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## Text:

- “Abstract Algebra: An Introduction, 3<sup>rd</sup> Edition” - Thomas W. Hungerford
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## Course Description and Objectives:

In order to have a proper foundational understanding of the numbers and algorithms we use every day one must have knowledge of groups, rings, and fields. This course will introduce the basic ideas in groups, rings, and fields. There will be particular emphasis on field extensions, factorization, and transformations. Students will learn about the fundamental algebraic structures behind the algorithms for arithmetic and polynomials. They will also understand how one number system is developed from another. Prerequisite: C or better in MAT 207 and C or better in MAT 272.

Upon successful completion of this course, students will be able to:

- State the definitions of ring, integral domain, field, and group, and give examples of different types of each of these structures.
- Demonstrate fluency with quotient structures, in particular an understanding of rings of integers modulo  $n$ .
- Describe the correspondence between the unique prime factorization of integers and factorization of polynomials into irreducible polynomials.
- Explain familiar facts about arithmetic in terms of ring and field properties.
- Explain the relationship between algorithms for arithmetic of numbers in decimal representation and the arithmetic of polynomials.
- Describe various groups of permutations such as symmetric and dihedral groups and their basic properties.
- Describe the algebraic ways in which some number systems are extensions of others, e.g., the extension of the ring of integers to a field yields the rational numbers.

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## Course Content:

- Unit 1 - Ring and Fields: Basic Examples and Definitions  
Chapters 1-3
- Unit 2 - Polynomials and Quotients over rings  
Chapter 4 (4.1-4.4), Chapter 5, Chapter 6
- Unit 3 - Groups and Their Properties  
Chapter 7, Chapter 8 (8.1-8.4)
- Unit 4 - Advanced Topics in Groups  
Chapter 9 (9.1-9.4)

## Grading:

Problem Solving Exercises	16%
Unit Exams (Best 3 of 4)	63% (21% per exam)
Final Exam	21%

**Problem Solving Exercises:** There will typically be 5 or 6 per unit and they all will require problem solving, these will not be algorithmic problems solved by rote memorization.

**Unit Exams:** You will have four unit exams, the top three count toward your grade. The purpose of these exams are to check that you know your fundamental definitions, examples, theorems, and (occasionally) algorithms. The content of these exams represents the fundamental knowledge you should get from the class. Definitions in particular are important to your understanding, as such 40% of each exam will be questions focussed on understanding definitions.

**Final Exam:** The final exam will be cumulative, but otherwise serves the same purpose and has the same form as your unit exams. It will also include material from the last unit on special topics which will not be covered in the other exams.

## Course Calendar:

MONDAY	TUESDAY	WEDNESDAY	THURSDAY
5/24 <b>1</b> Modular Arithmetic, Matrices, Geometric Transformations, and Complex Numbers	5/25 <b>2</b> <b>Begin Unit 1:</b> Rings and Fields: Intro and Examples	5/26 <b>3</b> Properties of Rings and Fields	5/27 <b>4</b> Morphisms of Rings and Fields
5/31 Memorial Day - No Classes	6/1 <b>5</b> <b>Begin Unit 2:</b> Polynomials over Rings Fundamental Theorem of Algebra	6/2 <b>6</b> Quotients and Homomorphisms, First Isomorphism Theorem for Rings	6/3 <b>7</b> <b>Unit 1: Rings and Fields Exam</b>
6/7 <b>8</b> <b>Begin Unit 3:</b> Groups and Subgroups: Intro and Examples	6/8 <b>9</b> Symmetric Groups and Cayley's Theorem Cosets and Lagrange's Theorem	6/9 <b>10</b> Normal Subgroups, Quotients, Homomorphisms	6/10 <b>11</b> <b>Unit 2: Polynomials and Morphisms Exam</b>
6/14 <b>12</b> First Isomorphism Theorem for Groups	6/15 <b>13</b> <b>Begin Unit 4:</b> Direct Products and Abelian Groups	6/16 <b>14</b> F.T. of Finitely Generated Abelian Groups	6/17 <b>15</b> <b>Unit 3: Groups and Morphisms Exam</b>
6/21 <b>16</b> Sylow's and Cauchy's Theorems	6/22 <b>17</b> <b>Unit 4: Advanced Topics in Groups</b>	6/23 <b>18</b> Review	6/24 <b>19</b> <b>Final Exam</b>

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## Course Outline:

### MAT 375: Algebraic Structures

#### 1. Introduction to Rings and Fields

- (a) Rings of Integers and Integers Modulo  $n$
- (b) Units Modulo  $n$
- (c) Rational Numbers
- (d) Real Numbers
- (e) Extensions:
  - i. Polynomial ring as extensions of a ring or field
  - ii. Complex numbers as an extension of the Real Numbers
  - iii. Extensions in general
- (f) Factorization:
  - i. Unique factorization of integers
  - ii. Unique factorization of polynomials
- (g) Subrings, Ideals and Quotient Rings:
  - i. Subrings and Ideals
  - ii. Integral Domains
  - iii. Principle Ideal Domains
- (h) Ring and Field Morphisms:
  - i. Linear transformations
  - ii. Homomorphisms for rings and fields
  - iii. Isomorphisms for rings and fields

#### 2. Introduction to Groups

- (a) Geometric Transformations
- (b) Group Structures in Number Theory and Linear Algebra
- (c) Group Structures in Integer, Rational, Real, and Complex Numbers
- (d) Permutation Groups
- (e) Subgroups, Normal Subgroups, and Quotient Groups:
  - i. Cosets
  - ii. Lagrange's Theorem

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## You and Your Grades:

- “A” (Exceptional) range 90% to 100%:  
The student has demonstrated significant mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and most nonstandard problems which require deeper insight.
  - “A”  $\iff 92.5\% \leq \textit{Grade} \leq 100\%$
  - “A-”  $\iff 90\% \leq \textit{Grade} < 92.5\%$
- “B” (Good) range 80% to 90%:  
The student has demonstrated mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve standard formulaic exercises and some nonstandard problems which require deeper insight.
  - “B+”  $\iff 87.5\% \leq \textit{Grade} < 90\%$
  - “B”  $\iff 82.5\% \leq \textit{Grade} < 87.5\%$
  - “B-”  $\iff 80\% \leq \textit{Grade} < 82.5\%$
- “C” (Adequate) range 70% to 80%:  
The student has demonstrated adequate mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve most standard formulaic exercises but struggles with nonstandard problems which require deeper insight.
  - “C+”  $\iff 77.5\% \leq \textit{Grade} < 80\%$
  - “C”  $\iff 72.5\% \leq \textit{Grade} < 77.5\%$
  - “C-”  $\iff 70\% \leq \textit{Grade} < 72.5\%$
- “D” (Inadequate) range 60% to 70%:  
The student has demonstrated inadequate or incomplete mastery of the appropriate knowledge and skills relevant to the course. The student is able to solve some standard formulaic exercises but few if any nonstandard problems which require deeper insight.
  - “D+”  $\iff 67.5\% \leq \textit{Grade} < 70\%$
  - “D”  $\iff 62.5\% \leq \textit{Grade} < 67.5\%$
  - “D-”  $\iff 60\% \leq \textit{Grade} < 62.5\%$
- “F” (Unacceptable) below 60%:  
The student has demonstrated essentially no mastery of the appropriate knowledge and skills relevant to the course. The student is unable to solve most standard formulaic exercises and essentially no nonstandard problems which require deeper insight.

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## End User Agreement:

**General Expectations:** As a student in this class you are expected to:

- show up for every class on time, prepared to learn,
- actively participate in class,
- take notes in class,
- review your notes on a regular basis,
- check your university email every day,
- check the class website at least every other day,
- begin studying for exams in a timely fashion,
- ask questions in class,
- attend office hours,
- seek help in the math clinic or tutoring center, and
- complete assignments and readings on time.

**Assignment Guidelines:** (These apply to *all out of class work*.)

- Out of class assignments should always look neat, legible, and professional; they must be written on loose leaf college ruled paper or be typed. Messy work, work on crumpled papers, or on paper torn from a notebook will be rejected and may be counted as late.
- Whenever appropriate, answers on all assignments should be given in complete sentences. I should be able to tell what your answer means without re-reading the problem. This does not mean you simply rewrite the question.
- An assignment is considered late after I have handed it back or gone over it in class. Late assignments are accepted but will receive at most 75% credit. Also, late assignments go to the absolute bottom of the stack of papers to be graded, *all on time work is graded first before any late work*.
- If you work on an assignment as part of a group, then there may be no more than three individuals in the group and you must hand in only one copy of the assignment with all your names on it; if you hand in multiple copies, I will deduct points.

**Email Etiquette Guidelines:** When sending an email you must include the course number and semester in the subject line. For example, if you are taking MAT 314 in Fall 1592 then the the subject line should begin with “[MAT 314 Fall 1592].” Also, you should always begin with a salutation “Dear Dr. Rocca” and end with a closing such as “Sincerely, I. Newton.”

**Exam Makeup Policy:** To qualify for a makeup exam you must have a valid reason for missing the exam and, if at all possible, let me know ahead of time that you are missing the exam. You will need to show up for class in person in order to arrange a time for the make up exam. If you do not have a valid reason, do not give prior notice when possible, or simply do not show up for an exam, you are not entitled to a makeup and will not be given one. If you fail to show up for your makeup exam, you will not be given a second opportunity.

**The 2% Exception:** Any quiz or in class work which is ultimately worth no more than 2% of your final grade can not be made up.

**Time on Task:** For all your classes you should be spending at least 2 hours working outside of the class for every 1 hour in the class. In particular for this class you should be doing 6 hours of work a week not including class time. Note that this is an average, if you are weak in the subject or under prepared you will need to spend more time on the class.

**Attendance:** There is no specific policy for attendance in this course. However please keep the following in mind:

- if you have three consecutive unexcused absences within the first half of the semester I am required to report to the University that you have stopped attending,
- some assignments may be started, if not completed, in class, and
- while most of the dates and assignments for the course will be posted on the website occasionally small assignments or quizzes may only be announced in class.

If you come in late, then *you are responsible for emailing me to let me know you were in class*.

**Devices:** If you wish to have an electronic device in class to help with learning the material, recording notes, or recording lectures that is fine. Please make an attempt to be polite and professional, do not use your device for personal reasons during class; that is the sort of behavior that can ruin things for everyone.

**Academic Honesty:** If on any assignment, quiz, or exam you turn in someone else’s work as if it were your own you will receive a zero on that assignment, quiz, or exam. If you are caught doing this three times you will receive an F in the course and the Dean will be informed of your academic dishonesty.

**Accommodations:** If you have need of an accommodation for testing or note taking, please visit AccessAbility Services, located in White Hall 005 (<http://www.wcsu.edu/accessability>).