Instructions

Below are 15 practice exam problems which you must turn in when you come in to take the exam; these must be written up neatly or typed on separate paper and in accordance with the guidelines in your syllabus. Your grade will be based on you completing all the questions and on the quality of your work. In addition there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam.

Logic and Stuff:

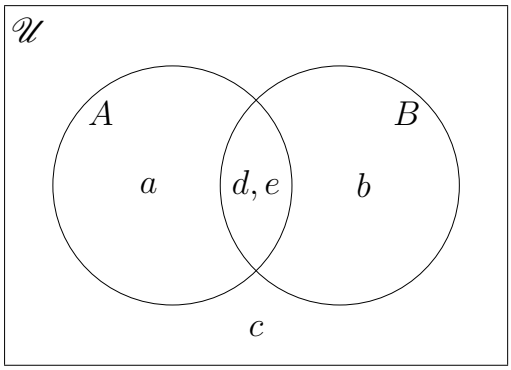
1. Find the truth set for the given predicate with the given domain.
   (a) $Q(d):\ 6d < 20$, if the domain is $\mathbb{N}$.
   (b) $Q(d):\ 6d < 20$, if the domain is $\mathbb{R}$.
2. Write the negation of
$$\exists\ x \in \mathbb{R} : x^2 = 2 \wedge x \in \mathbb{Q},$$
   and then write both statements as sentences in English.
3. Rewrite *"cats are aliens"* as an "if - then" statement and then write its contrapositive, converse, inverse, and negation.

Sequences and Stuff:

4. Write the first four terms of the sequence defined below, begin with c_1.
$$c_n = \frac{(-3)^n}{n}$$
5. Write an explicit formula for the squence below.
$$\frac{1}{2},\ -\frac{1}{5},\ \frac{1}{10},\ -\frac{1}{17},\ \frac{1}{26},\ -\frac{1}{37},\ \frac{1}{50},\ -\frac{1}{65}, \frac{1}{82}, -\frac{1}{101}, \ldots$$
6. *Use the formula*
$$\sum_{i=0}^{n} r^n = \frac{r^{n+1} - 1}{r - 1}$$
   to find the sum of $S = 16 + 64 + 256 + 1024 + \cdots + 4^{10}$.
7. *Use iteration* to find an explicit formula for the recursively defined sequence $a_k = 2 + 4a_{k-1}$ where $a_0 = 2$.

Sets and Stuff:

8. Given the Venn diagram find each of the following:[1]



   (a) $\mathscr{U}$
   (b) $\mathscr{P}(A \cap B)$
   (c) $A \backslash B$
   (d) $A \times B^c$
   (e) $A^c \cap B$

9. Let $\mathbb{S} = (\mathbb{Z} \times \mathbb{Z}) \setminus \{(0,0)\}$ and define $p : \mathbb{S} \to \mathbb{Z}$ by $\forall (a,b) \in \mathbb{S} : \ p(a,b) = a^b$.
   (a) What is the *domain* of p?
   (b) What is the *co-domain* of p?
   (c) Give a counter example to show that p is not one-to-one.
   (d) Prove that p is onto.

10. Let $\mathcal{D}$ be the relation defined on $\mathbb{R}$ as follows:

$$\forall\, x, y \in \mathbb{R} : x \mathcal{D} y \iff x + y \geq 0.$$

    Determine if the relation $\mathcal{D}$ is *reflexive*, *symmetric*, *transitive*, or none of these.

Counting and Stuff:

11. ***Sea-Air-Land Pizza***™ offers pizzas with your choice of 2 out of 10 vegetables, 2 out of 4 cheeses, 1 out of 3 types of dough, and either anchovies, duck, or pork. How many differnt pizzas can you order?

12. Suppose that a student council consists of 15 members, 8 men and 7 women.
    (a) In how many ways can you form a six person committee with 3 men and 3 women?
    (b) In how many ways can you form a six person committee with at least 1 women and 1 man?

13. If a store sells seven differnt varieties of soda and they have at least 24 of each in stock, then how many ways can you pick out 24 sodas?

14. Assuming face cards are worth 10 points, aces are worth 1 point, and all other cards have the point value listed on them, how many cards must you select from a standard deck of 52 cards to guarantee that the total of all the cards you have drawn is over 10?

15. Use the *Binomial Theorem* to expand $(v - u)^4$

[1] $\mathscr{P}$ indicates the power set and $\mathscr{U}$ is the universal set.

Graphs and Trees:

Use the graphs in figure 1 when answering questions about specific graphs.

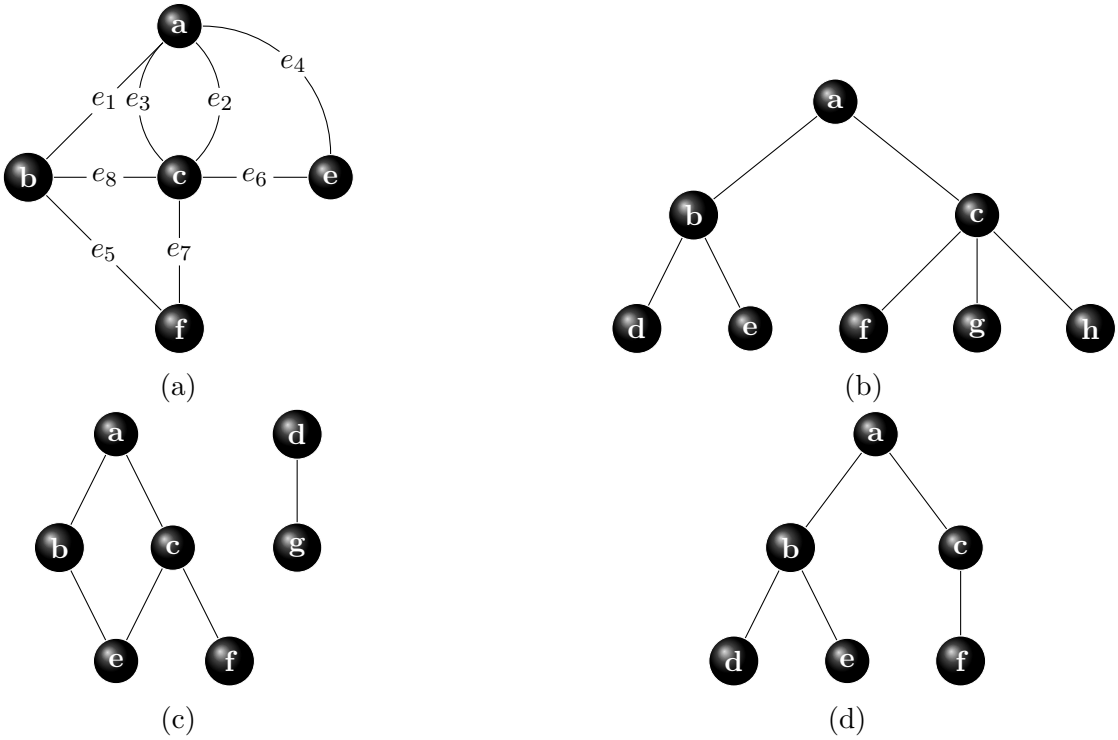


Figure 1: Reference Graphs

16. What is the *total degree* of graph 1c?
17. Does the graph 1a have an *Euler Circuit*? Justify your answer.
18. Write the *adjacency matrix* for graph 1b.
19. Sketch a graph with *adjacency matrix*

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

20. Which graph(s) in figure 1 are *binary trees*? Explain your conclusion.