

**Definitions:** You should be able to define and give examples of, or tell me something extra about, each of the following:

- Group,
- Normal Subgroup,
- Ring
- Field
- Integral Domain
- Ideal
- Zero Divisors
- Ring or Group Homomorphism,
- Kernel of a Homomorphism,
- Equivalence Modulo a Subgroup,
- Quotient Group or Ring
- Irreducible Polynomial
- Characteristic of a Field
- Order of a Group
- Field Extension
- Degree of a Field Extension
- Vector Space
- Algebraic Element of Degree  $n$

**Theorems:** You should be familiar enough with these to be able to use them to solve problems and you should be able to state the theorems which have names or in italics.

- *Lagrange's Theorem,*
- *Cauchy's Theorem,*
- Given that  $G$  is a group, then  $G/N$  is group if and only if  $N$  is a normal subgroup of  $G$ .
- Given that  $R$  is a commutative ring with an unit, then  $R/I$  is a field if and only if  $I$  is a maximal ideal.
- Homomorphism Theorems for Ring and Group Homomorphisms, particularly the *First Homomorphism Theorem* and the *Correspondence Theorem*.
- Given a field  $F$  and a polynomial  $p(x) \in F[x]$ ,  $(p(x))$  is a maximal ideal in  $F[x]$  if and only if  $p(x)$  is irreducible.
- *The Division Algorithm for Polynomials.*
- *Eisenstein Criterion*
- *Theorem 5.3.1: Given fields  $F$ ,  $K$ , and  $L$ , if  $K$  is a finite extension of  $F$  and  $L$  is a finite extension of  $K$ , then  $[L : K] \cdot [K : F] = [L : F]$ .*
- *Theorem 5.3.5: Let  $K \supset F$ , both fields, and suppose that  $a \in K$  is algebraic over  $F$  of degree  $n$ , then  $F(a)$ , the field obtained by adjoining  $a$  to  $F$ , is a finite extension of  $F$ , and  $[F(a) : F] = n$ .*

## Problems:

- p.46 - #'s 1, 9,
- p.54 - #'s 1, 4, 8, 14,
- p.63 - #'s 1, 5,
- p.73 - #'s 1, 2,
- p.82 - #'s 1, 3,
- p.87 - #'s 2, 4
- p.91 - #'s 5
- p.101 - #'s 1
- p.117 - #'s 1, 2, 3, 6,
- p.123 - #'s 1,
- p.133 - #'s 1,3,
- p.139 - #'s 1(use induction), 2, 3,
- p.146 - #'s 1,2,3,
- p.150 - #'s 1,10,
- p.163 - #'s 1,3,12
- p.170 - #'s Look at Examples 1-5
- p.171 - #'s 1,2,3,4 (see example 4), 5 (see example 5)
- p.179 - #'s 7
- p.189 - #'s 5
- p.197 - #'s 1,2,3,4,5,7 (Note that for some of these you will need de Moivre's Identity and how to sum a geometric sum)