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MAT 599 - 71: Multivariable Mathematics for Machine Learning
On Ground: Higgins 104, W 5:30-8pm
Credits: 3 credit
Grading: Standard A-F

## Office Hours:

Office hours are on ground for the Fall 2024 Semester. If you need to meet virtually we can make an appointment to do so via my WebEx Virtual Office:
Higgins 101-DV (https://wcsu.webex.com/meet/roccac)

- Monday & Thursday: 11am - 12pm & 3:30pm - 4:30pm
- Wednesday & Friday: 1pm - 2pm
- or by appointment

## Course Materials:

- *"Mathematics for Machine Learning 1st Edition"* by Deisenroth, Faisal, and Ong (ISBN-13: 978-1108455145). This text is freely available as a PDF at `https://mml-book.github.io/`.

## Course Description:

In this course students will extend their knowledge of Calculus and Linear Algebra in preparation for understanding and utilizing machine learning algorithms. Building on prior knowledge of Linear Algebra students will study inner products and norms beyond the basic dot product, orthogonal projections, matrix decompositions (in particular diagonalization and singular value decomposition) and be able to carry out linear regression. Their knowledge in Calculus will be extended to multiple dimensions allowing them to use derivatives to create approximations of higher dimensional functions, solve optimization problems analytically or through gradient descent, and solve constrained optimization problems with Lagrange multipliers. **Prerequisites:** MAT 272: Linear Algebra and MAT 182: Calculus II (or equivalents)

## Student Learning Outcomes:

After successful completion of this course students will be able to:

- find the eigenvalues and eigenvectors of a matrix and use them, where possible to diagonalize the matrix,
- use the Gram-Schmidt orthogonalization process to create an orthonormal basis,
- use orthogonal projection for dimensional reduction and linear regression,
- give the singular value decomposition of a matrix,
- evaluate partial derivatives and iterated integrals,
- explain and utilize gradient descent to find minimal values for a function or data set,
- use analytic techniques to find exact solutions to basic optimization problems, and
- use Lagrange multipliers to find solutions to constrained optimization problems.

## Course Content:

| *Topic* | *Chapters* |
|---|---|
| Linear Algebra: Review and Applications | Chapters 2 - 4 |
| Multivariable Calculus | Chapters 5 and 7 |

## Grading:

- Four Assignments 28% (7% Each)
- Two Unit Exams 46% (23% Each)
- Final Exam 26%

***Assignments (28%):*** Assignments will be a mix of problems from the text as well as some that I write up for you. They will be more difficult than the exercises that you will see on the exams; both computationally and conceptually difficult. You will have two assignments per unit for a total of four. Each assignment will be worth 7% of your grade. ***As out of class work assignments must be typed, in complete sentences, and in your own words.***

***Unit Exams (46%):*** You will have two unit exams, each worth 23% of your grade, with exercises focused on knowledge and skills. These will be comprehensive exams covering all the content in a unit. For these exams, you will be allowed to redo some of the questions you get wrong in order to regain up to 33% of the points you lost. ***Since redos are out of class work, they must be typed, in complete sentences, and in your own words.***

***Final Exam (26%):*** You will have a final exam during final exam week, as with the other exams, this will focus on knowledge and skills. This will be a comprehensive exam covering all the content from the semester, it is 26% of your grade.

## Course Calendar:

| Wednesday | |
|---|---|
| 8/28<br>Syllabus, Vector and Matrix Calculations | **1** |
| 9/4<br>Basis and Change of Basis/Coordinates Matrices | **2** |
| 9/11<br>Eigen - values, vectors & spaces, Diagonalization | **3** |
| 9/18<br>Inner Products, Norms, Orthogonalization, and Least-Squares | **4** |
| 9/25<br>Symmetric Matrices, Quadratic Forms, and Constrained Optimization with Matrices | **5** |
| 10/2<br>Symmetric Matrices and Singular Value Decomposition | **6** |
| 10/9<br>Review and the Unit I Exam on Linear Algebra | **7** |
| 10/16<br>Review of Calculus I & II | **8** |
| 10/23<br>Extending Derivatives to Higher Dimensions | **9** |
| 10/30<br>Optimization with Analytic Techniques and Lagrange Multipliers | **10** |
| 11/6<br>Directional Derivatives, Gradients, Optimization by Gradient Descent | **11** |
| 11/13<br>Higher Order Integrals, Jacobians and Change of Variables | **12** |
| 11/20<br>Hessians and Extensions of Taylor Series | **13** |
| 11/27<br>***Thanksgiving Break - No Class*** | |
| 12/4<br>Review and the Unit II Exam on Multivariable Calculus | **14** |
| 12/11<br>Final Exam | **15** |

## Course Outline:

Review and Extension of Linear Algebra

1. Basic Matrix Operations
   (a) Addition, Multiplication, Transpose
   (b) Determinants
   (c) Trace
2. Vectors
   (a) Geometric and Numerical Representations
   (b) Elementary Operations with Vectors
   (c) Dot Product and Orthogonality
   (d) Vector Projections
3. Bases for a Vector Space
   (a) Definition of Bases and Linear Independence
   (b) Dimension of a Vector Space
   (c) Change of Bases Matrices
4. Eigenvalues and Eigenvectors
   (a) Finding Eigenvalues and Eigenvectors
   (b) Diagonalizing Matrices
   (c) Eigenspaces
5. Orthonormal Bases
   (a) Definition
   (b) Orthogonal Complements
   (c) Gram-Schmidt Orthogonalization
   (d) Least-Squares Regression
6. Symmetric Matrices
   (a) Definition and Characteristics
   (b) Quadratic Forms
   (c) Singular Value Decomposition
   (d) Constrained Optimization with Matrices
7. Inner Products
   (a) Definition of an Inner Product
   (b) Dot Product as an Inner Product
   (c) Inner Products which are not the Dot Product
   (d) Inner Products, Norms, and Orthogonality

Multivariable Calculus

1. Functions of Several Variables
   (a) 3-Dimensional Graphs
   (b) Contour Diagrams I
2. Partial Differentiation
   (a) Review of Differentiation and Extension to Multiple Dimensions
   (b) Finding Partial Derivatives
   (c) The Chain Rule
   (d) Linearization and Taylor Series
   (e) Directional Derivatives
   (f) The Gradient and Paths of Steepest Accent
   (g) The Jacobian Matrix
   (h) The Hessian Matrix
3. Optimization
   (a) Analytic Techniques and Exact Solutions
   (b) Constrained Optimization with Lagrange Multipliers
   (c) Using Gradient Descent to Find Minimal Values
4. Integration
   (a) Integrating Over Multiple Variables
   (b) Determinant of the Jacobian and a Change of Variables

## You and Your Grades:

- "A" (Exceptional) range 90% to 100%:
  The student has demonstrated significant mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and most nonstandard problems which require deeper insight.
  - "A" $\Longleftrightarrow 92.5\% \leq Grade \leq 100\%$
  - "A-" $\Longleftrightarrow 90\% \leq Grade < 92.5\%$
- "B" (Good) range 80% to 90%:
  The student has demonstrated mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and some nonstandard problems which require deeper insight.
  - "B+" $\Longleftrightarrow 87.5\% \leq Grade < 90\%$
  - "B" $\Longleftrightarrow 82.5\% \leq Grade < 87.5\%$
  - "B-" $\Longleftrightarrow 80\% \leq Grade < 82.5\%$
- "C" (Adequate) range 70% to 80%:
  The student has demonstrated adequate mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve most standard formulaic exercises but struggles with nonstandard problems which require deeper insight.
  - "C+" $\Longleftrightarrow 77.5\% \leq Grade < 80\%$
  - "C" $\Longleftrightarrow 72.5\% \leq Grade < 77.5\%$
  - "C-" $\Longleftrightarrow 70\% \leq Grade < 72.5\%$
- "D" (Inadequate) range 60% to 70%:
  The student has demonstrated inadequate or incomplete mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve some standard formulaic exercises but few if any nonstandard problems which require deeper insight.
  - "D+" $\Longleftrightarrow 67.5\% \leq Grade < 70\%$
  - "D" $\Longleftrightarrow 62.5\% \leq Grade < 67.5\%$
  - "D-" $\Longleftrightarrow 60\% \leq Grade < 62.5\%$
- "F" (Unacceptable) below 60%:
  The student has demonstrated essentially no mastery of the appropriate knowledge and skills relevant to the course. The student is unable to solve most standard formulaic exercises and essentially no nonstandard problems which require deeper insight.

## End User Agreement:

**General Expectations:** As a student in this class you are expected to:

- attend class and take notes,
- actively read material in each section, taking notes,
- review your notes on a regular basis,
- check your university email every day,
- check the class site ***at least*** every other day,
- begin studying for exams in a timely fashion,
- ask questions early and often,
- attend office hours,
- seek help in the math clinic or tutoring center, and
- complete assignments and readings on time.

**Assignment Guidelines:** (These apply to ***all out of class work***.)

- Work handed in must always look neat, legible, and professional. Work must be very neatly written or preferably typed. The quality of your work will be factored into your grade, up to 10%, in extreme cases work may be rejected and then counted as late.
- Answers on all assignments should be given in complete sentences. I should be able to tell what your answer means without re-reading the problem. This does not mean you simply rewrite the question.
- An assignment is considered late after I have handed it back or gone over it in class. Late assignments are accepted but may receive at most 75% credit. Late assignments go to the absolute bottom of the stack of papers to be graded; ***all on time work is graded before any late work.***
- If you work on an assignment as part of a group, then there may be no more than three individuals in the group and all your names must be on the assignment. You should hand in only one copy of the work.
- All work must be submitted in the manner directed.

**Email Etiquette Guidelines:** When sending an email you must include the course number and semester in the subject line. For example, if you are taking MAT 314 in Fall 1592 then the the subject line should begin with "[MAT 314 Fall 1592]." Also, you should always begin with a salutation such as "Dear Dr. Rocca" and end with a closing such as "Sincerely, I. Newton."

**Exam Makeup Policy:** To qualify for a makeup exam you must have a valid reason for missing the exam and, if at all possible, let me know ahead of time that you are missing the exam. You will need to meet with me in order to arrange a time for the make up exam. If you do not have a valid reason, do not give prior notice when possible, or simply do not show up for an exam, you are not entitled to a makeup and will not be given one. If you fail to show up for your makeup exam, you will not be given a second opportunity.

**The 2% Exception:** Any quiz or class work which is ultimately worth no more then 2% of your final grade can not be made up.

**Time on Task:** As a 3 credit class you should expect to average 7.5 to 8.5 hours of work a week including class time. Some weeks you may get away with less and some may require more.

**Attendance:** There is no specific policy for attendance in this course. However, if you have ***three consecutive unexcused absences*** within the first half of the semester I am required to report to the University that you have ***stopped attending***.

**Academic Honesty:** If on any assignment, quiz, or exam you turn in someone else's work, regardless of the source, as if it were your own you will receive a zero on that assignment, quiz, or exam. If you are caught doing this three times you will receive an F in the course and the Dean will be informed of your academic dishonesty. (https://www.wcsu.edu/faculty-handbook/2019-2020/policies-pertaining-to-students/academic-honesty-policy/)

**Accommodations:** If you have need of an accommodation for testing or note taking, please visit AccessAbility Services, located in the HAAS Library room 406 (http://www.wcsu.edu/accessability).