Instructions

Below are the practice exam problems which you must turn in when you come in to take the exam; these must be written up neatly or typed on separate paper and in accordance with the guidelines in your syllabus. Your grade will be based on you completing all the questions and on the quality of your work. In addition there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam.

Practice Exam Problems:

1. Given $A = \{1, 2\}$ and $B = \{2, 3\}$ find each of the following:[1]

   (a) $\mathscr{P}(A)$ (b) $\mathscr{P}(A \cap B)$ (c) $\mathscr{P}(A \times B)$

2. Let the universal set be the real numbers, $\mathbb{R}$, and let

   $$A = \{x \in \mathbb{R} | 0 < x \le 2\} \qquad B = \{x \in \mathbb{R} | 1 \le x < 4\} \qquad C = \{x \in \mathbb{R} | 3 \le x < 9\}$$

   Find the following:

   (a) $A \cap B$ (b) $(A \cup C)^c$ (c) $(B^c \cap C)$

3. Student C defines a function $h : \mathbb{Q} \to \mathbb{Q}$ by the rule:

   $$h\left(\frac{m}{n}\right) = \frac{m^2}{n}$$

   Student D claims this function is not well defined. Justify student D's claim.

4. Let $\mathbb{S}$ be the set of all strings of 0's and 1's and define $l : \mathbb{S} \to \mathbb{Z}$ by

   $$\forall s \in \mathbb{S} : \ l(s) = the \ length \ of \ s.$$

   (a) Is l one-to-one? Why or Why Not?
   (b) Is l onto? Why or Why Not?

5. If $f : X \to Y$ and $g : Y \to Z$ are both functions and $g \circ f$ is one-to-one, must g be one-to-one? Why or Why Not?

6. Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let $\mathcal{R}$ be the less than relation on $A \times B$, i.e.:

   $$\forall \ (a, b) \in A \times B : \ a\mathcal{R}b \Leftrightarrow a < b.$$

   State explicitly which pairs from $A \times B$ are in $\mathcal{R}$ and $\mathcal{R}^{-1}$.

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[1] $\mathscr{P}$ indicates the power set.

7. Let A be the set of all strings consisting solely of a's and b's of length 4. Define the relation $\mathcal{R}$ on A by

$$\forall\ s, t \in A :\ s\mathcal{R}t \Leftrightarrow s \text{ and } t \text{ have the same first two letters}$$

Is $\mathcal{R}$ *reflexive, symmetric,* and *transitive*? (Justify your answers.)

8. Define a relation P on the set $\mathbb{R} \times \mathbb{R}$ as follows:

$$\forall\ (w, x), (y, z) \in \mathbb{R} \times \mathbb{R} :\ (w, x)P(y, z) \Leftrightarrow w = y.$$

Show that P is an equivalnce relation and describe the equivalence classes formed by P.

Additional Practice Problems:

(listed by section and problem number)

§ 6.1: 3ab, 5, 11, 19, 28, 31
§ 7.1: 1, 6a, 7ac, 15, 17ac, 18ac, 19, 23, 33
§ 7.2: 6, 7a, 10, 14, 15, 16, 21
§ 7.3: 1, 3, 9a, 24

§ 8.1: 2, 10, 21

§ 8.2: 9, 12, 15, 25, 34

§ 8.3: 1, 5, 8, 21, 29