

Some TAUTOLOGIES¹

Let p, q, r , and s be arbitrary statements, let t be a tautology, and let c be a contradiction, then:

The four basic truth tables:

(a) Negation

p	$\sim p$
T	F
F	T

(b) Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(c) Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(d) Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Laws involving Conjunctions, Disjunctions, and Negations:

Tautology	Name	Abbreviation
0. p and $q \Rightarrow p \wedge q$	Conjunctive inference	(Conj.)
1. $p \Rightarrow p \vee q$	Law of Addition	(Add.)
2. $p \wedge q \Rightarrow p$	Laws of Simplification	(Simp.)
3. $(p \vee q) \wedge \sim p \Rightarrow q$	Disjunctive Syllogism	(D.S.)
4. $\sim(\sim p) \equiv p$	Law of Double Negation	(D.N.)
5. $p \wedge q \equiv q \wedge p, p \vee q \equiv q \vee p$	Commutative Laws	(Com.)
6. (a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ (b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws	(Assoc.)
7. (a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Laws	(Dist.)
8. (a) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ (b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$	DeMorgan's Laws	(DeM.)
9. (a) $(p \wedge t) \Leftrightarrow p$ (b) $(p \vee c) \Leftrightarrow p$	Identity Laws	(I.L.)
10. (a) $(p \vee t) \Leftrightarrow t$ (b) $(p \wedge c) \Leftrightarrow c$	Domination Laws	(D.L.)
11. (a) $p \vee \sim p \equiv t$ (b) $p \wedge \sim p \equiv c$	Inverse Laws	(Inv.)
12. (a) $p \wedge p \equiv p$ (b) $p \vee p \equiv p$	Laws of Idempotency	(Idemp.)
13. (a) $p \vee (p \wedge q) \Leftrightarrow p$ (b) $p \wedge (p \vee q) \Leftrightarrow p$	Absorption Laws	(A.L.)

¹Originally created by Vasily C. Cateforis for Set Theory and Logic at SUNY Postdam based on the text *Set Theory with Applications* by Lin & Lin

Laws involving Implications:

Tautology	Name	Abbreviation
14. $p \rightarrow q \equiv \sim (p \wedge \sim q)$	Conditional Law	(Def. \rightarrow)
15. $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$	Contrapositive Law	(Contrap.)
16. $(p \rightarrow q) \wedge p \Rightarrow q$	Modus Ponens	(M.P.)
17. $(p \rightarrow q) \wedge \sim q \Rightarrow \sim p$	Modus Tollens	(M.T.)
18. $(p \rightarrow q) \Leftrightarrow (p \wedge \sim q \rightarrow c)$	Reductio ad absurdum	(R.A.)
19. $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$	Transitive Law	(Trans.)
20. $p \rightarrow t \equiv t$		
21. $c \rightarrow p \equiv t$		
22. (a) $(p \rightarrow q) \wedge (r \rightarrow s) \Rightarrow (p \vee r \rightarrow q \vee s)$ (b) $(p \rightarrow q) \wedge (r \rightarrow s) \Rightarrow (p \wedge r \rightarrow q \wedge s)$	Constructive Dilemmas	(C.D.)
23. (a) $(p \rightarrow q) \wedge (r \rightarrow s) \Rightarrow (\sim q \vee \sim s \rightarrow \sim p \vee \sim r)$ (b) $(p \rightarrow q) \wedge (r \rightarrow s) \Rightarrow (\sim q \wedge \sim s \rightarrow \sim p \wedge \sim r)$	Destructive Dilemmas	(D.D.)
24. $(p \wedge q \rightarrow r) \equiv (p \rightarrow (q \rightarrow r))$	Exportation Law	(Exp.)
25. $p \rightarrow q$	“ p is sufficient for q ”	
26. $p \rightarrow q$	“ q is necessary for p ”	
27. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	“ p is necessary and sufficient for q ”	

28. Converse Error $(P \rightarrow Q) \wedge Q \not\equiv P$:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

29. Inverse Error $(P \rightarrow Q) \wedge \sim P \not\equiv \sim Q$:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

30. Quantifier Negation (QN):

(a) $\sim [(\forall x) (p(x))] \equiv (\exists x) (\sim p(x))$

(b) $\sim [(\exists x) (p(x))] \equiv (\forall x) (\sim p(x))$

31. Principle of Mathematical Induction (PMI): If $P(n)$ is a statement about or involving the natural number n such that

(1) $P(k_0)$ is true for some k_0 , and

(2) $P(k) \Rightarrow P(k + 1)$ for all natural numbers $k \geq k_0$, then

$P(n)$ is true for all natural numbers n .