



Dr. Charles Rocca
Higgins Hall 101D
roccac@wcsu.edu
<http://sites.wcsu.edu/roccac>

MAT 207 : Proofs
MR: 12:30 pm - 1:45 pm,
Higgins 116



Office Hours:

- M: 2:00pm - 3:30pm
- W: 1:00pm - 3:30pm
- R: 2:00pm - 3:00pm
- or by appointment

Course Materials:

Textbook: *How to Think Like a Mathematician:
A Companion to Undergraduate Mathematics*
by Kevin Houston

Course Description and Objectives:

An introduction to the theory and practice of reading and writing mathematical proofs, using theorems and problems in number theory as a source of examples. Prerequisite: C or better in MAT 141: Foundational Discrete Mathematics

1. Successful students in this course will be able to read and write well-organized and correct proofs of mathematical statements. In particular, students will be able to:
 - (a) recognize reasoning and proof as fundamental aspects of mathematics
 - (b) make and investigate mathematical conjectures
 - (c) develop and evaluate mathematical arguments and proofs
 - (d) select and use various types of reasoning and methods of proof
2. Students will demonstrate knowledge of introductory number theory. In particular, students will be able to demonstrate an understanding of:
 - (a) the fundamental properties of the integers, in particular primes
 - (b) modular arithmetic
 - (c) significant number theoretic functions

Course Content:

Unit	Chapters
Reading Math and Thinking Logically	1-13
Analysis of Statements	14-19
Direct Proof	20-22
Indirect Proofs	23,26
Induction Proofs	24,25
Some (More) Number Theory	27-31 & Supplements

Grading:

2 × Reading and Writing Exams	20% (10% each)
3 × Proof Exams	30% (10% each)
1 × Number Theory Exam	10%
Various Assignments	30%
Proofs Portfolio	10%

Reading and Writing Exams: On these exams you will need to demonstrate a basic ability to read and write mathematics, and to understand logical statements. To do this you will be asked to

- rewrite or reformat some statements,
- proofread and when necessary correct samples of writing, and
- solve some basic problems and write up their solutions adhering to standard conventions.

Proof Exams: For each proof technique you will have a two part exam. For the first part you will be given an example or examples of the proof technique and you will need to fill in details which have been left out. For the second part of the exam you will be given a theorem statement and you will need to construct a proof of that theorem from scratch using the specified technique. After each exam you will be allowed to hand in corrections in order to earn back up to 33% of the points lost on the exam.

Number Theory Exam: This exam will be a test of your knowledge of number theory. About half of the exam will focus on the last unit we will cover in class; the remainder will pull material we touch on throughout the semester. This exam will be given during final exam week.

Assignments: Throughout the semester you will be given assignments to help you practice the techniques we learn in class. Typically you will be given opportunities to start these in class but will need to finish them, or at least write them up neatly, outside of class. For assignments which require you to write proofs you may revise them up to two times making corrections in order to raise your grade, your best attempt at the proof is the one I will record in the grade book. Most assignments may be done as part of a group, however your group can not have more than three people in it and if you complete an assignment as part of a group then you must only turn in one copy of the assignment. If you turn in multiple copies of a group assignment I will deduct points. Final submissions for all assignments must be typed and in complete sentences. I should be able to tell what your question was without rereading it and without you copying the question. The quality of your work can count for up to 10% of the assignment grade.

Proofs Portfolio: At the end of the semester you must compile an example of each type of proof which you have written and revised. Thus you will need to have examples of a:

1. direct proof,
2. proof by cases,
3. proof by contradiction,
4. proof by contrapositive, and
5. proof by induction.

For each one you must turn in your rough work, all drafts of your proof, and a final draft of the proof. **Thus, you must hold onto all of your work throughout the semester!!!**

Course Calendar:

MONDAY	THURSDAY
8/29 Syl., Intro., & Building Up Ideas	9/1 Reading Mathematics
9/5 Labor Day - No Class	9/8 Writing Mathematics
9/12 Proofreading Mathematics, the Rewrite	9/15 Solving Problems
9/19 Statements, Implications, and Their Kin	9/22 Review of Reading and Writing
9/26 <i>Reading, Writing, and Statements Exam</i>	9/29 Reading Definitions and Theorems
10/3 What is a Proof and How to Read It	10/6 Pythagorean Case Study
10/10 Review Definitions, Theorems, and Proofs	10/13 <i>Definitions, Theorems, and Proofs Exam</i>
10/17 Direct Proofs	10/20 Common Mistakes
10/24 Proof By Cases and Review	10/27 <i>Direct Proof Exam</i>
10/31 Indirect Proof: Proof by Contradiction	11/3 Indirect Proof: Proof by Contrapositive
11/7 Review of Indirect Proofs	11/10 <i>Indirect Proof Exam</i>
11/14 Basic Proofs By Induction	11/17 Practice with Induction
11/21 <i>Induction Exam</i>	11/24 Thanksgiving Day - No Class
11/28 Divisibility and the Euclidean Algorithm	12/1 Chinese Remainder Theorem
12/5 Euler's φ - Function	12/8 Theorems of Wilson, Fermat, and Euler

Number Theory Exam - December 15th, 11:00am - 1:30pm

Departmental Course Outline:

1. Proof Topics

(a) Getting Started:

- i. Reading the statement of a theorem
- ii. Quantification
- iii. Getting from the statement to the start of the proof

(b) Types of Proofs:

- i. Direct proof
- ii. Examination of cases
- iii. Proof by contradiction
- iv. Proof by contrapositive
- v. Equivalence (or if and only if) proofs
- vi. Mathematical induction and the well-ordering principle (and infinite descent)

(c) Other Significant Topics:

- i. Negation and DeMorgan's law
- ii. Contrapositive
- iii. Contrapositive vs. Converse
- iv. Mathematical experiments "how do you know what to try to prove?"

2. Number Theory Topics:

(a) Primes:

- i. Properties of prime numbers
- ii. Prime factorization
- iii. Gaussian primes

(b) Divisibility:

- i. gcd
- ii. Euclidean algorithm
- iii. lcm

(c) Properties of Integers:

- i. Sums of squares
- ii. Figurate numbers
- iii. Pythagorean triples

(d) Modular arithmetic and algebra

(e) Number theoretic functions

The content in this course addresses the *Expertise in Content Knowledge* portion of the WCSU Education Program Conceptual Framework. When you have successfully completed this course you will have added to the body of knowledge necessary to teach mathematics in Connecticut public schools.

You and Your Grades:

- “A” (Exceptional) range 90% to 100%:
The student has demonstrated significant mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and most nonstandard problems which require deeper insight.
 - “A” $\iff 92.5\% \leq \text{Grade} \leq 100\%$
 - “A-” $\iff 90\% \leq \text{Grade} < 92.5\%$
- “B” (Good) range 80% to 90%:
The student has demonstrated mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and some nonstandard problems which require deeper insight.
 - “B+” $\iff 87.5\% \leq \text{Grade} < 90\%$
 - “B” $\iff 82.5\% \leq \text{Grade} < 87.5\%$
 - “B-” $\iff 80\% \leq \text{Grade} < 82.5\%$
- “C” (Adequate) range 70% to 80%:
The student has demonstrated adequate mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve most standard formulaic exercises but struggles with nonstandard problems which require deeper insight.
 - “C+” $\iff 77.5\% \leq \text{Grade} < 80\%$
 - “C” $\iff 72.5\% \leq \text{Grade} < 77.5\%$
 - “C-” $\iff 70\% \leq \text{Grade} < 72.5\%$
- “D” (Inadequate) range 60% to 70%:
The student has demonstrated inadequate or incomplete mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve some standard formulaic exercises but few if any nonstandard problems which require deeper insight.
 - “D+” $\iff 67.5\% \leq \text{Grade} < 70\%$
 - “D” $\iff 62.5\% \leq \text{Grade} < 67.5\%$
 - “D-” $\iff 60\% \leq \text{Grade} < 62.5\%$
- “F” (Unacceptable) below 60%:
The student has demonstrated essentially no mastery of the appropriate knowledge and skills relevant to the course. The student is unable to solve most standard formulaic exercises and essentially no nonstandard problems which require deeper insight.

End User Agreement:

General Expectations: As a student in this class you are expected to:

- attend class and take notes,
- actively read material in each section, taking notes,
- review your notes on a regular basis,
- check your university email every day,
- check the class site *at least* every other day,
- begin studying for exams in a timely fashion,
- ask questions early and often,
- attend office hours,
- seek help in the math clinic or tutoring center, and
- complete assignments and readings on time.

Assignment Guidelines: (These apply to *all out of class work*.)

- Work handed in must always look neat, legible, and professional. Work must be very neatly written or preferably typed. The quality of your work will be factored into your grade, up to 10%, in extreme cases work may be rejected and then counted as late.
- Answers on all assignments should be given in complete sentences. I should be able to tell what your answer means without re-reading the problem. This does not mean you simply rewrite the question.
- An assignment is considered late after I have handed it back or gone over it in class. Late assignments are accepted but may receive at most 75% credit. Late assignments go to the absolute bottom of the stack of papers to be graded; *all on time work is graded before any late work*.
- If you work on an assignment as part of a group, then there may be no more than three individuals in the group and all your names must be on the assignment. You should hand in only one copy of the work.
- All work must be submitted in the manner directed.

Email Etiquette Guidelines: When sending an email you must include the course number and semester in the subject line. For example, if you are taking MAT 314 in Fall 1592 then the the subject line should begin with “[MAT 314 Fall 1592].” Also, you should always begin with a salutation such as “Dear Dr. Rocca” and end with a closing such as “Sincerely, I. Newton.”

Exam Makeup Policy: To qualify for a makeup exam you must have a valid reason for missing the exam and, if at all possible, let me know ahead of time that you are missing the exam. You will need to meet with me in order to arrange a time for the make up exam. If you do not have a valid reason, do not give prior notice when possible, or simply do not show up for an exam, you are not entitled to a makeup and will not be given one. If you fail to show up for your makeup exam, you will not be given a second opportunity.

The 2% Exception: Any quiz or class work which is ultimately worth no more than 2% of your final grade can not be made up.

Time on Task: As a 3 credit class you should expect to average 7 to 8 hours of work a week including class time. Some weeks you may get away with less and some may require more.

Attendance: There is no specific policy for attendance in this course. However, if you have *three consecutive unexcused absences* within the first half of the semester I am required to report to the University that you have *stopped attending*.

Academic Honesty: If on any assignment, quiz, or exam you turn in someone else’s work, regardless of the source, as if it were your own you will receive a zero on that assignment, quiz, or exam. If you are caught doing this three times you will receive an F in the course and the Dean will be informed of your academic dishonesty.

(<https://www.wcsu.edu/faculty-handbook/2019-2020/policies-pertaining-to-students/academic-honesty-policy/>)

Accommodations: If you have need of an accommodation for testing or note taking, please visit AccessAbility Services, located in White Hall 005 (<http://www.wcsu.edu/accessability>).