Dr. Charles Rocca
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MAT 207-01: Proofs
MR: 12:30 pm - 1:45 pm,
Higgins 228

WESTERN CONNECTICUT STATE UNIVERSITY
MACRICOSTAS SCHOOL OF ARTS & SCIENCES

Office Hours:

Office hours are on ground for the Fall 2024 Semester. If you need to meet virtually we can make an appointment to do so via my WebEx Virtual Office: Higgins 101-DV (https://wcsu.webex.com/meet/roccac)

- Monday & Thursday: 11am - 12pm & 3:30pm - 4:30pm
- Wednesday & Friday: 1pm - 2pm
- or by appointment

Course Materials:

- Required Text: *Number Theory Through Inquiry* by Marshall, Odell, & Starbird (ISBN: 978-1-4704-6159-1)
- Recommenced Text: *How to Think Like a Mathematician* by Houston (ISBN 978-0521719780)

Course Description and Objectives:

An introduction to the theory and practice of reading and writing mathematical proofs, using theorems and problems in number theory as a source of examples. Prerequisite: C or better in MAT 141: Foundational Discrete Mathematics

1. Successful students in this course will be able to read and write well-organized and correct proofs of mathematical statements. In particular, students will be able to:
   (a) recognize reasoning and proof as fundamental aspects of mathematics
   (b) make and investigate mathematical conjectures
   (c) develop and evaluate mathematical arguments and proofs
   (d) select and use various types of reasoning and methods of proof
2. Students will demonstrate knowledge of introductory number theory. In particular, students will be able to demonstrate an understanding of:
   (a) the fundamental properties of the integers, in particular primes
   (b) modular arithmetic
   (c) significant number theoretic functions

Course Content:

Unit	**Chapters**
Reading, Writing, and Analyzing Math	Supplemental Material
Proof Types: Direct, Indirect, & Induction	Chap. 1 & 2, Appendix A, & Supplemental Material
Modular Arithmetic	Chapters 4, 5, & 6

Grading:

	Option 1	Option 2
Various Assignments	20%	18%
Reading, Writing, and Analyzing Exam	20%	18%
Proofs Exam	20%	18%
Number Theory Exam	20%	18%
Proofs Portfolio	20%	18%
Class Participation	0%	10% (using write ups or note checks)

Assignments: Throughout the semester you will be given assignments to help you practice the techniques we learn in class, some of these we will begin during class. You may revise each assignment up to two times making corrections in order to raise your grade, your best attempt is the one which will count in the grade book. Most assignments may be done as part of a group, however your group can not have more than three people in it and if you complete an assignment as part of a group then you must only turn in one copy of the assignment. ***Final submissions for all assignments must be typed and be in complete sentences unless very specifically told otherwise; 10% of the grade is for quality.***

Reading, Writing, and Analyzing Exam: On this exams you will need to demonstrate a basic ability to read and write mathematics, and to understand logical statements. To do this you will be asked to

- demonstrate how to read and familiarize yourself with a new concept,
- rewrite or reformat some statements,
- proofread and when necessary correct samples of writing, and
- solve some basic problems and write up their solutions adhering to standard conventions.

Proofs Exams: This will be a two part exam. On part one you will be given an example or examples of proof techniques and you will need to fill in details which have been left out or reformat the proof. For the second part of the exam you will be given a theorem statement and you will need to construct a proof of that theorem from scratch using the specified technique.

Number Theory Exam: This exam will cover fundamental topics from Number Theory. The exact content will depend on what we can cover in class. Generally we will try to cover:

- ☐ Divisibility
- ☐ Well Ordering Principle
- ☐ Division Algorithm
- ☐ Greatest Common Divisor
- ☐ Bezout's Lemma
- ☐ Euclidean Algorithm
- ☐ Least Common Multiple
- ☐ Fund. Theorem of Arithmetic
- ☐ Modular Arithmetic
- ☐ Chinese Remainder Theorem
- ☐ Euler's ϕ - Function
- ☐ Wilson's Theorem
- ☐ Fermat's Little Theorem
- ☐ Euler's Theorem
- ☐ RSA Encryption[1]

This exam will be given during final exam week.

After each exam you will be allowed to hand in corrections in order to earn back up to 33% of the points lost on the exam. ***Corrections must be typed and be in complete sentences; 10% of the grade is for quality.***

[1] Coverage will depend on how well we progress through the other material.

Proofs Portfolio: At the end of the semester you must compile an example of each type of proof which you have written and revised. Thus you will need to have examples of a:

1. direct proof,
2. proof by cases,
3. proof by contradiction,
4. proof by contrapositive, and
5. proof by induction.

For each type of proof you must turn in rough work, all drafts of the proofs from earlier in the semester, and a final draft of the proof. This means you need to hold onto all of your work throughout the semester!!! ***The portfolio of should be "bound" in a report folder and, except for scrap work, must be typed in complete sentences; 10% of the grade is for quality.***

Course Calendar:

Monday	Thursday
8/26	8/29 **1** Syl., Intro., & Building Up Ideas
9/2 Labor Day - No Class	9/5 **2** Reading Mathematics
9/9 **3** Writing Mathematics	9/12 **4** Proofreading Mathematics, the Rewrite
9/16 **5** Solving Problems	9/19 **6** Reading Definitions and Theorems
9/23 **7** What is a Proof and How to Read It	9/26 **8** Review Reading, Writing, and Analyzing Mathematics
9/30 **9** ***Reading, Writing, and Analyzing Exam***	10/3 **10** Statements, Implications, and Their Kin
10/7 **11** Divisibility, Non-Divisibility, and Congruences (Direct and Contradiction, pp.7-10)	10/10 **12** Congruences and Modular Arithmetic (Direct, Induction, pp.10-14)
10/14 **13** Well Ordering, Division Alg., Evens, & Odds (Direct, pp.14-16)	10/17 **14** GCD, Euclidean Alg., Bezout's Lemma, and LCM (Various, pp. 16-23)
10/21 **15** GCD, Euclidean Alg., Bezout's Lemma, and LCM (Various, pp. 16-23)	10/24 **16** Primes, Powers, and Divisibility (Contrapositive and Induction)

Monday	Thursday
10/28 **17** Primes and the Fundamental Theorem of Arithmetic (Strong Induction, pp.28-31)	10/31 **18** An Infinity of Primes; From n to $n!$ (Contradiction-ish, pp.35-37)
11/4 **19** Clean up and Review	11/7 **20** ***Proofs Exam***
11/11 **21** Solving Linear Congruences (pp.43-49)	11/14 **22** Chinese Remainder Theorem (pp.50-51)
11/18 **23** Fermat's Theorem (pp.53-57)	11/21 **24** Euler's φ - Function and Theorem (pp.59-60)
11/25 **25** Some more Euler and Wilson's Theorem (pp.61-62)	11/28 Thanksgiving Break - No Class
12/2 **26** Applying the CRT and Euler's Theorem with RSA (pp.65-69)	12/5 **27** Applying the CRT and Euler's Theorem with RSA (pp.65-69)
12/9	12/12 **28** ***Number Theory Exam, 11:00am - 1:30pm***

Departmental Course Outline:

1. Proof Topics
   (a) Getting Started:
      i. Reading the statement of a theorem
      ii. Quantification
      iii. Getting from the statement to the start of the proof
   (b) Types of Proofs:
      i. Direct proof
      ii. Examination of cases
      iii. Proof by contradiction
      iv. Proof by contrapositive
      v. Equivalence (or if and only if) proofs
      vi. Mathematical induction and the well-ordering principle (and infinite descent)
   (c) Other Significant Topics:
      i. Negation and DeMorgan's law
      ii. Contrapositive
      iii. Contrapositive vs. Converse
      iv. Mathematical experiments â€“ how do you know what to try to prove?
2. Number Theory Topics:
   (a) Primes:
      i. Properties of prime numbers
      ii. Prime factorization
      iii. Gaussian primes
   (b) Divisibility:
      i. gcd
      ii. Euclidean algorithm
      iii. lcm
   (c) Properties of Integers:
      i. Sums of squares
      ii. Figurate numbers
      iii. Pythagorean triples
   (d) Modular arithmetic and algebra
   (e) Number theoretic functions

The content in this course addresses the *Expertise in Content Knowledge* portion of the WCSU Education Program Conceptual Framework. When you have successfully completed this course you will have added to the body of knowledge necessary to teach mathematics in Connecticut public schools.

You and Your Grades:

- "A" (Exceptional) range 90% to 100%:
  The student has demonstrated significant mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and most nonstandard problems which require deeper insight.

  - "A" $\Longleftrightarrow 92.5\% \leq Grade \leq 100\%$
  - "A-" $\Longleftrightarrow 90\% \leq Grade < 92.5\%$

- "B" (Good) range 80% to 90%:
  The student has demonstrated mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and some nonstandard problems which require deeper insight.

  - "B+"$\Longleftrightarrow 87.5\% \leq Grade < 90\%$
  - "B" $\Longleftrightarrow 82.5\% \leq Grade < 87.5\%$
  - "B-"$\Longleftrightarrow 80\% \leq Grade < 82.5\%$

- "C" (Adequate) range 70% to 80%:
  The student has demonstrated adequate mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve most standard formulaic exercises but struggles with nonstandard problems which require deeper insight.

  - "C+"$\Longleftrightarrow 77.5\% \leq Grade < 80\%$
  - "C"$\Longleftrightarrow 72.5\% \leq Grade < 77.5\%$
  - "C-"$\Longleftrightarrow 70\% \leq Grade < 72.5\%$

- "D" (Inadequate) range 60% to 70%:
  The student has demonstrated inadequate or incomplete mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve some standard formulaic exercises but few if any nonstandard problems which require deeper insight.

  - "D+"$\Longleftrightarrow 67.5\% \leq Grade < 70\%$
  - "D" $\Longleftrightarrow 62.5\% \leq Grade < 67.5\%$
  - "D-" $\Longleftrightarrow 60\% \leq Grade < 62.5\%$

- "F" (Unacceptable) below 60%:
  The student has demonstrated essentially no mastery of the appropriate knowledge and skills relevant to the course. The student is unable to solve most standard formulaic exercises and essentially no nonstandard problems which require deeper insight.

End User Agreement:

General Expectations: As a student in this class you are expected to:

- attend class and take notes,
- actively read material in each section, taking notes,
- review your notes on a regular basis,
- check your university email every day,
- check the class site ***at least*** every other day,
- begin studying for exams in a timely fashion,
- ask questions early and often,
- attend office hours,
- seek help in the math clinic or tutoring center, and
- complete assignments and readings on time.

Assignment Guidelines: (These apply to ***all out of class work***.)

- Work handed in must always look neat, legible, and professional. Work must be very neatly written or preferably typed. The quality of your work will be factored into your grade, up to 10%. In extreme cases work may be rejected and then counted as late.
- Answers on all assignments should be given in complete sentences. I should be able to tell what your answer means without re-reading the problem. This does not mean you simply rewrite the question.
- An assignment is considered late after I have handed it back or gone over it in class. Late assignments are accepted but may receive at most 75% credit. Late assignments go to the absolute bottom of the stack of papers to be graded; ***all on time work is graded before any late work.***
- If you work on an assignment as part of a group, then there may be no more than three individuals in the group and all your names must be on the assignment. You should hand in only one copy of the work.
- All work must be submitted in the manner directed.

Email Etiquette Guidelines: When sending an email you must include the course number and semester in the subject line. For example, if you are taking MAT 314 in Fall 1592 then the the subject line should begin with "[MAT 314 Fall 1592]." Also, you should always begin with a salutation such as "Dear Dr. Rocca" and end with a closing such as "Sincerely, I. Newton."

Exam Makeup Policy: To qualify for a makeup exam you must have a valid reason for missing the exam and, if at all possible, let me know ahead of time that you are missing the exam. You will need to meet with me in order to arrange a time for the make up exam. If you do not have a valid reason, do not give prior notice when possible, or simply do not show up for an exam, you are not entitled to a makeup and will not be given one. If you fail to show up for your makeup exam, you will not be given a second opportunity.

The 2% Exception: If a class has any quiz or class work which is ultimately worth no more then 2% of your final grade can not be made up.

Time on Task: As a 3 credit class you should expect to average 7.5 to 8.5 hours of work a week including class time. Some weeks you may get away with less and some may require more.

Attendance: There is no specific policy for attendance in this course. However, if you have ***three consecutive unexcused absences*** within the first half of the semester I am required to report to the University that you have ***stopped attending***.

Academic Honesty: If on any assignment, quiz, or exam you turn in someone else's work, regardless of the source, as if it were your own you will receive a zero on that assignment, quiz, or exam. If you are caught doing this three times you will receive an F in the course and the Dean will be informed of your academic dishonesty.
(https://www.wcsu.edu/faculty-handbook/2019-2020/policies-pertaining-to-students/academic-honesty-policy/)

Accommodations: If you have need of an accommodation for testing or note taking, please visit AccessAbility Services, located in the HAAS Library room 406 (http://www.wcsu.edu/accessability).