MATHEMATICAL NOTES

EDITED BY ROY DUBISCH, Fresno State College

Material for this department should be sent to Roy Dubisch, Department of Mathematics, Fresno State College, Fresno 26, California

ANOTHER PROOF OF CAUCHY'S GROUP THEOREM

JAMES H. MCKAY, Seattle University

Since $ab=1$ implies $ba=b(ab)b^{-1}=1$, the identities are symmetrically placed in the group table of a finite group. Each row of a group table contains exactly one identity and thus if the group has even order, there are an even number of identities on the main diagonal. Therefore, $x^2=1$ has an even number of solutions. Since the 1^2 is on the diagonal, the pigeon hole principle dictates that there is at least one other x^2 on the diagonal.

Generalizing this observation, we obtain a simple proof of Cauchy's theorem. For another proof see [1].

CAUCHY'S THEOREM. *If the prime p divides the order of a finite group G, then G has kp solutions to the equation $x^p=1$.*

Let G have order n and denote the identity of G by 1. The set

There are p-1 degrees of freedom.

$$S = \{(a_1, \cdots, a_p) \mid a_i \in G, a_1a_2 \cdots a_p = 1\}$$

has n^{p-1} members. Define an equivalence relation on S by saying two p-tuples are equivalent if one is a cyclic permutation of the other.

If all components of a p-tuple are equal then its equivalence class contains only one member. Otherwise, if two components of a p-tuple are distinct, there are p members in the equivalence class. At least two. r is at least one because of 1.

Let r denote the number of solutions to the equation $x^p=1$. Then r equals the number of equivalence classes with only one member. Let s denote the number of equivalence classes with p members. Then $r+sp=n^{p-1}$ and thus $p \mid r$.

If dla and dl(a+b) then dlb.

Reference

1. G. A. Miller, On an extension of Sylow's theorem, Bull. Amer. Math. Soc., vol. 4, 1898, pp. 323–327.

A REMARK ON BOUNDED FUNCTIONS

V. F. COWLING, University of Kentucky

Denote by E the class of functions regular and bounded by unity in $|z|<1$. Denote by E^* the subclass of functions of E which are in addition univalent in $|z|<1$. Analogies of various inequalities which are known to hold for functions in the class E have been obtained for functions of the class E^*. For example, it is known [3] that there exist functions in E for which the sequence $\{a_0+\cdots+a_n\}$ $(f(z)=\sum a_nz^n)$ is unbounded. On the other hand, it is shown by Fejér in [1] that if $f\in E^*$ then $|a_0+\cdots+a_n|<1+(1/\sqrt{2})$ for all n.