Sets
Concepts and Laws[1]

In the lists on these pages:

(a) A, B, C, X, Γ, and A_γ, for each $\gamma \in \Gamma$ are all sets.

(b) $\mathscr{F}$ is a set of sets.

(c) $\mathscr{U}$ is the universal set (i.e. a set containing all the elements under consideration).

(d) $p(x)$ is a statement about the variable x.

I. Concepts

Symbols	Meaning (Definition or Statement in English)	Abbreviation
1. $x \in A$	x is an element of the set A	—
2. $x \notin A$	x is not an element of the set A	—
3. $\{x \in A \mid p(x)\}$	the set of all x in A such that $p(x)$ is true	—
4. $\{x \mid p(x)\}$	the set of all x in $\mathscr{U}$ such that $p(x)$ is true	—
5. $\varnothing$	the empty set $\equiv$ the set with no elements	—
6. $A \subseteq B$	A is a subset of $B \equiv \forall x\ (x \in A) \to (x \in B)$	Def. $\subseteq$
7. $A = B$	A is equal to $B \equiv \forall x\ (x \in A) \leftrightarrow (x \in B)$ $\equiv (A \subseteq B) \wedge (B \subseteq A)$	Def. $=$
8. $A \subset B$	A is a proper subset of $B \equiv (A \subseteq B) \wedge (A \neq B)$	Def. $\subset$
9. $\mathscr{P}(A)$	the power set of $A \equiv$ the set of all subsets of A $\equiv \{X \mid X \subseteq A\}$	Def. of Power Set
10. $A \cup B$	A union $B \equiv \{x \mid (x \in A) \vee (x \in B)\}$	Def. $\cup$
11. $A \cap B$	A intersection $B \equiv \{x \mid (x \in A) \wedge (x \in B)\}$	Def. $\cap$
12. $A - B$	A minus $B \equiv \{x \mid (x \in A) \wedge (x \notin B)\}$	Def. $-$
13. A^c	the compliment of $A \equiv \{x \mid x \notin A\}$, sometimes denoted A' or A^c	Def. Comp.
14. $\{A_\gamma \mid \gamma \in \Gamma\}$	the family of sets A_γ indexed by the set Γ	—
15. $\bigcup_{\gamma \in \Gamma} A_\gamma$	the union of the sets A_γ indexed by the set Γ $\equiv \{x \mid \exists \gamma \in \Gamma\ \ (x \in A_\gamma)\}$	—
16. $\bigcap_{\gamma \in \Gamma} A_\gamma$	the intersection of the sets A_γ indexed by the set Γ $\equiv \{x \mid \forall \gamma \in \Gamma\ \ (x \in A_\gamma)\}$	—
17. $\bigcup_{A \in \mathscr{F}} A$	the union of the sets A in the family $\mathscr{F}$ $\equiv \{x \mid \exists A \in \mathscr{F}\quad (x \in A)\}$	—
18. $\bigcap_{A \in \mathscr{F}} A$	the intersection of the sets A in the family $\mathscr{F}$ $\equiv \{x \mid \forall A \in \mathscr{F}\quad (x \in A)\}$	—

[1]Originally created by Vasily C. Cateforis for Set Theory and Logic at SUNY Postdam based on the text *Set Theory with Applications* by Lin & Lin

II. Laws

Symbols	Name
1a. $\forall A\ \varnothing \subseteq A$ and 1b. $\forall A\ A \subseteq A$	—
2. $A \subseteq B \wedge B \subseteq C \Rightarrow A \subseteq C$	Transitivity of $\subseteq$
3a. $A \cup \varnothing = A$ and 3b. $A \cap \mathscr{U} = A$	Identity Laws
4a. $A \cap \varnothing = \varnothing$ and 4b. $A \cup \mathscr{U} = \mathscr{U}$	Domination Laws
5a. $A \cap A = A$ and 5b. $A \cup A = A$	Idempotency Laws
6a. $A \cup B = B \cup A$ and 6b. $A \cap B = B \cap A$	Commutativity Laws
7a. $A \cup (B \cup C) = (A \cup B) \cup C$ and 7b. $A \cap (B \cap C) = (A \cap B) \cap C$	Associativity Laws
8. $A \subseteq A \cup B \ \wedge\ B \subseteq A \cup B$	Addition Laws
9. $A \cap B \subseteq A \ \wedge\ A \cap B \subseteq B$	Simplification Laws
10a. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 10b. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive Laws
11. $A - B = A \cap B^c$	—
12. $(A^c)^c = A$	Double Compliment
13a. $\varnothing^c = \mathscr{U}$ and 13b. $\mathscr{U}^{\,c} = \varnothing$	—
14a. $A \cap A^c = \varnothing$ and 14b. $A \cup A^c = \mathscr{U}$	Inverse Laws
15a. $(A \cap B)^c = A^c \cup B^c$ and 15b. $(A \cup B)^c = A^c \cap B^c$	DeMorgan's Laws
16. $A \subseteq B \Leftrightarrow B^c \subseteq A^c$	Contrapositive Law
17a. $A \cup (A \cap B) = A$ and 17b. $A \cap (A \cup B) = A$	Absorbtion Laws