



Dr. Charles Rocca  
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<http://sites.wcsu.edu/roccac>

MAT 412/512-01: Group Theory  
R: 5:30 pm - 8:00 pm,  
Higgins 103



## Office Hours:

Office hours are on ground for the Fall 2023 Semester. If you need to meet virtually we can make an appointment to do so via my WebEx Virtual Office:

Higgins 101-DV (<https://wcsu.webex.com/meet/roccac>)

- Monday & Thursday: 11am - 12pm, 3:30pm - 4:30pm
- Friday: 1pm - 2pm
- or by appointment

## Course Materials:

Textbook: *A First Course in Abstract Algebra, 8<sup>th</sup> ed.* by Fraleigh and Brand

## Course Description and Objectives:

Groups are one of the fundamental mathematical objects that help us to understand how and why things work as they do in mathematics. They are significant in their own right, but are also important in applications of mathematics to physics, chemistry, and information security. As such, students can greatly benefit from a clear understanding of their properties and structures. Prerequisite: MAT 375: Algebraic Structures, or equivalent.

After successful completion of this course students should be able to:

- Define and give examples of the fundamental algebraic structures,
- State and describe the importance of Lagrange's Theorem, Cauchy's Theorem, Cayley's Theorem, the Isomorphism Theorems, Sylow's Theorems, and the Fundamental Theorem of Finite Abelian Groups,
- Construct proofs, or outlines of proofs for, Lagrange's Theorem, Cayley's Theorem, Cauchy's Theorem, and the First Isomorphism Theorem
- Analyze hypotheses of theorems and definitions and through examples and non-examples explain why they are written the way they are, and
- Solve problems by choosing and applying appropriate theorems.

## Course Content:

Unit	Sections Referenced
Review and Overview of Algebraic Structures	0,2,22,23,30
Groups, Properties, Homomorphisms	1-8, 10-12
Advanced Topics in Group Theory	9,13,14,16,17,19

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## Grading:

11 × “Daily” Quizzes	28% (The top 7 count 4% each)
3 × Unit Exams	42% (14% each)
3 × Unit Assignments	30% (10% each)

**Daily Quizzes:** In classes when we do not have an exam, there will be a quiz. The content of the quiz will be based on the material covered in the most recent lecture. Quizzes will have three to five questions focusing on basic knowledge and skills.

**Unit Exams:** Unit exams will be comprehensive exams covering all the material from the unit. Questions on exams will mostly focus on basic knowledge and skills. Graduate students will also be expected to outline major theorems. You will be allowed to redo selected problems on each exam in order to recover up to 33% of the points you lost on the exam.

**Key Definitions and Theorems:** Here is a list of the major theorems (the ones that graduate students should be able to outline) and key definitions. This is not a completely comprehensive list of everything we will be covering, but is the list of what should know well.

### Major Theorems:

- |   |  |  |
|---|--|--|
| <input type="checkbox"/> Cayley’s Theorem   | <input type="checkbox"/> Subring         | <input type="checkbox"/> Abelian           |
| <input type="checkbox"/> Lagrange’s Theorem                                       | <input type="checkbox"/> Ideal           | <input type="checkbox"/> Non-Abelian       |
| <input type="checkbox"/> Cauchy’s Theorem   | <input type="checkbox"/> Integral Domain | <input type="checkbox"/> Dihedral Group    |
| <input type="checkbox"/> Sylow’s Theorems   | <input type="checkbox"/> Field           | <input type="checkbox"/> Symmetric Group   |
| <input type="checkbox"/> Isomorphism Theorems                                     | <input type="checkbox"/> Characteristic  | <input type="checkbox"/> Homomorphism      |
| <input type="checkbox"/> Fundamental Theorem of Finitely Generated Abelian Groups | <input type="checkbox"/> Zero Divisor    | <input type="checkbox"/> Surjection (onto) |
|   | <input type="checkbox"/> Group           | <input type="checkbox"/> Injection (1-1)   |
|   | <input type="checkbox"/> Subgroup        | <input type="checkbox"/> Kernel            |
|   | <input type="checkbox"/> Normal Subgroup | <input type="checkbox"/> Isomorphism       |

### Definitions:

- |  |  |  |
|--|--|--|
| <input type="checkbox"/> Equivalence Relations | <input type="checkbox"/> Normalizer                | <input type="checkbox"/> Endomorphism        |
| <input type="checkbox"/> Divisibility          | <input type="checkbox"/> Center of a Group         | <input type="checkbox"/> Automorphism        |
| <input type="checkbox"/> Modular Equivalence   | <input type="checkbox"/> Centralizer of an Element | <input type="checkbox"/> Group Action        |
| <input type="checkbox"/> Ring                  | <input type="checkbox"/> Order                     | <input type="checkbox"/> Quotient Structures |
|  | <input type="checkbox"/> Index                     |  |

**Unit Assignments:** Unit assignments will consist of exercises from the text. Assignment questions will involve problem solving; you won’t generally be able to solve these simply by plugging numbers into a formula or reciting a definition. Assignment answers **must be typed and in complete sentences**. For each question a reader should be able to tell what the question was without having to look it up; do not just repeat the question. Proper presentation and formatting will account for up to 10% of each assignment grade.

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## Course Calendar:

THURSDAY	
8/31 Syllabus, Introductions, & Equivalence Relations (Section 0)	1
9/7 Overview of Structures (Sections 2,22,23)	2
9/14 Ideals, Quotient Rings, and Transformations (Sections 23,30)	3
9/21 <i>Exam: Review/Overview of Structures</i>	4
9/28 Varieties of Groups and Subgroups (Sections 1-6, 9) & <i>Assignment 1 Due</i>	5
10/5 Permutations and Cayley's Theorem (Sections 7 & 8)	6
10/12 Lagrange's Theorem and the First Isomorphism Theorem (Section 10 & 12)	7
10/19 Groups acting on Spaces (Section 11)	8
10/26 <i>Exam: Basics of Groups</i>	9
11/2 Factor Groups and Computations (Section 13) & <i>Assignment 2 Due</i>	10
11/9 Group Actions and Cauchy's Theorem (Section 14)	11
11/16 Isomorphism Theorems (Section 16)	12
11/23 <i>Thanksgiving Day - No Class</i>	
11/30 Sylow's Theorems (Section 17)	13
12/7 The Fundamental Theorem of Finitely-Generated Abelian Groups (Sections 9 & 19)	14
12/14 <i>Exam: Group Structure</i>	15

*Assignment 3 Due 12/15/2023*

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## Departmental Course Outline:

### 1. Review of Algebraic Structures

#### (a) Rings

- i. Subrings
- ii. Ideals
- iii. Quotient Rings
- iv. Homomorphisms

#### (b) Domains and Fields

- i. Zero Divisors versus Units
- ii. Integral Domains
- iii. Finite and Infinite Fields

### 2. Common Group Examples

#### (a) Abelian Groups

#### (b) Matrix Groups

#### (c) Dihedral Groups

#### (d) Symmetric Groups

### 3. Structure Theorems

#### (a) Lagrange's Theorem

#### (b) Sylow's Theorems

#### (c) Cauchy's Theorem

#### (d) Fundamental Theorem of Finite Abelian Groups

### 4. Group Actions

#### (a) Cayley Graphs

- i. Representing Groups Graphically
- ii. Groups Acting on Graphs
- iii. Groups acting as permutations
- iv. Cayley's Theorem

#### (b) Groups Acting on Spaces

- i. Linear Transformations: Rotations, Projections, Dilations, Reflections
- ii. Affine Transformations: Linear + Translation

### 5. Morphisms of Groups

#### (a) Homomorphisms (Revisited)

- i. Domain and Codomain
- ii. Range and Kernel

#### (b) Isomorphisms

- i. General Definitions
- ii. Isomorphism Theorems

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## You and Your Grades:

- “A” (Exceptional) range 90% to 100%:  
The student has demonstrated significant mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and most nonstandard problems which require deeper insight.
  - “A”  $\iff 92.5\% \leq \textit{Grade} \leq 100\%$
  - “A-”  $\iff 90\% \leq \textit{Grade} < 92.5\%$
- “B” (Good) range 80% to 90%:  
The student has demonstrated mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and some nonstandard problems which require deeper insight.
  - “B+”  $\iff 87.5\% \leq \textit{Grade} < 90\%$
  - “B”  $\iff 82.5\% \leq \textit{Grade} < 87.5\%$
  - “B-”  $\iff 80\% \leq \textit{Grade} < 82.5\%$
- “C” (Adequate) range 70% to 80%:  
The student has demonstrated adequate mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve most standard formulaic exercises but struggles with nonstandard problems which require deeper insight.
  - “C+”  $\iff 77.5\% \leq \textit{Grade} < 80\%$
  - “C”  $\iff 72.5\% \leq \textit{Grade} < 77.5\%$
  - “C-”  $\iff 70\% \leq \textit{Grade} < 72.5\%$
- “D” (Inadequate) range 60% to 70%:  
The student has demonstrated inadequate or incomplete mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve some standard formulaic exercises but few if any nonstandard problems which require deeper insight.
  - “D+”  $\iff 67.5\% \leq \textit{Grade} < 70\%$
  - “D”  $\iff 62.5\% \leq \textit{Grade} < 67.5\%$
  - “D-”  $\iff 60\% \leq \textit{Grade} < 62.5\%$
- “F” (Unacceptable) below 60%:  
The student has demonstrated essentially no mastery of the appropriate knowledge and skills relevant to the course. The student is unable to solve most standard formulaic exercises and essentially no nonstandard problems which require deeper insight.

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## End User Agreement:

**General Expectations:** As a student in this class you are expected to:

- attend class and take notes,
- actively read material in each section, taking notes,
- review your notes on a regular basis,
- check your university email every day,
- check the class site *at least* every other day,
- begin studying for exams in a timely fashion,
- ask questions early and often,
- attend office hours,
- seek help in the math clinic or tutoring center, and
- complete assignments and readings on time.

**Assignment Guidelines:** (These apply to *all out of class work*.)

- Work handed in must always look neat, legible, and professional. Work must be very neatly written or preferably typed. The quality of your work will be factored into your grade, up to 10%, in extreme cases work may be rejected and then counted as late.
- Answers on all assignments should be given in complete sentences. I should be able to tell what your answer means without re-reading the problem. This does not mean you simply rewrite the question.
- An assignment is considered late after I have handed it back or gone over it in class. Late assignments are accepted but may receive at most 75% credit. Late assignments go to the absolute bottom of the stack of papers to be graded; *all on time work is graded before any late work*.
- If you work on an assignment as part of a group, then there may be no more than three individuals in the group and all your names must be on the assignment. You should hand in only one copy of the work.
- All work must be submitted in the manner directed.

**Email Etiquette Guidelines:** When sending an email you must include the course number and semester in the subject line. For example, if you are taking MAT 314 in Fall 1592 then the the subject line should begin with “[MAT 314 Fall 1592].” Also, you should always begin with a salutation such as “Dear Dr. Rocca” and end with a closing such as “Sincerely, I. Newton.”

**Exam Makeup Policy:** To qualify for a makeup exam you must have a valid reason for missing the exam and, if at all possible, let me know ahead of time that you are missing the exam. You will need to meet with me in order to arrange a time for the make up exam. If you do not have a valid reason, do not give prior notice when possible, or simply do not show up for an exam, you are not entitled to a makeup and will not be given one. If you fail to show up for your makeup exam, you will not be given a second opportunity.

**The 2% Exception:** Any quiz or class work which is ultimately worth no more than 2% of your final grade can not be made up.

**Time on Task:** As a 3 credit class you should expect to average 9 to 10 hours of work a week including class time. Some weeks you may get away with less and some may require more.

**Attendance:** There is no specific policy for attendance in this course. However, if you have *three consecutive unexcused absences* within the first half of the semester I am required to report to the University that you have *stopped attending*.

**Academic Honesty:** If on any assignment, quiz, or exam you turn in someone else’s work, regardless of the source, as if it were your own you will receive a zero on that assignment, quiz, or exam. If you are caught doing this three times you will receive an F in the course and the Dean will be informed of your academic dishonesty.

(<https://www.wcsu.edu/faculty-handbook/2019-2020/policies-pertaining-to-students/academic-honesty-policy/>)

**Accommodations:** If you have need of an accommodation for testing or note taking, please visit AccessAbility Services, located in White Hall 005 (<http://www.wcsu.edu/accessability>).