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Another Proof of Cauchy's Group Theorem

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# MATHEMATICAL NOTES

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## ANOTHER PROOF OF CAUCHY'S GROUP THEOREM

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Since  $ab = 1$  implies  $ba = b(ab)b^{-1} = 1$ , the identities are symmetrically placed in the group table of a finite group. Each row of a group table contains exactly one identity and thus if the group has even order, there are an even number of identities on the main diagonal. Therefore,  $x^2 = 1$  has an even number of solutions. Since the 1<sup>st</sup> is on the diagonal, the pigeon hole principle dictates that there is at least one other  $x^2 = 1$  on the diagonal.

Generalizing this observation, we obtain a simple proof of Cauchy's theorem. For another proof see [1].

**CAUCHY'S THEOREM.** *If the prime  $p$  divides the order of a finite group  $G$ , then  $G$  has  $kp$  solutions to the equation  $x^p = 1$ .*

Let  $G$  have order  $n$  and denote the identity of  $G$  by 1. The set

There are  $p-1$  degrees of freedom.

$$S = \{(a_1, \dots, a_p) \mid a_i \in G, a_1 a_2 \dots a_p = 1\}$$

has  $n^{p-1}$  members. Define an equivalence relation on  $S$  by saying two  $p$ -tuples are equivalent if one is a cyclic permutation of the other.

If all components of a  $p$ -tuple are equal then its equivalence class contains only one member. Otherwise, if two components of a  $p$ -tuple are distinct, there are  $p$  members in the equivalence class. At least two.  $r$  is at least one because of 1.

Let  $r$  denote the number of solutions to the equation  $x^p = 1$ . Then  $r$  equals the number of equivalence classes with only one member. Let  $s$  denote the number of equivalence classes with  $p$  members. Then  $r + sp = n^{p-1}$  and thus  $p \mid r$ .

### Reference

If  $d \mid a$  and  $d \mid (a+b)$  then  $d \mid b$ .

1. G. A. Miller, On an extension of Sylow's theorem, Bull. Amer. Math. Soc., vol. 4, 1898, pp. 323-327.

## A REMARK ON BOUNDED FUNCTIONS

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Denote by  $E$  the class of functions regular and bounded by unity in  $|z| < 1$ . Denote by  $E^*$  the subclass of functions of  $E$  which are in addition univalent in  $|z| < 1$ . Analogies of various inequalities which are known to hold for functions in the class  $E$  have been obtained for functions of the class  $E^*$ . For example, it is known [3] that there exist functions in  $E$  for which the sequence  $\{a_0 + \dots + a_n\}$  ( $f(z) = \sum a_n z^n$ ) is unbounded. On the other hand, it is shown by Fejér in [1] that if  $f \in E^*$  then  $|a_0 + \dots + a_n| < 1 + (1/\sqrt{2})$  for all  $n$ .