

## Instructions

Below is the practice exam which you must turn in when you come in to take your fourth exam on Thursday 12/2/2021; this makes up 5% of your exam grade and must be written up or typed up neatly on separate paper and in accordance with the guidelines in your syllabus. Your exam will have the same format as the practice exam. In addition there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam.

## Additional Practice Problems:

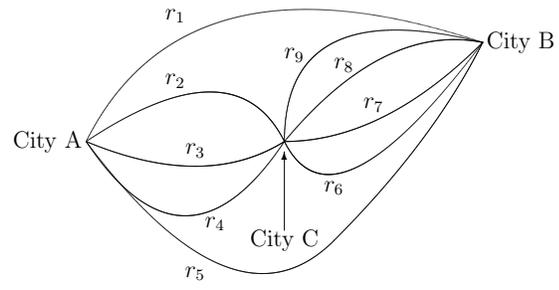
(listed by section and problem number)

- § 9.1: 3, 11a, 13a, 16, 18, 21, 24, 26, 28;
- § 9.2: 1, 4, 6, 9, 11ab, 14abd, 16, 19, 21, 34;
- § 9.3: 3, 6, 11, 14, 33;
- § 9.4: 1, 3, 9, 10, 14, 20, 24;
- § 9.5: 3, 6, 8, 15, 19;
- § 9.6: 3, 5, 10, 11;
- § 9.7: 1, 3, 5, 10, 19, 23, 25, 29, 31, 37;

## Practice Exam

### General Counting and Probability:

1. What is the probability of the event that a single card chosen from a standard deck of 52 cards is black and has an even number on it?
2. How many positive two digit integers are multiples of three and what is the probability that a randomly chosen positive two digit integer is a multiple of 3?
3. Suppose that in a certain state all license plates consist of four letters followed by three digits.
  - (a) How many different license plates are possible?
  - (b) How many plates begin with A and end with 0?
  - (c) On how many plates are all the letters and digits distinct?
4. Use the map below to answer the following:
  - (a) In how many ways can you travel from *City A* to *City B*?
  - (b) In how many ways can you travel from *City A* to *City B* with a stop in *City C*?



**Combinations with and without replacement:**

5. Suppose that a student council consists of 15 members, 8 men and 7 women.
  - (a) In how many ways can you form a six person committee with 3 men and 3 women?
  - (b) In how many ways can you form a six person committee with at least 1 women?
6. How many distinguishable ways can the letters of the word *HULLABALOO* be arranged?
7. If  $n$  is a positive integer, in how many ways can you pick 4 integers,  $i, j, k, m$ , so that  $1 \leq i \leq j \leq k \leq m \leq n$ ? (Note that the integers can be equal.)
8. If  $n$  is a positive integer how many solutions are there to the equation

$$x_1 + x_2 + x_3 = n$$

if  $x_i \geq 0 \forall i$ ? What if  $x_i \geq 1 \forall i$ ? (Note that your answer will be in terms of  $n$ .)

**Pigeon Hole Principle:**

9. How many cards must you select from a standard deck of 52 cards in order to guarantee that two of them are the same suit? For a 5 point bonus, how many cards must you select to have a better than 50% chance of getting the same suit twice?
10. In repeated divisions by 2373 how many distinct remainders can be obtained? If you were to write the decimal expansion of  $173/2373$  what is the longest possible length of the repeating section of the representation? (Don't actually calculate it!)
11. Show that in any set of thirteen integers chosen from 2 through 40 there must be at least two with a divisor greater than 1.

**Binomial Theorem:**

12. Use the *Binomial Theorem* to expand  $(p - 2q)^4$
13. Use the *Binomial Theorem* to find the coefficient for  $a^5b^7$  in  $(a - 2b)^{12}$
14. Use the *Binomial Theorem* to prove that for all integers  $n \geq 0$ ,

$$3^n = \binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n}$$

15. (Challenge Problem worth 15 points) Find the coefficient of  $x^3y^2z^5$  in the expansion of  $(x + y + z)^{12}$ .