a, d), (d, a), (t, o), (o, t)\}.$$

Fill in the ***truth set*** for each of the following predicates.

1. The truth set for $G(x)$ is

   $$T_G = \left\{ \qquad\qquad\qquad\qquad \right\}$$

2. The truth set for $H(x)$ is

   $$T_H = \left\{ \qquad\qquad\qquad\qquad \right\}$$

3. Fill in the Venn Diagram for each of the previous truth sets.

4. Where would the set T_Z fit in figure 3? What does that tell us about the statement

   $$\forall x : Z(x) \to G(x)?$$

5. The truth set for $L(x, y)$ is

   $$T_L = \left\{ \qquad\qquad\qquad\qquad\qquad\qquad \right\}$$

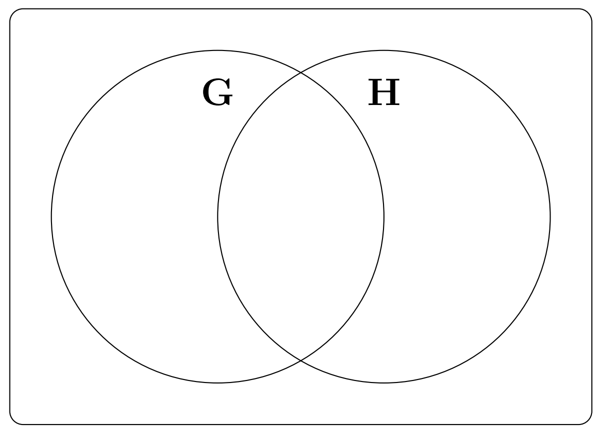

Figure 3: Venn Diagram of T_G and T_H

Quantified Statements

Recall that if we write

$$\exists x : V(x) \wedge P(x)$$

we can read that as *"there exists a vowel which is also a pentagon"* or in plainer English *"some vowels are pentagons."* (Is this true or false? How can we see it in the Venn Diagram in figure 2?) And if we write

$$\forall x : V(x) \vee P(x)$$

we read it as *"all the shapes are vowels or pentagons."* (Is this true or false? How can we see it in the Venn diagram in figure 2?) Recall that we call $\exists$ the ***existential quantifier*** and $\forall$ the ***universal quantifier***. In addition to using them one at a time we can combine them like this

$$\forall x\, \exists y : C(x) \to L(x, y)$$

which we read as *"every consonant shares a column with some letter,"* or like this

$$\exists y\, \forall x : C(x) \to L(x, y)$$

which we read as *"some letter shares a column with every consonant."* (Which of the previous two statements is true and which is false? How do we know?)

Which of the following predicates are true and which are false? Try to write each in plain English and negate the false predicates.

1. $\forall x : H(x) \to C(x)$

2. $\forall x\, \exists y : C(x) \to V(y) \wedge L(x, y)$

3. $\exists x : V(x) \wedge R(x)$

4. $\forall x\, \forall y : C(x) \wedge V(y) \to L(x, y) \vee W(x, y)$

5. $\forall x \in T_Z, \exists y \in T_N : W(x, y)$

6. $\exists y \in T_N, \forall x \in T_Z : W(x, y)$

Finally, look at figure 1 and construct three quantified statements of your own using the given predicates. Make two of them true and one a lie. Make at least one of them doubly quantified.

- Truth:

- Truth:

- Lie: