

Instructions

Below are 15 practice exam problems which you must turn in when you come in to take your final exam on Monday December 13th, Exam Period 3: 2pm - 4:30pm; as stated on the syllabus, these make up 10% of your exam grade and must be written or typed up neatly on separate paper and in accordance with the guidelines on the syllabus. In addition *take time to look at the additional exercises* that have been listed on previous practice exams; they are similar to what you will see and their answers are in the text.

Logic and Stuff:

- Find the truth set for the given predicate with the given domain.

(a) $Q(d) : 6d < 20$, if the domain is \mathbb{N} .

(b) $Q(d) : 6d < 20$, if the domain is \mathbb{R} .

- Write the negation of

$$\exists x \in \mathbb{R} : x^2 = 2 \wedge x \in \mathbb{Q},$$

and then write both statements as sentences in English.

- Rewrite “*cats are aliens*” as an “if - then” statement and then write its contrapositive, converse, inverse, and negation.

Sequences and Stuff:

- Write the first four terms of the sequence defined below, begin with c_1 .

$$c_n = \frac{(-3)^n}{n}$$

- Write an explicit formula for the sequence below.

$$\frac{1}{2}, -\frac{1}{5}, \frac{1}{10}, -\frac{1}{17}, \frac{1}{26}, -\frac{1}{37}, \frac{1}{50}, -\frac{1}{65}, \frac{1}{82}, -\frac{1}{101}, \dots$$

- Use the formula

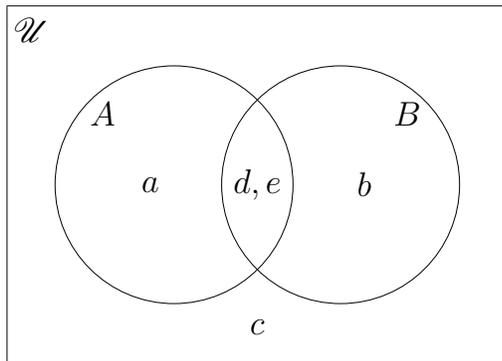
$$\sum_{i=0}^n r^i = \frac{r^{n+1} - 1}{r - 1}$$

to find the sum of $S = 16 + 64 + 256 + 1024 + \dots + 4^{10}$.

- Use iteration to find an explicit formula for the recursively defined sequence $a_k = 2 + 4a_{k-1}$ where $a_0 = 2$.

Sets and Stuff:

8. Given the Venn diagram find each of the following:¹



- (a) \mathcal{U}
- (b) $\mathcal{P}(A \cap B)$
- (c) $A \setminus B$
- (d) $A \times B^c$
- (e) $A^c \cap B$

9. Let $\mathbb{S} = (\mathbb{Z} \times \mathbb{Z}) \setminus \{(0, 0)\}$ and define $p : \mathbb{S} \rightarrow \mathbb{Z}$ by $\forall (a, b) \in \mathbb{S} : p(a, b) = a^b$.

- (a) What is the *domain* of p ?
- (b) What is the *co-domain* of p ?
- (c) Give a counter example to show that p is not one-to-one.
- (d) Prove that p is onto.

10. Let \mathcal{D} be the relation defined on \mathbb{R} as follows:

$$\forall x, y \in \mathbb{R} : x \mathcal{D} y \iff x + y \geq 0.$$

Determine if the relation \mathcal{D} is *reflexive*, *symmetric*, *transitive*, or none of these.

Counting and Stuff:

- 11. **Sea-Air-Land Pizza**TM offers pizzas with your choice of 2 out of 10 vegetables, 2 out of 4 cheeses, 1 out of 3 types of dough, and either anchovies, duck, or pork. How many different pizzas can you order?
- 12. Suppose that a student council consists of 15 members, 8 men and 7 women.
 - (a) In how many ways can you form a six person committee with 3 men and 3 women?
 - (b) In how many ways can you form a six person committee with at least 1 woman and 1 man?
- 13. If a store sells seven different varieties of soda and they have at least 24 of each in stock, then how many ways can you pick out 24 sodas?
- 14. Assuming face cards are worth 10 points, aces are worth 1 point, and all other cards have the point value listed on them, how many cards must you select from a standard deck of 52 cards to guarantee that the total of all the cards you have drawn is over 10?
- 15. Use the *Binomial Theorem* to expand $(v - u)^4$

¹ \mathcal{P} indicates the power set and \mathcal{U} is the universal set.