Writing Up a Math Problem

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Table of Contents

Table of Contents

The Problem

Projectile Motion

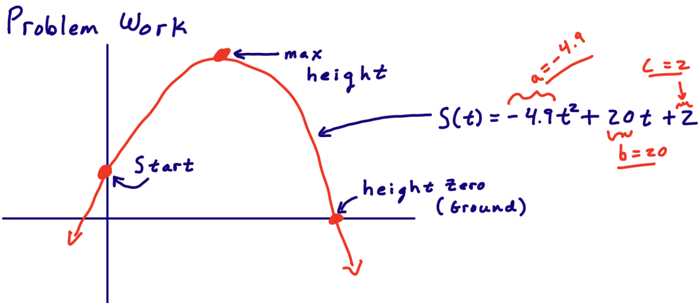
The height of a ball thrown in the air is given by the equation

$$s(t) = -4.9t^2 + 20t + 2 \text{ meters},$$

were t measures time in seconds. Using this find the time at which the ball reaches its maximum height and the time at which it lands on the ground.

Table of Contents

Picture


Problem Work
max
height
Start
height zero
(Ground)
S(t) = -4.9t² + 20t + 2
a = -4.9
b = 20
c = 2

Algebra

Vertex at $t = -b/2a = \dfrac{-20}{2(-4.9)} = \dfrac{-20}{-9.8} \approx 2.041$ seconds

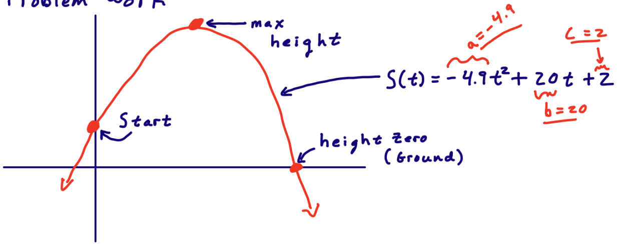
Roots at $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-20 \pm \sqrt{400 + 39.2}}{-9.8}$

$$= \frac{20 \pm \sqrt{439.2}}{9.8}$$

$$\approx 4.179 \text{ or } -0.098$$

All Together

Problem Work



Vertex at $t = -b/2a = \frac{-20}{2(-4.9)} = \frac{-20}{-9.8} \approx 2.041$ seconds

Roots at $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{400 + 39.2}}{-9.8}$

$$= \frac{20 \pm \sqrt{439.2}}{9.8}$$

$$\approx 4.179 \text{ or } -0.098$$

Table of Contents

Flip the Question

Answer

The ball whose height is given by

$$s(t) = -4.9t^2 + 20t + 2 \text{ meters}$$

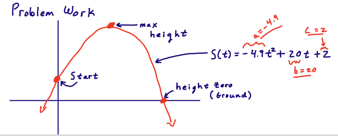
reaches its maximum height at $t \approx 2.041$ seconds and lands on the ground after $t \approx 4.179$ seconds.

Add Details

Answer

The path of the ball is a parabola, the maximum height is the vertex at

$$t = \frac{-b}{2a} = \frac{-20}{-9.8} \approx 2.041 \text{ sec.}$$



The ball lands on the ground when the height is 0 meters; the roots of the parabola. Using the quadratic equation we get

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{400 + 39.2}}{-9.8} \approx 4.179 \text{ or } -0.098.$$

Only the positive value makes sense so $t \approx 4.179$ sec.

Final Solution All Together

Answer

The ball whose height is given by

$$s(t) = -4.9t^2 + 20t + 2 \text{ meters}$$

reaches its maximum height at $t \approx 2.041$ seconds and lands on the ground after $t \approx 4.179$ seconds.

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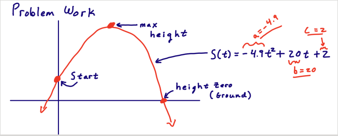
$$t = \frac{-b}{2a} = \frac{-20}{-9.8} \approx 2.041 \text{ sec.}$$



The ball lands on the ground when the height is 0 meters; the roots of the parabola. Using the quadratic equation we get

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{400 + 39.2}}{-9.8} \approx 4.179 \text{ or } -0.098.$$

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Table of Contents

The Problem

Horsepower Exercise

The horsepower (hp) that a shaft can safely transmit varies jointly with its speed (in revolutions per minute (rpm)) and the cube of the diameter. If the shaft of a certain material 3 inches in diameter can transmit 45 hp at 100 rpm, what must the diameter be in order to transmit 60 hp at 150 rpm?

The Scrap Work

$$h.p. = k \cdot rpm \cdot d^3 \quad \Big\} \text{ From text}$$

$$k = \frac{hp}{rpm\, d^3} = \frac{45}{100 \cdot 3^3} = \frac{1}{60}$$

$$60 = \frac{1}{60} \cdot 100 \cdot d^3 \longrightarrow d^3 = \frac{120}{5} = 24$$

$$d = \sqrt[3]{24} \approx 2.88$$

The Write-Up

Horsepower Solution

Horse power transmitted by a shaft varies jointly with speed and the cube of its diameter

$$\mathrm{hp} = \mathrm{k} \cdot \mathrm{rpm} \cdot (\mathrm{d}^3).$$

A 3 inch diameter shaft transmitting 45 hp at 100 rpm gives

$$\mathrm{k} = \frac{\mathrm{hp}}{\mathrm{rpm} \cdot (\mathrm{d}^3)} = \frac{45}{100 \cdot (3^3)} = \frac{1}{60}.$$

Therefore, to generate 60 hp at 150 rpm we need a shaft with diameter

$$\begin{aligned} \mathrm{d} = \sqrt[3]{\frac{\mathrm{hp}}{\mathrm{k} \cdot \mathrm{rpm}}} &= \sqrt[3]{\frac{60}{(1/60) \cdot 150}} \\ &= \sqrt[3]{24} \\ &= 2\sqrt[3]{3}\,\text{inch} \\ &\approx 2.88\,\text{inch}. \end{aligned}$$

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