# Euclid and Axioms

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MACRICOSTAS SCHOOL OF ARTS & SCIENCES

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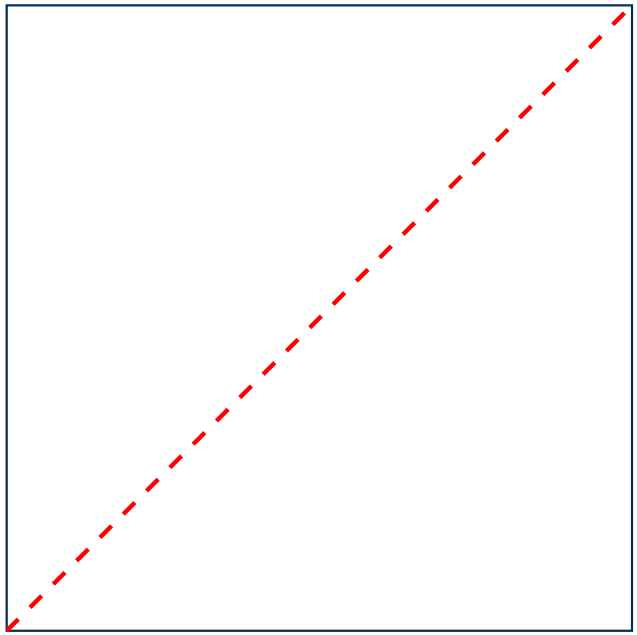

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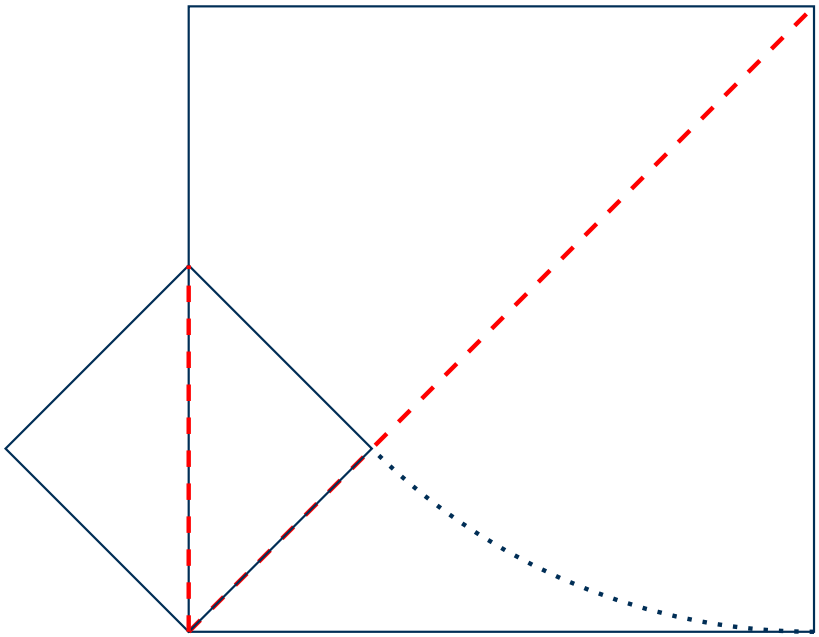

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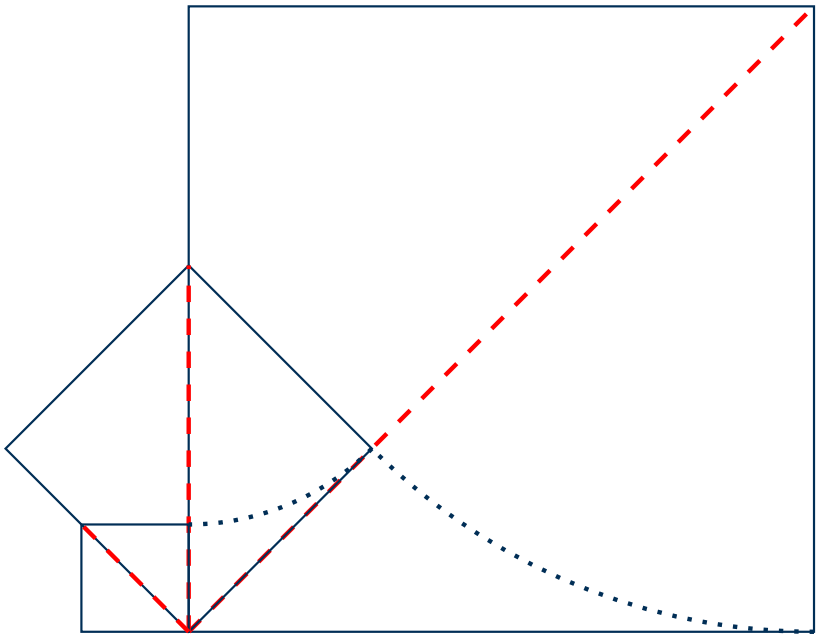

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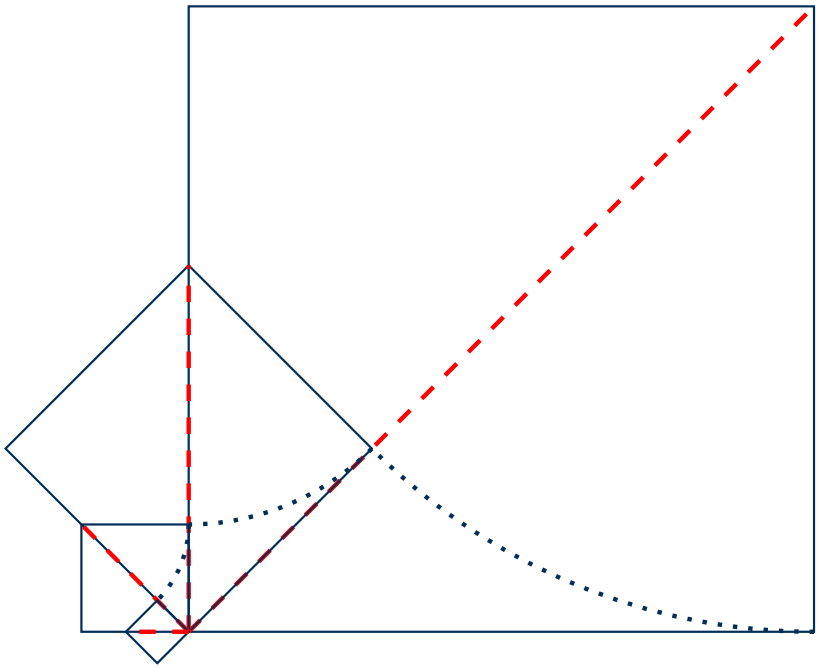

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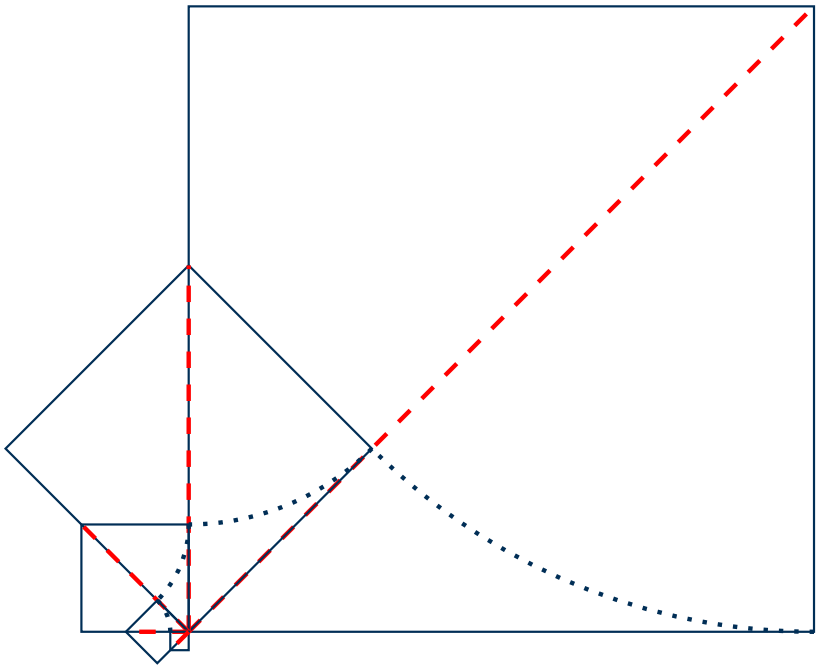

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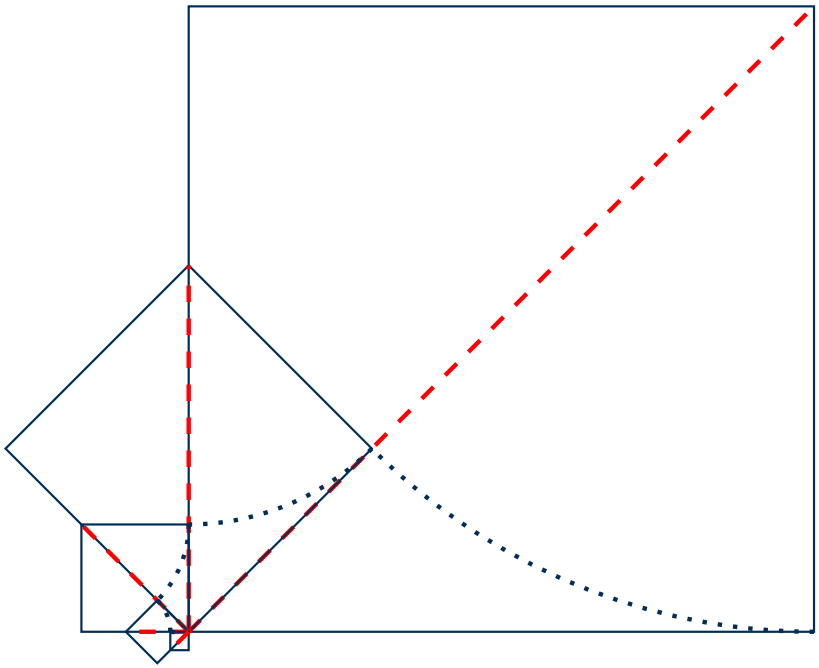

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$$S_N = a\left(1-\sqrt{2}\right)^N \approx a\left(0.4142\right)^N \longrightarrow 0$$

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Using only compass and straightedge ...

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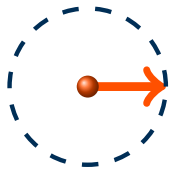

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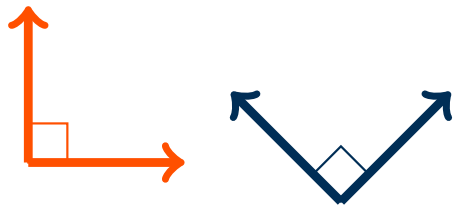

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  4. That all right angles equal one another.
  5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

# Postulate 5

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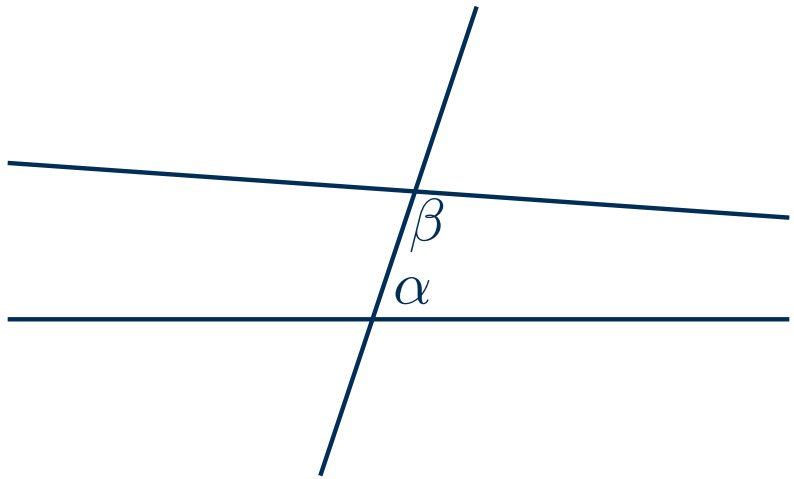


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To construct an equilateral triangle on a given finite straight line.

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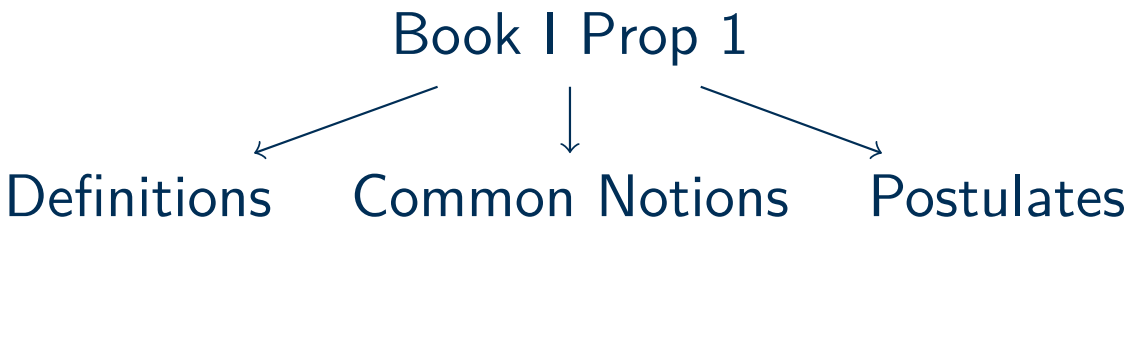


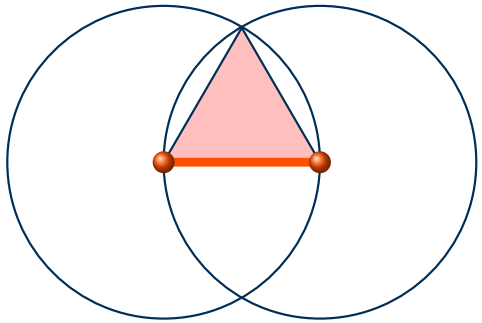

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In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.

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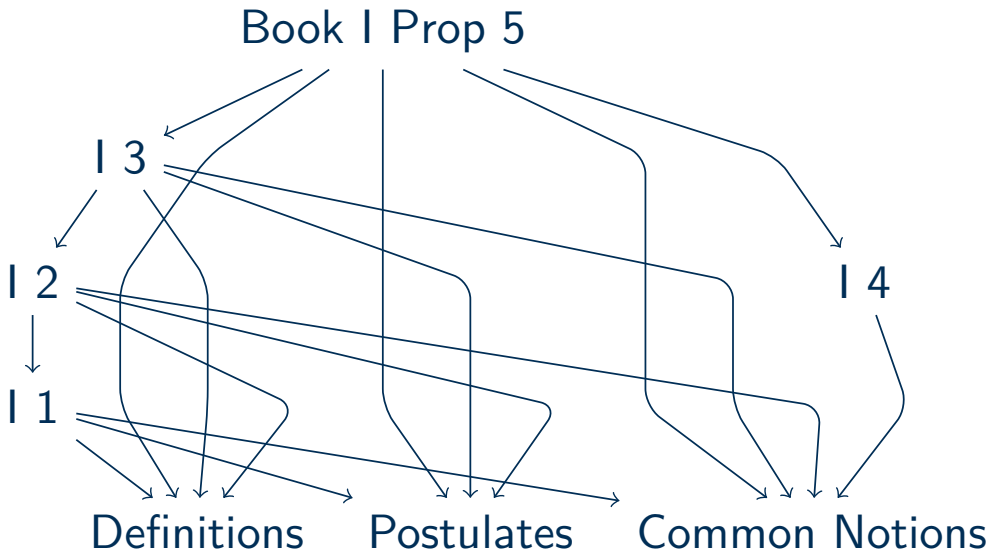
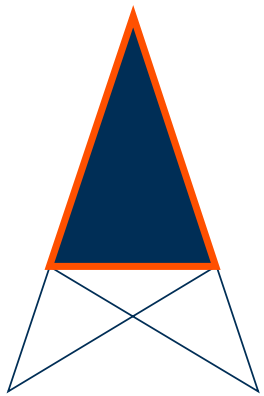
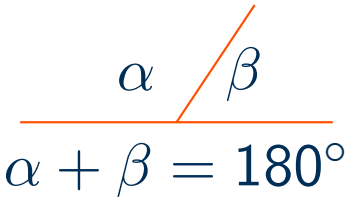


# Proposition 13

**Proposition 13**

If a straight line stands on a straight line, then it makes either two right angles or angles whose sum equals two right angles.

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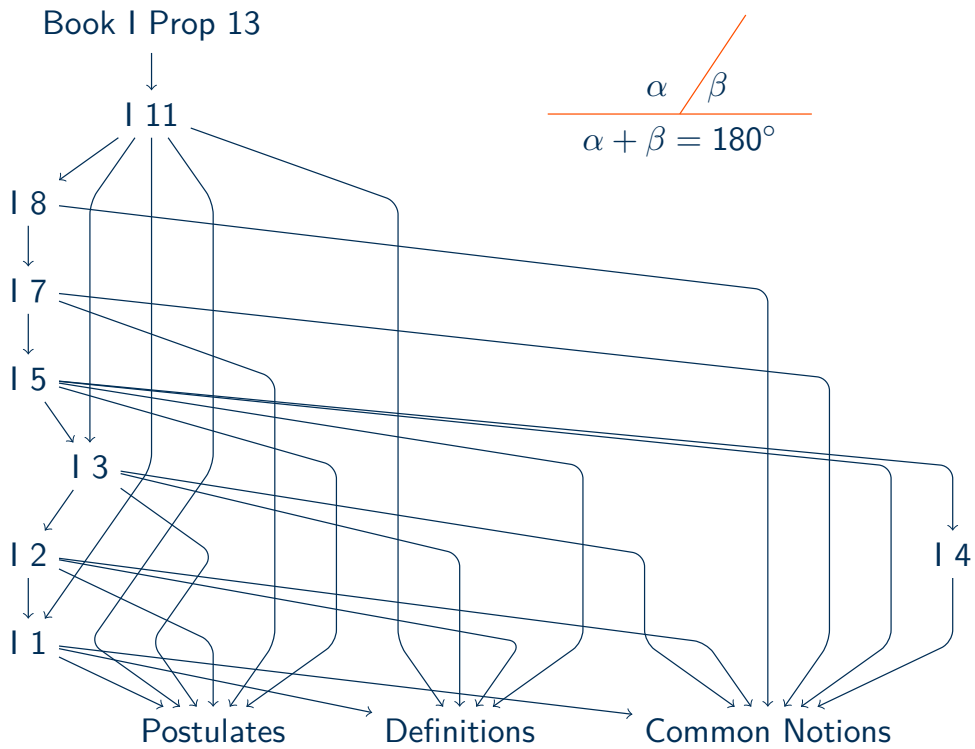





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