

Dr. Charles Rocca Higgins Hall 101D roccac@wcsu.edu http://sites.wcsu.edu/roccac MAT 375 : Algebraic Structures MR: 2-3:15 p.m., H116 Credits: 3 credit Grading: Standard A-F



Office Hours:

• MR 11am-12pm & 1-2pm • T 1-2pm

• or by appointment

Course Materials:

• "Abstract Algebra: An Introduction, 3^{rd} Edition" - Thomas W. Hungerford

Course Description:

In order to have a proper foundational understanding of the numbers and algorithms we use every day one must have knowledge of groups, rings, and fields. This course will introduce the basic ideas in groups, rings, and fields. There will be particular emphasis on field extensions, factorization, and transformations. Students will learn about the fundamental algebraic structures behind the algorithms for arithmetic and polynomials. They will also understand how one number system is developed from another. Prerequisite: C or better in MAT 207 and C or better in MAT 272. Upon successful completion of this course, students will be able to:

- State the definitions of ring, integral domain, field, and group, and give examples of different types of each of these structures.
- Demonstrate fluency with quotient structures, in particular an understanding of rings of integers modulo n.
- Describe the correspondence between the unique prime factorization of integers and factorization of polynomials into irreducible polynomials.
- Explain familiar facts about arithmetic in terms of ring and field properties.
- Explain the relationship between algorithms for arithmetic of numbers in decimal representation and the arithmetic of polynomials.
- Describe various groups of permutations such as symmetric and dihedral groups and their basic properties.
- Describe the algebraic ways in which some number systems are extensions of others, e.g., the extension of the ring of integers to a field yields the rational numbers.

Course Content:

Unit 1 - Ring and Fields: Basic Examples and Definitions Chapters 1-3
Unit 2 - Polynomials and Quotients over rings Chapter 4 (4.1-4.4), Chapter 5, Chapter 6
Unit 3 - Groups and Their Properties Chapter 7
Unit 4 - Cosets and Normal Subgroups in Groups Chapter 8 (8.1-8.4)

Grading:

Problem Solving Exercises	20%
Unit Exams	64% (16% per exam)
Final Exam	16%

- **Problem Solving Exercises:** These exercises will almost always come from your text unless we run across something interesting. There will typically be 5 or 6 per unit. They all will require problem solving, these will not be algorithmic problems solved by wrote memorization. You will be allowed to do a second draft on each assignment in order to raise your grade. While you may work in groups on these assignments, cheating will not be tolerated. If you cheat on any part of an assignment you will get a zero on the entire assignment and you will not be allowed to make it up. All of these must be typed and in complete sentences.
- Unit Exams: You will have four unit exams. The purpose of these exams are to check that you know your fundamental definitions, examples, theorems, and (occasionally) algorithms. The content of these exams represents the fundamental knowledge you should get from the class. Definitions in particular are important to your understanding, as such 40% of each exam will be definitions. You will be given the opportunity to redo some of the questions on each exam in order to earn back up to 33% of your missed points. Since redos are out of class work they must be typed up and in complete sentences.
- **Final Exam:** The final exam will be cumulative, but otherwise serves the same purpose and has the same form as your unit exams.

Course Calendar:

THURSDAY	Monday	
1/19 1	1/23	2
Syllabus and Course Overview	Review Modular Arithmetic and Matrices	
1/26 3	1/30	4
Rings and Fields (introduction and examples)	Rings and Fields (properties)	
2/2 5	2/6	6
Homomorphisms and Isomorphisms	Homomorphisms and Isomorphisms	
2/9 7	2/13	8
Cleanup and Review	Exam 1:Rings - Properties and Morphisms	
2/16 9	2/20	
Polynomials over Rings	Presidents Day - No Class	
2/23 10	2/27	11
Irreducible Polynomials and Extensions	Fundamental Theorem of Algebra	
3/2 12	3/6	13
Quotients and Homomorphisms	First Isomorphism Theorem for Rings	
3/9 14	3/13	
Cleanup and Review	Spring Break - No Class	
3/16	3/20	15
Spring Break - No Class	Exam 2: Polynomials and Quotients	
3/23 16	3/27	17
Groups	Subgroups	
3/30 18	4/3	19
Symmetric Groups	Symmetric Groups and Cayley's Theorem	
4/6 20	4/10	21
Cleanup and Review	Exam 3: Groups and Their Properties	
4/13 22	4/17	23
Cosets and Langrange's Theorem	Normal Subgroups and Quotients	
4/20 24	4/24	25
Quotients and Homomorphisms	First Isomorphism Theorem for Groups	
4/27 26	5/1	27
Cleanup and Review	Exam 4: Cosets and Normal Subgroups	
5/4 28	5/8	29
Cleanup and Review	Final Exam Period 3 – 2:00pm – 4:30pm	

Course Outline:

- 1. Introduction to Rings and Fields
 - (a) Rings of Integers and Integers Modulo n
 - (b) Units Modulo n
 - (c) Rational Numbers
 - (d) Real Numbers
 - (e) Extensions:
 - i. Polynomial ring as extensions of a ring or field
 - ii. Complex numbers as an extension of the Real Numbers
 - iii. Extensions in general
 - (f) Factorization:
 - i. Unique factorization of integers
 - ii. Unique factorization of polynomials
 - (g) Subrings, Ideals and Quotient Rings:
 - i. Subrings and Ideals
 - ii. Integral Domains
 - iii. Principle Ideal Domains
 - (h) Ring and Field Morphisms:
 - i. Linear transformations
 - ii. Homomorphisms for rings and fields
 - iii. Isomorphisms for rings and fields
- 2. Introduction to Groups
 - (a) Geometric Transformations
 - (b) Group Structures in Number Theory and Linear Algebra
 - (c) Group Structures in Integer, Rational, Real, and Complex Numbers
 - (d) Permutation Groups
 - (e) Subgroups, Normal Subgroups, and Quotient Groups:
 - i. Cosets
 - ii. Lagrange's Theorem

You and Your Grades:

• "A" (Exceptional) range 90% to 100%:

The student has demonstrated significant mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and most nonstandard problems which require deeper insight.

- "A" $\iff 92.5\% \le Grade \le 100\%$

- "A-" $\iff 90\% \leq Grade < 92.5\%$

• "B" (Good) range 80% to 90%:

The student has demonstrated mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and some nonstandard problems which require deeper insight.

- $"B+" \iff 87.5\% \le Grade < 90\%$
- $\text{``B''} \iff 82.5\% \le Grade < 87.5\%$
- − "B-" $\iff 80\% \le Grade < 82.5\%$
- "C" (Adequate) range 70% to 80%:

The student has demonstrated adequate mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve most standard formulaic exercises but struggles with nonstandard problems which require deeper insight.

- "C+" \iff 77.5% \leq Grade < 80%
- $\text{``C''} \Longleftrightarrow 72.5\% \le Grade < 77.5\%$
- $\text{``C-''} \Longleftrightarrow 70\% \le Grade < 72.5\%$
- "D" (Inadequate) range 60% to 70%:

The student has demonstrated inadequate or incomplete mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve some standard formulaic exercises but few if any nonstandard problems which require deeper insight.

- "D+" $\iff 67.5\% \leq Grade < 70\%$
- "D" $\iff 62.5\% \leq Grade < 67.5\%$
- − "D-" $\iff 60\% \le Grade < 62.5\%$
- "F" (Unacceptable) below 60%:

The student has demonstrated essentially no mastery of the appropriate knowledge and skills relevant to the course. The student is unable to solve most standard formulaic exercises and essentially no nonstandard problems which require deeper insight.

General Expectations: As a student in this class you are expected to:

- attend class and take notes,
- actively read material in each section, taking notes,
- review your notes on a regular basis,
- check your university email every day,
- check the class site *at least* every other day,
- begin studying for exams in a timely fashion,
- ask questions early and often,
- attend office hours,
- seek help in the math clinic or tutoring center, and
- complete assignments and readings on time.

Assignment Guidelines: (These apply to all out of class work.)

- Work handed in must always look neat, legible, and professional. Work must be very neatly written or preferably typed. The quality of your work will be factored into your grade, up to 10%, in extreme cases work may be rejected and then counted as late.
- Answers on all assignments should be given in complete sentences. I should be able to tell what your answer means without re-reading the problem. This does not mean you simply rewrite the question.
- An assignment is considered late after I have handed it back or gone over it in class. Late assignments are accepted but may receive at most 75% credit. Late assignments go to the absolute bottom of the stack of papers to be graded; *all on time work is graded before any late work.*
- If you work on an assignment as part of a group, then there may be no more than three individuals in the group and all your names must be on the assignment. You should hand in only one copy of the work.
- All work must be submitted in the manner directed.
- **Email Etiquette Guidelines:** When sending an email you must include the course number and semester in the subject line. For example, if you are taking MAT 314 in Fall 1592 then the the subject line should begin with "[MAT 314 Fall 1592]." Also, you should always begin with a salutation such as "Dear Dr. Rocca" and end with a closing such as "Sincerely, I. Newton."
- **Exam Makeup Policy:** To qualify for a makeup exam you must have a valid reason for missing the exam and, if at all possible, let me know ahead of time that you are missing the exam. You will need to meet with me in order to arrange a time for the make up exam. If you do not have a valid reason, do not give prior notice when possible, or simply do not show up for an exam, you are not entitled to a makeup and will not be given one. If you fail to show up for your makeup exam, you will not be given a second opportunity.
- The 2% Exception: Any quiz or class work which is ultimately worth no more than 2% of your final grade can not be made up.
- **Time on Task:** As a 3 credit class you should expect to average 7.5 to 8.5 hours of work a week including class time. Some weeks you may get away with less and some may require more.
- Attendance: There is no specific policy for attendance in this course. However, if you have *three consecutive unexcused absences* within the first half of the semester I am required to report to the University that you have *stopped attending*.
- Academic Honesty: If on any assignment, quiz, or exam you turn in someone else's work, regardless of the source, as if it were your own you will receive a zero on that assignment, quiz, or exam. If you are caught doing this three times you will receive an F in the course and the Dean will be informed of your academic dishonesty. (https://www.wcsu.edu/faculty-handbook/2019-2020/policies-pertaining-to-students/academic-honesty-policy/)
- Accommodations: If you have need of an accommodation for testing or note taking, please visit AccessAbility Services, located in White Hall 005 (http://www.wcsu.edu/accessability).