Some TAUTOLOGIES[1]

Let p, q, r, and s be arbitrary statements, let t be a tautology, and let c be a contradiction, then:

	Tautology	Name	Abbreviation
0.	$p \to q \equiv \sim(p \wedge \sim q)$	Conditional Law	(Def. $\to$)
1.	$p \Rightarrow p \vee q$	Law of Addition	(Add.)
2.	(a) $p \wedge q \Rightarrow p$	Laws of Simplification	(Simp.)
	(b) $p \wedge q \Rightarrow q$		
3.	$(p \vee q) \wedge \sim p \Rightarrow q$	Disjunctive Syllogism	(D.S.)
4.	$\sim(\sim p) \equiv p$	Law of Double Negation	(D.N.)
5.	(a) $p \wedge q \equiv q \wedge p$	Commutative Laws	(Com.)
	(b) $p \vee q \equiv q \vee p$		
6.	(a) $p \wedge p \equiv p$	Laws of Idempotency	(Idemp.)
	(b) $p \vee p \equiv p$		
7.	$(p \to q) \equiv (\sim q \to \sim p)$	Contrapositive Law	(Contrap.)
8.	(a) $\sim(p \wedge q) \equiv \sim p \vee \sim q$	DeMorgan's Laws	(DeM.)
	(b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$		
9.	(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws	(Assoc.)
	(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$		
10.	(a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws	(Dist.)
	(b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$		
11.	$(p \to q) \wedge (q \to r) \Rightarrow (p \to r)$	Transitive Law	(Trans.)
12.	(a) $(p \to q) \wedge (r \to s) \Rightarrow (p \vee r \to q \vee s)$	Constructive Dilemmas	(C.D.)
	(b) $(p \to q) \wedge (r \to s) \Rightarrow (p \wedge r \to q \wedge s)$		
13.	(a) $(p \to q) \wedge (r \to s) \Rightarrow (\sim q \vee \sim s \to \sim p \vee \sim r)$	Destructive Dilemmas	(D.D.)
	(b) $(p \to q) \wedge (r \to s) \Rightarrow (\sim q \wedge \sim s \to \sim p \wedge \sim r)$		
14.	(a) $(p \to q) \wedge p \Rightarrow q$	Modus Ponens	(M.P.)
	(b) $(p \to q) \wedge \sim q \Rightarrow \sim p$	Modus Tollens	(M.T.)
	(c) $(p \to q) \Leftrightarrow (p \wedge \sim q \to c)$	Reductio ad absurdum	(R.A.)
15.	(a) $(p \wedge t) \Leftrightarrow p$	Identity Laws	(I.L.)
	(b) $(p \vee c) \Leftrightarrow p$		
16.	(a) $(p \vee t) \Leftrightarrow t$	Domination Laws	(D.L.)
	(b) $(p \wedge c) \Leftrightarrow c$		
17.	(a) $p \vee \sim p \to t$	Inverse Laws	(Inv.)
	(b) $p \wedge \sim p = c$		
18.	(a) $p \vee (p \wedge q) \Leftrightarrow p$	Absorption Laws	(A.L.)
	(b) $p \wedge (p \vee q) \Leftrightarrow p$		

[1] Originally created by Vasily C. Cateforis for Set Theory and Logic at SUNY Postdam based on the text *Set Theory with Applications* by Lin & Lin

	Tautology	Name	Abbreviation
19.	(a) $c \Rightarrow p$		
	(b) $p \Rightarrow t$		
20.	p and $q \;\Rightarrow\; p \wedge q$	Conjunctive inference	(Conj.)
21.	$(p \wedge q \rightarrow r) \equiv (p \rightarrow (q \rightarrow r))$	Exportation Law	(Exp.)

Further Notes

22. p is sufficient for $q \equiv p \rightarrow q \;\equiv\; p$, only if q.

23. p is necessary for $q \equiv q \rightarrow p \;\equiv\; p$, if q.

24. p is necessary and sufficient for $q \equiv p \leftrightarrow q \;\equiv\; p$, iff q.

25. Quantifier Negation (QN):

(a) $\sim [(\forall x)\,(p(x))] \equiv (\exists x)\,(\sim p(x))$

(b) $\sim [(\exists x)\,(p(x))] \equiv (\forall x)\,(\sim p(x))$

26. Principle of Mathematical Induction (PMI): If $P(n)$ is a statement about or involving the natural number n such that

(1) $P(1)$ is true, and

(2) $P(k) \Rightarrow P(k+1)$ for all natural numbers k, then

$P(n)$ is true for all natural numbers n.

26. The four basic truth tables:

(a) Negation

p	$\sim p$
T	F
F	T

(b) Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(c) Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(d) Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T