Problem 1.9 from Stewart:

Given a polynomial equation of the form

$$y^3 + py + q = 0$$

Cardano's formula tells us that

$$y = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

is a solution to the equation. Further if

$$\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

then, by the observation on page 27 of Stewart,

$$y = \omega\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \omega^2\sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

and,

$$y = \omega^2\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \omega\sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

are also roots.

Now, consider the equation

$$t^3 - 15t - 4 = 0.$$

If we let $p = -15$ and $q = -4$, then we get the roots:

$$t = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}},$$

$$t = \omega\sqrt[3]{2 + \sqrt{-121}} + \omega^2\sqrt[3]{2 - \sqrt{-121}},$$

and

$$t = \omega^2\sqrt[3]{2 + \sqrt{-121}} + \omega\sqrt[3]{2 - \sqrt{-121}},$$

which we can then simplify to get

$$t = 4,\ t = -2 - \sqrt{3},\ \text{and}\, t = -2 + \sqrt{3}.$$