Instructions

Below is the practice exam which you must turn in when you come in to take your third exam on Monday 11/1/2021; these make up 5% of your exam grade and must be written up or typed up neatly on separate paper and in accordance with the guidelines in your syllabus. Your exam will have the same format as the practice exam. In addition there is a long list of practice problems from the text which you do not need to turn in but are representative of the sorts of questions which may be on the exam.

Additional Practice Problems:

§6.1: 3ab, 5, 11, 19, 28, 31
§6.2: 2a, 3, 25, 27
§7.1: 1, 6a, 7ac, 15, 17ac, 18ac, 19, 21, 23, 33
§7.2: 6a, 7a, 10, 14, 15, 16, 21
§7.3: 1, 3, 9, 16, 23
§8.1: 2, 10, 21
§8.2: 9, 12, 15, 25, 34
§8.3: 1, 5, 8, 20, 29

Practice Exam

Chapter 6: Complete all the problems to the best of your ability.

1. Given $A = \{1, 2\}$ and $B = \{2, 3\}$ find each of the following:[1]

   (a) $\mathcal{P}(A)$ (b) $\mathcal{P}(A \cap B)$ (c) $\mathcal{P}(A \times B)$

2. Let the universal set be the real numbers, $\mathbb{R}$, and let

   $$A = \{x \in \mathbb{R} | 0 < x \leq 2\} \qquad B = \{x \in \mathbb{R} | 1 \leq x < 4\} \qquad C = \{x \in \mathbb{R} | 3 \leq x < 9\}$$

   Find the following:

   (a) $A \cap B$ (b) $(A \cup C)^c$ (c) $(B^c \cap C)$

3. Prove that

   $$(A \cap B) \cap (A \cap B^c) = \emptyset$$

Chapter 7: Complete all the problems to the best of your ability.

1. Student C defines a function $h : \mathbb{Q} \to \mathbb{Q}$ by the rule:

   $$h\left(\frac{m}{n}\right) = \frac{m^2}{n}$$

   Student D claims this function is not well defined. Justify student D's claim.

[1] $\mathcal{P}$ indicates the power set.

2. Let $\mathbb{S}$ be the set of all strings of 0's and 1's and define $l : \mathbb{S} \to \mathbb{Z}$ by

$$\forall s \in \mathbb{S}: \; l(s) = the \; length \; of \; s.$$

   (a) Is l one-to-one? Prove or give a counter example.

   (b) Is l onto? Prove or give a counter example.

3. If $f : X \to Y$ and $g : Y \to Z$ are both functions and $g \circ f$ is one-to-one, must g be one-to-one? Prove or give a counter example.

Chapter 8: Complete all the problems to the best of your ability.

1. Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let $\mathcal{R}$ be the less than relation on $A \times B$, i.e.:

$$\forall \; (a, b) \in A \times B : \; a\mathcal{R}b \Leftrightarrow a < b.$$

   State explicitly which pairs from $A \times B$ are in $\mathcal{R}$ and $\mathcal{R}^{-1}$.

2. Let A be the set of all strings consisting solely of $a's$ and $b's$ of length 4. Define the relation $\mathcal{R}$ on A by

$$\forall \; s, t \in A : \; s\mathcal{R}t \Leftrightarrow s \text{ and } t \text{ have the same first two letters}$$

   Is $\mathcal{R}$ *reflexive, symmetric,* and *transitive*? (Justify your answers.)

3. Define a relation P on the set $\mathbb{R} \times \mathbb{R}$ as follows:

$$\forall \; (w, x), (y, z) \in \mathbb{R} \times \mathbb{R} : \; (w, x)P(y, z) \Leftrightarrow w = y.$$

   Show that P is an equivalnce relation and describe the equivalence classes formed by P.