

# Mathematics and Games

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- 1 The Games
- 2 The Math
- 3 The Resources
- 4 Catanbinatorics
- 5 Catan and Probabilities
- 6 Dice Handicap
- 7 Dice and Combinatorics



# Sample of Games Written About

- Chutes and Ladders
- Monopoly
- Risk
- Settlers of Catan
- Mancala
- Rock-Paper-Scissors-Lizard-Spock
- CMJ Puzzles and Games Issues



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# Combinatorics

- Permutations and Combinations
- Combinations with Replacement
- Multinomial Distributions
- Inclusion-Exclusion Principle



# Probability

- Binomial and Multinomial Probabilities
- Expected Values and Variance
- Odds and Probabilities
- Conditional Probabilities



# Others (Mostly Linear Algebra)

- Paths and Graph Theory
- Adjacency Matrices
- State Vectors
- Eigenvectors and Eigenvalues
- Markov Chains



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# Articles

- [1] Jathan Austin, Brian G. Kronenthal, and Susanna Molitoris Miller.  
The settlers of “catanbinatorics”.  
Mathematics Magazine, 92(3):187–198, 2019.
- [2] Jathan Austin and Susanna Molitoris-Miller.  
The settlers of catan: Using settlement placement strategies in the probability classroom.  
The College Mathematics Journal, 46(4):275–282, 2015.
- [3] B. R. Johnson.  
Dice handicap.  
Mathematics Magazine, 54(3):135–139, 1981.
- [4] Janet M. McShane and Michael I. Ratliff.  
Dice distributions using combinatorics, recursion, and generating functions.  
The College Mathematics Journal, 34(5):370–376, 2003.



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# Catanbinatorics

## The settlers of “Catanbinatorics”

Jathan Austin and Brian G. Kronenthal and Susanna Molitoris Miller

### Description

In this paper the authors look at the number of ways the game board in Catan can be set up. In particular they look at the number of ways in which number and resource token arrangements can be achieved.



# The Mathematics

## Math Covered

- Combinatorics



# The Mathematics

## Math Covered

- Combinatorics
- Binomial Coefficients



# The Mathematics

## Math Covered

- Combinatorics
- Binomial Coefficients
- Multinomial Coefficients



# The Mathematics

## Math Covered

- Combinatorics
- Binomial Coefficients
- Multinomial Coefficients
- Basic Dice Probabilities



# The Mathematics

## Math Covered

- Combinatorics
- Binomial Coefficients
- Multinomial Coefficients
- Basic Dice Probabilities
- Symmetry





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# Catan in the Probability Classroom

## The Settlers of Catan: Using Settlement Placement Strategies in the Probability Classroom

Jathan Austin and Susanna Molitoris-Miller

### Description

In this article the authors study the game Settlers of Catan. They use math to examine different Catan settlement placement strategies. The goal is to develop lessons centered on the game which are suitable for teaching basic concepts in probability and expected value.



# The Mathematics

## Math Covered

- Probability



# The Mathematics

## Math Covered

- Probability
- Expected Value



# The Mathematics

## Math Covered

- Probability
- Expected Value
- **Weighted Averages**



# The Mathematics

## Math Covered

- Probability
- Expected Value
- Weighted Averages
- Logical Strategies



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# Dice Handicap

## Dice Handicap B. R. Johnson

### Description

The author considers a two-player game in which Player A rolls  $n + 1$  fair  $r$ -sided dice and scores the sum of the highest  $n$  and Player B rolls  $n$  fair  $r$ -sided dice and scores the sum; the higher score wins, with ties being awarded to Player B. They then try to answer the questions:

- Who is the favorite?





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- Does the answer depend on  $n$  and/or  $r$ ?



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- Who is the favorite?
- Does the answer depend on  $n$  and/or  $r$ ?
- And, as a function of  $n$  and  $r$ , what is the probability that Player A will win?



# The Mathematics

## Math Covered

- Probabilities



# The Mathematics

## Math Covered

- Probabilities
- Independent Random Variables



# The Mathematics

## Math Covered

- Probabilities
- Independent Random Variables
- Disjoint Events



# The Mathematics

## Math Covered

- Probabilities
- Independent Random Variables
- Disjoint Events
- Expected Values and Variance



# The Mathematics

## Math Covered

- Probabilities
- Independent Random Variables
- Disjoint Events
- Expected Values and Variance
- Central Limit Theorem



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# Dice and Combinatorics

## Dice Distributions Using Combinatorics, Recursion, and Generating Functions

Janet M. McShane and Michael I. Ratliff

### Description

In this article the authors examine the questions:

- 1 Given five dice to toss, what is the probability of tossing a sum of 17?
- 2 Given  $n$  dice,  $n > 1$ , what is the probability of tossing a sum of  $k$ ?
- 3 And, if we have  $m$  dice and randomly select  $n$  of them ( $m > n > 1$ ) to toss, what is the probability of obtaining a sum of  $k$ ?

The first two are answered using three different approaches, each of which may be encountered in a typical undergraduate curriculum.



# The Mathematics

## Math Covered

- Combinations with Replacement



# The Mathematics

## Math Covered

- Combinations with Replacement
- Multiplication Principle



# The Mathematics

## Math Covered

- Combinations with Replacement
- Multiplication Principle
- Recursive Sequences



# The Mathematics

## Math Covered

- Combinations with Replacement
- Multiplication Principle
- Recursive Sequences
- Probability Generating Functions



# The Mathematics

## Math Covered

- Combinations with Replacement
- Multiplication Principle
- Recursive Sequences
- Probability Generating Functions
- Joint Probability Function



# The Mathematics

## Math Covered

- Combinations with Replacement
- Multiplication Principle
- Recursive Sequences
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- Joint Probability Function
- Expected Values and Variance



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