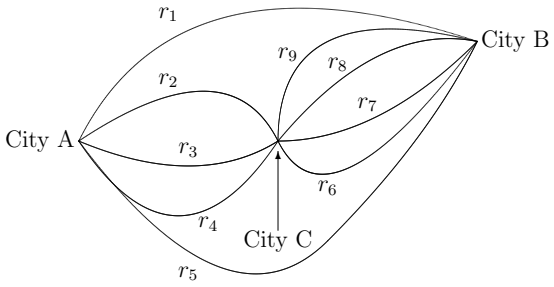
Instructions

Below are the practice exam problems which you must turn in when you come in to take the exam; these must be written up neatly or typed on separate paper and in accordance with the guidelines in your syllabus. Your grade will be based on you completing all the questions and on the quality of your work. In addition there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam.

Practice Exam Problems:

1. How do we know there are 30 positive two digit integers that are multiples of three?
2. Suppose that in a certain state all license plates consist of four letters followed by three digits.
   (a) How do we know there are 456,976,000 different license plates?
   (b) How do we know there are 1,757,600 plates that begin with A and end with 0?
   (c) How do we know there are 258,336,000 plates on which all the letters and digits are distinct?
3. Use the map below to answer the following:
   (a) How do we know there are 11 ways we can travel from *City A* to *City B*?
   (b) How do we know there are 9 ways we can travel from *City A* to *City B* with a stop in *City C*?



4. Suppose that a student council consists of 15 members, 8 men and 7 women.
   (a) Why are there 1960 ways to form a six person committee with 3 men and 3 women?
   (b) Why are there 4977 ways to form a six person committee with at least 1 women?
5. Why are there 151,200 distinguishable ways to arrange the letters of the word *HULLABALOO*?
6. If n is a positive integer, in how many ways can you pick 4 integers, i, j, k, m, so that $1 \leq i \leq j \leq k \leq m \leq n$? (Note that the integers can be equal.)

7. If n is a positive integer how many solutions are there to the equation

$$x_1 + x_2 + x_3 = n$$

if $x_i \geq 0\ \forall i$? What if $x_i \geq 1\ \forall i$? (Note that your answer will be in terms of n.)

8. How many cards must you select from a standard deck of 52 cards in order to guarantee that two of them are the same suit? For a 5 point bonus, how many cards must you select to have a better than 50% chance of getting the same suit twice?

9. In repeated divisions by 2373 how many distinct remainders can be obtained? If you were to write the decimal expansion of 173/2373 what is the longest possible length of the repeating section of the representation? (Don't actually calculate it!)

10. Show that in any set of thirteen integers chosen from 2 through 40 there must be at least two with a common divisor greater than 1.

11. Use the *Binomial Theorem* to expand $(p - 2q)^4$

12. Use the *Binomial Theorem* to find the coefficient for a^5b^7 in $(a - 2b)^{12}$

13. Use the *Binomial Theorem* to prove that for all integers $n \geq 0$,

$$3^n = \binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n}$$

14. (Challenge Problem worth 15 points) Find the coefficient of $x^3y^2z^5$ in the expansion of $(x + y + z)^{10}$.

Additional Practice Problems:

(listed by section and problem number)

§ 9.2: 1, 4, 6, 9, 11ab, 14abd, 16, 19, 21, 34;
§ 9.3: 3, 6, 11, 14;
§ 9.4: 1, 3, 9, 10, 14, 20, 24;
§ 9.5: 3, 6, 8, 15, 19;
§ 9.6: 3, 5, 10, 11;
§ 9.7: 1, 3, 5, 10, 19, 23, 25, 29, 31, 37;