# Instructions

Below are the practice exam problems which you must turn in when you come in to take the exam; these must be written up neatly or typed on separate paper and in accordance with the guidelines in your syllabus. Your grade will be based on you completing all the questions and on the quality of your work. In addition there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam.

Practice Exam Problems:

1. How many positive two digit integers are multiples of three?

2. Suppose that in a certain state all license plates consist of four letters followed by three digits.

   (a) How many different license plates are possible?

   (b) How many plates begin with A and end with 0?

   (c) On how many plates are all the letters and digits distinct?

3. Use the map below to answer the following:

   (a) In how many ways can you travel from *City A* to *City B*?

   (b) In how many ways can you travel from *City A* to *City B* with a stop in *City C*?
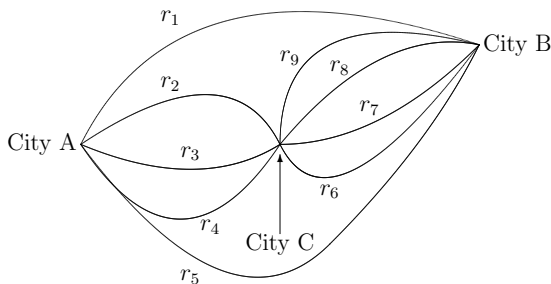
4. Suppose that a student council consists of 15 members, 8 men and 7 women.

   (a) In how many ways can you form a six person committee with 3 men and 3 women?

   (b) In how many ways can you form a six person committee with at least 1 women?

5. How many distinguishable ways can the letters of the word *HULLABALOO* be arranged?

6. If n is a positive integer, in how many ways can you pick 4 integers, i, j, k, m, so that $1 \leq i \leq j \leq k \leq m \leq n$? (Note that the integers can be equal.)

7. If n is a positive integer how many solutions are there to the equation

$$x_1 + x_2 + x_3 = n$$

if $x_i \geq 0 \ \forall i$? What if $x_i \geq 1 \ \forall i$? (Note that your answer will be in terms of n.)

8. How many cards must you select from a standard deck of 52 cards in order to guarantee that two of them are the same suit? For a 5 point bonus, how many cards must you select to have a better than 50% chance of getting the same suit twice?

9. In repeated divisions by 2373 how many distinct remainders can be obtained? If you were to write the decimal expansion of $173/2373$ what is the longest possible length of the repeating section of the representation? (Don't actually calculate it!)

10. Show that in any set of thirteen integers chosen from 2 through 40 there must be at least two with a common divisor greater than 1.

11. Use the *Binomial Theorem* to expand $(p - 2q)^4$

12. Use the *Binomial Theorem* to find the coefficient for $a^5 b^7$ in $(a - 2b)^{12}$

13. Use the *Binomial Theorem* to prove that for all integers $n \geq 0$,

$$3^n = \binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n}$$

14. (Challenge Problem worth 15 points) Find the coefficient of $x^3 y^2 z^5$ in the expansion of $(x + y + z)^{10}$.

Additional Practice Problems:

(listed by section and problem number)

§ 9.2: 1, 4, 6, 9, 11ab, 14abd, 16, 19, 21, 34;
§ 9.3: 3, 6, 11, 14;
§ 9.4: 1, 3, 9, 10, 14, 20, 24;
§ 9.5: 3, 6, 8, 15, 19;
§ 9.6: 3, 5, 10, 11;
§ 9.7: 1, 3, 5, 10, 19, 23, 25, 29, 31, 37;