Instructions

Below are 11 practice exam problems which you must turn in when you come in to take the exam; these must be written up neatly or typed on separate paper and in accordance with the guidelines in your syllabus. Your grade will be based on you completing all the questions and on the quality of your work. In addition there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam.

Practice Exam Problems:

1. Write the first four terms of the sequence defined below.

$$b_j = \frac{5-j}{5+j}$$

2. Write an explicit formula for the squence below.

$$\left(1-\frac{1}{2}\right),\ \left(\frac{1}{2}-\frac{1}{3}\right),\ \left(\frac{1}{3}-\frac{1}{4}\right),\ \left(\frac{1}{4}-\frac{1}{5}\right),\ \left(\frac{1}{5}-\frac{1}{6}\right),\ \ldots$$

3. Compute the value of the finite sum below.

$$\sum_{k=0}^{5} 2k+1$$

4. Compute the value of the finite product below.

$$\prod_{i=1}^{4} \left(\frac{1}{2}\right)^i$$

5. Write the following using summation notation:

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2.$$

6. Use

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

to find the sum of $S = 3 + 4 + 5 + 6 + \cdots + 1000$.

7. Rewrite the summation by separating off the final term.

$$\sum_{i=1}^{n+1} \frac{1}{i^2+1}$$

8. Make a change of variable (i.e. re-index) in the summation using $j = i - 1$:

$$\sum_{i=1}^{n+1} \frac{(i-1)^2}{i \cdot n}.$$

9. Write out the first five terms of the recursive sequence defined by

$$s_0 = 1,\ s_1 = 1,\ s_n = \frac{1}{s_{n-1} + s_{n-2}}.$$

10. Use iteration to find an explicit formula for the recursive relation $h_k = 2^k - h_{k-1}$ where $h_0 = 1$.

11. Verify that $F_k = 3F_{k-3} + 2F_{k-4}$ for all integers $n \geq 4$, where $F_n = F_{n-1} + F_{n-2}$ is the Fibonacci Sequence.

Extra: Assume the sequence satisfies the given recurance relation and initial conditions and find an explicit formula for the sequence.

$$s_0 = 0,\ s_1 = 1\ s_k = -4s_{k-1} - 4s_{k-2},\ \text{for } k > 1$$

Extra: Let b_0, b_1, b_2, ... be a sequence defined by the explicit formula

$$b_n = C \cdot 3^n + D \cdot (-2)^n \text{ for all } n \geq 0,$$

where C and D are real numbers. Show that for any choice of C and D,

$$b_k = b_{k-1} + 6b_{k-2} \text{ for } k \geq 2.$$

Additional Practice Problems:

(listed by section and problem number)

§ 5.1: 1,3,5,11,12,14,20,23,37,46,51,55
§ 5.6: 1,3,5,11,17ac,26
§ 5.7: 1ab,5,10,19,24
Extra - 5.8: 1,5,8,11,13,22