



Dr. Charles Rocca  
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<http://sites.wcsu.edu/roccac>

MAT 512/467 - 01: Group Theory  
On Ground: Higgins 228, T 5:30-8pm  
Credits: 3 credit  
Grading: Standard A-F



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## Office Hours:

Office hours are on ground for the Fall 2021 Semester. If you need to meet virtually we can make an appointment to do so via my WebEx Virtual Office:

Higgins 101-DV (<https://wcsu.webex.com/meet/roccac>)

- Monday: 1pm - 2pm, 3:30pm - 4:30pm
- Tuesday: 2:30pm - 4:30pm
- Thursday: 1pm - 2pm
- or by appointment

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## Course Materials:

- "Contemporary Abstract Algebra, 10<sup>th</sup> Edition" - Joseph A. Gallian (ISBN-13 : 978-0367651787) The 9<sup>th</sup> edition would be fine as well we would just need to work out the difference in exercises.

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## Course Description and Outcomes:

Groups are one of the fundamental mathematical objects that help us to understand how and why things work as they do in mathematics. They are significant in their own right, but are also important in applications of mathematics to physics, chemistry, and information security. As such, students can greatly benefit from a clear understanding of their properties and structures. Prerequisite: MAT 375: Algebraic Structures, or equivalent

Upon successful completion of this course, students will be able to:

- Define and give examples of the fundamental algebraic structures,
- State and describe the importance of Lagrange's Theorem, Cauchy's Theorem, Cayley's Theorem, the Isomorphism Theorems, Sylow's Theorems, and the Fundamental Theorem of Finite Abelian Groups,
- Construct proofs, or outlines of proofs for, Lagrange's Theorem, Cayley's Theorem, Cauchy's Theorem, and the First Isomorphism Theorem
- Analyze hypotheses of theorems and definitions and through examples and non-examples explain why they are written the way they are, and
- Solve problems by choosing and applying appropriate theorems.

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## Course Content:

Part I: Basics	
Introduction, Definitions, and Examples -	Chapters 0-2
Finite and Cyclic Groups-	Chapters 3 & 4
Part II: Subgroups	
Permutation Groups and Cayley's Theorem -	Chapters 5 & 6
Cosets, Subgroups, and Products -	Chapters 7-9
Part III: Group Structure	
Morphisms -	Chapters 6 & 10
Group Structure -	Chapters 11, 23, & 25

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## Grading:

Problem Solving Exercises	14%
Unit Skills Exercises	14%
Exams	72% (24% per exam)

**Problem Solving Exercises:** These exercises will come from your text unless we run across something interesting. You need to do four of these per part. They will all require problem solving, these will not be algorithmic problems solved by wrote memorization.

**Skills Exercises:** These exercises will come from your text unless we run across something interesting. You need to do six of these per part. They are largely algorithmic problems solved by wrote memorization or algorithm.

**Exams:** You will have three exams, one for each part. The purpose of these exams are to check that you know your fundamental definitions, examples, theorems, and (occasionally) algorithms. The content of these exams represents the fundamental knowledge you should get from the class. Definitions in particular are important to your understanding, as such 30% of each exam will be definitions.

## Course Calendar:

TUESDAY		
8/31		1
Syllabus, course overview, review of some Proofs stuff		<b>Chapter 0</b>
9/7		2
Overview of Algebraic Structures, basic group definitions and examples		<b>Chapters 1 &amp; 2</b>
9/14		3
More on groups, properties, and finite groups		<b>Chapters 2 &amp; 3</b>
9/21		4
Subgroups, and cyclic groups		<b>Chapters 3 &amp; 4</b>
9/28		5
<b>Part I Exam</b>		
10/5		6
Permutations, cycles, and transpositions		<b>Chapter 5</b>
10/12		7
Definitions and properties of isomorphisms, Cayley's Theorem		<b>Chapter 6</b>
10/19		8
Cosets and Lagrange's Theorem		<b>Chapter 7</b>
10/26		9
External and internal direct products		<b>Chapters 8 &amp; 9</b>
11/2		10
Normal subgroups and factor groups (aka quotient groups)		<b>Chapter 9</b>
11/9		11
<b>Part II Exam</b>		
11/16		12
Homomorphisms, isomorphisms, kernels, and the First Isomorphism Theorem		<b>Chapters 6 &amp; 10</b>
11/23		13
Review internal and external direct products, and the Fundamental Theorem of Finite Abelian Groups		<b>Chapter 11</b>
11/30		14
Sylow's First Theorem and Cauchy's Theorem		<b>Chapter 23</b>
12/7		15
Generators, relations, and the Universal Mapping Theorem		<b>Chapter 25</b>
12/14		16
<b>Part III Exam</b>		

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## Course Outline:

### 1. Review of Algebraic Structures

#### (a) Rings

- i. Subrings
- ii. Ideals
- iii. Quotient Rings
- iv. Homomorphisms

#### (b) Domains and Fields

- i. Zero Divisors versus Units
- ii. Integral Domains
- iii. Finite and Infinite Fields

### 2. Common Group Examples

#### (a) Abelian Groups

#### (b) Matrix Groups

#### (c) Dihedral Groups

#### (d) Symmetric Groups

### 3. Structure Theorems

#### (a) Lagrange's Theorem

#### (b) Sylow's Theorems

#### (c) Cauchy's Theorem

#### (d) Fundamental Theorem of Finite Abelian Groups

### 4. Group Actions

#### (a) Cayley Graphs

- i. Representing Groups Graphically
- ii. Groups Acting on Graphs
- iii. Groups acting as permutations
- iv. Cayley's Theorem

#### (b) Groups Acting on Spaces

- i. Linear Transformations: Rotations, Projections, Dilations, Reflections
- ii. Affine Transformations: Linear + Translation

### 5. Morphisms of Groups

#### (a) Homomorphisms (Revisited)

- i. Domain and Codomain
- ii. Range and Kernel

#### (b) Isomorphisms

- i. General Definitions
- ii. Isomorphism Theorems

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## You and Your Grades:

- “A” (Exceptional) range 90% to 100%:  
The student has demonstrated significant mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and most nonstandard problems which require deeper insight.
  - “A”  $\iff 92.5\% \leq \textit{Grade} \leq 100\%$
  - “A-”  $\iff 90\% \leq \textit{Grade} < 92.5\%$
  
- “B” (Good) range 80% to 90%:  
The student has demonstrated mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and some nonstandard problems which require deeper insight.
  - “B+”  $\iff 87.5\% \leq \textit{Grade} < 90\%$
  - “B”  $\iff 82.5\% \leq \textit{Grade} < 87.5\%$
  - “B-”  $\iff 80\% \leq \textit{Grade} < 82.5\%$
  
- “C” (Adequate) range 70% to 80%:  
The student has demonstrated adequate mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve most standard formulaic exercises but struggles with nonstandard problems which require deeper insight.
  - “C+”  $\iff 77.5\% \leq \textit{Grade} < 80\%$
  - “C”  $\iff 72.5\% \leq \textit{Grade} < 77.5\%$
  - “C-”  $\iff 70\% \leq \textit{Grade} < 72.5\%$
  
- “D” (Inadequate) range 60% to 70%:  
The student has demonstrated inadequate or incomplete mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve some standard formulaic exercises but few if any nonstandard problems which require deeper insight.
  - “D+”  $\iff 67.5\% \leq \textit{Grade} < 70\%$
  - “D”  $\iff 62.5\% \leq \textit{Grade} < 67.5\%$
  - “D-”  $\iff 60\% \leq \textit{Grade} < 62.5\%$
  
- “F” (Unacceptable) below 60%:  
The student has demonstrated essentially no mastery of the appropriate knowledge and skills relevant to the course. The student is unable to solve most standard formulaic exercises and essentially no nonstandard problems which require deeper insight.

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## End User Agreement:

**General Expectations:** As a student in this class you are expected to:

- attend class and take notes,
- actively read material in each section, taking notes,
- review your notes on a regular basis,
- check your university email every day,
- check the class site **at least** every other day,
- begin studying for exams in a timely fashion,
- ask questions early and often,
- attend office hours,
- seek help in the math clinic or tutoring center, and
- complete assignments and readings on time.

**Assignment Guidelines:** (These apply to **all out of class work**.)

- Work handed in must always look neat, legible, and professional. Work must be very neatly written or preferably typed. The quality of your work will be factored into your grade, up to 10%, in extreme cases work may be rejected and then counted as late.
- Answers on all assignments should be given in complete sentences. I should be able to tell what your answer means without re-reading the problem. This does not mean you simply rewrite the question.
- An assignment is considered late after I have handed it back or gone over it in class. Late assignments are accepted but may receive at most 75% credit. Late assignments go to the absolute bottom of the stack of papers to be graded; **all on time work is graded before any late work.**
- If you work on an assignment as part of a group, then there may be no more than three individuals in the group and all your names must be on the assignment. You should hand in only one copy of the work.
- All work must be submitted in the manner directed.

**Email Etiquette Guidelines:** When sending an email you must include the course number and semester in the subject line. For example, if you are taking MAT 314 in Fall 1592 then the the subject line should begin with “[MAT 314 Fall 1592].” Also, you should always begin with a salutation such as “Dear Dr. Rocca” and end with a closing such as “Sincerely, I. Newton.”

**Exam Makeup Policy:** To qualify for a makeup exam you must have a valid reason for missing the exam and, if at all possible, let me know ahead of time that you are missing the exam. You will need to meet with me in order to arrange a time for the make up exam. If you do not have a valid reason, do not give prior notice when possible, or simply do not show up for an exam, you are not entitled to a makeup and will not be given one. If you fail to show up for your makeup exam, you will not be given a second opportunity.

**The 2% Exception:** Any quiz or class work which is ultimately worth no more than 2% of your final grade can not be made up.

**Time on Task:** As a 3 credit class you should expect to average 7.5 to 8.5 hours of work a week including class time. Some weeks you may get away with less and some may require more.

**Attendance:** There is no specific policy for attendance in this course. However, please keep the following in mind, if you have three consecutive unexcused absences within the first half of the semester I am required to report to the University that you have stopped attending.

**Academic Honesty:** If on any assignment, quiz, or exam you turn in someone else’s work, regardless of the source, as if it were your own you will receive a zero on that assignment, quiz, or exam. If you are caught doing this three times you will receive an F in the course and the Dean will be informed of your academic dishonesty.

(WCSU Honesty Policy: <https://www.wcsu.edu/faculty-handbook/2019-2020/policies-pertaining-to-students/academic-honesty-policy/>)

**Accommodations:** If you have need of an accommodation for testing or note taking, please visit AccessAbility Services, located in White Hall 005 (<http://www.wcsu.edu/accessability>).