# Cryptology Through History and Inquiry

# Cryptology Through History and Inquiry

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### Introduction

Goals and Aims of the text:

- Present a \textit{brief} overview of the history of cryptology
- Present the cryptology as it was not with significantly newer or older concepts
- Exception to the previous statement will be the use of some modern terminology where appropriate
- Present a variety of mathematical topics: Statistical analysis, Linear Algebra, Number Theory, Group Theory
- Present the material from an inquiry based perspective when possible
- Present the text as a battle back and forth between making and breaking codes

\noindent What should be covered in terms of content can be summarized by the list:

- 500 B.C.E. (and before): Atabash, Scytale, Demaratus, Histiaeus
- 50 B.C.E.: Caesar
- 700-1000: Arab Cryptology\*
- 140"0-1850: Foiling Frequencies\*: Alberti's, Vigen\'{e}re's , Jefferson's, and Playfair's Ciphers
- 1846: Babbage / Kasiski Test\*
- 1914-1918: World War I: Zimmermann's Telegram, ADFGVX Cipher, etc.
- 1919: One Time Pad\*
- 1929: Hill's Cipher\*
- 1934: Enigma\* in the service of the German military
- 1978:" RSA\*
- 1991: Zimmermann and PGP
- 2000: Quantum Cryptology

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# Chapter 1 Caesar's Shifty Idea

In this chapter discuss ancient encryption systems:

- Caesar's Shift
- Skytle
- Others?
- Slight improvements: Nomenclatures, Nulls, etc.

#### 1.1 Simple Early Ciphers (and a little math)

**Definition 1.1.1 Cipher.** A **cipher** is a system by which a letter or block of letters in a message is replaced by another letter or block of letters in a systematic way. In this chapter we will focus on **monoalphabetic substitution ciphers** in which the substitution is always carried out in the same way with one-to-one correspondence between letters.

One of the earliest types of cipher (sometimes spelled *cypher* in older documents) that we have a record of was the Caesar shift cipher used by Julius Caesar around 100 BCE. Here it is as described by Suetonius who lived about 150 to 200 years after Julius Caesar. Augustus Caesar a short time later also employed a shift cipher, though arguably a much less effective one:

"When he had occasion to write in cypher he put b for a, c for b [and] so forth and instead of z, aa." [15, p. 134]

#### Comprehension Check:

- In Julius Caesar's cipher what letter was substituted for a? What about in Augustus' cipher, what letter was substituted for a?
- In Julius Caesar's cipher what letters would replace b, c, and d? What letter should replace n?
- In Julius Caesar's cipher how would you need to handle the letters x, y, and z?
- For each **plaintext** letter write in the corresponding **CIPHERTEXT** letter using Julius Caesar's cipher system. (We will generally follow the convention that plaintext is lowercase and ciphertext will be uppercase.)

Table 1.1.2 Monoalphabetic Substitution Table

plain	a	b	с	d	е	f	g	h	i
CIPHER									
plain	j	k	1	m	n	О	р	q	r
CIPHER									
plain	s	t	u	v	w	х	У	z	
CIPHER									

Now use the table you filled in to check you understanding.

**Checkpoint 1.1.3** Use your table to convert this to CIPHERTEXT: "hello world"

Answer. KHOOR ZRUOG

 ${\bf Checkpoint}$  1.1.4 Use your table to convert this to plaintext: WKLV LV D WHVW

Answer. this is a test

 ${\bf Checkpoint}~1.1.5$  Use your table to convert this to plaintext: CRPELHV OLNH CHEUDV

Answer. zombies like zebras

Both of the ciphers described above are called **shift ciphers** because of how they move letters along in the alphabet, but do not change their order.

Try to figure out the message in the passage below which has been **enciphered** using a different shift.

#### Checkpoint 1.1.6 Cipher Text:

OLYL PZ H SVUNLY TLZZHNL MVY FVB AV WYHJAPJL VU. AOL ZOPMA MVY AOPZ TLZZHNL DHZ IF H MHJAVY VM ZLCLU, ZV AOL H NVLZ AV AOL O. AOPZ KVLZU'A YLHSSF VMMLY TBJO TVYL ZLJBYPAF, LZWLJPHSSF ZPUJL AOLYL HYL VUSF ADLUAF ZPE WVZZPISL ZOPMA JPWOLYZ HUK VUL VM AOVZL KVLZU'A JOHUNL HUFAOPUN. DOPSL FVB DLYL KLJPWOLYPUN AOPZ DOHA WHAALYUZ KPK FVB UVAPJL? DOHA DVYKZ ZAVVK VBA? OVD JVBSK DL THRL AOPZ H TVYL ZLJBYL (OHYKLY AV JYHJR) JPWOLY? AOLZL HYL HSS XBLZAPVUZ AOHA DL DPSS PUCLZAPNHAL PU TVYL KLAHPS AOYVBNOVBA AOL ALEA.

#### Hint.

- Which letters stand alone, i.e. are there one letter words? Can this tell you something about which letters they represent?
- What about two or three letter words? What might they be?
- Do some of the letters appear more often than others?
- Can you think of other properties of words and letters that could help us understand what a message says?

**Answer**. Here is a longer message for you to practice on. The shift for this message was by a factor of seven, so the a goes to the h. This doesn't really offer much more security, especially since there are only twenty six possible shift ciphers and one of those doesn't change anything. While you were deciphering this what patterns did you notice? What words stood out? How could we make this a more secure (harder to crack) cipher? These are all questions that we will investigate in more detail throughout the text.

Were you able to **decrypt** the message? Throughout this text we will present material through increasingly more difficult ciphers and codes. Now, take some time to try and hone your skills on the next few passages that demonstrate ways in which ciphers may be made more secure. As you go through each be sure to consider the following questions:

- How does the new method compare to those which we have previously considered, what does it do to make the message more secure?
- Each time we change the method in which messages are enciphered what doesn't change?
- Are there any properties that can not change? That is, are there rules about how English is put together that we can make use of?
- If you have suspicions about what a message, or part of a message, might say how can you use this to help you?
- What sort of data might we keep track of or how can we organize our thoughts to help us solve these puzzles?

#### Checkpoint 1.1.7 Cipher Text:

IL JVWF UVD AV TLU VM NYVZZLY ISVVK, HUK ALHJO AOLT OVD AV DHY. HUK FVB, NVVK FLVTHU, DOVZL SPTIZ DLYL THKL PU LUNSHUK, ZOVD BZ OLYL AOL TLAASL VM FVBY WHZABYL; SLA BZ ZDLHY AOHA FVB HYL DVYAO FVBY IYLLKPUN; DOPJO P KVBIA UVA; MVY AOLYL PZ UVUL VM FVB ZV TLHU HUK IHZL, AOHA OHAO UVA UVISL SBZAYL PU FVBY LFLZ. P ZLL FVB ZAHUK SPRL NYLFOVBUKZ PU AOL ZSPWZ, ZAYHPUPUN BWVU AOL ZAHYA. AOL NHTL'Z HMVVA: MVSSVD FVBY ZWPYPA, HUK BWVU AOPZ JOHYNL JYF 'NVK MVY OHYYF, LUNSHUK, HUK ZHPUA NLVYNL! - OLUYF C, DPSSPHT ZOHRLZWLHYL

#### Answer.

"Be copy now to men of grosser blood, And teach them how to war. And you, good yeoman, Whose limbs were made in England, show us here The mettle of your pasture; let us swear That you are worth your breeding; which I doubt not; For there is none of you so mean and base, That hath not noble lustre in your eyes. I see you stand like greyhounds in the slips, Straining upon the start. The game's afoot: Follow your spirit, and upon this charge Cry 'God for Harry, England, and Saint George!" - Henry V, William Shakespeare

#### Checkpoint 1.1.8 Cipher Text:

BJKJB BJMFU UDKJB BJGFS ITKGW TYMJW XKTWM JYTIF DYMFY XMJIX MNXGQ TTIBN YMRJX MFQQG JRDGW TYMJW GJMJS JJWXT ANQJY MNXIF DXMFQ QLJSY QJMNX HTSIN YNTSF SILJS YQJRJ SNSJS LQFSI STBFG JIXMF QQYMN SPYMJ RXJQA JXFHH ZWXJI YMJDB JWJST YMJWJ FSIMT QIYMJ NWRFS MTTIX HMJFU BMNQJ XFSDX UJFPX YMFYK TZLMY BNYMZ XZUTS XFNSY HWNXU NSXIF DMJSW DAGDB NQQNF RXMFP JXUJF WJ

#### Hint.

- This is a quote by the same author as the previous cipher.
- The most common letters in the cipher text are  $J,\,M,\,{\rm and}\,\,X.$

#### Answer.

"We few, we happy few, we band of brothers; For he to-day that sheds his blood with me Shall be my brother; be he ne'er so vile, This day shall gentle his condition: And gentlemen in England now a-bed Shall think themselves accursed they were not here, And hold their manhoods cheap whiles any speaks That fought with us upon Saint Crispin's day." - Henry V, by William Shakespeare

Each of the previous messages used a simple shift cipher of which, in English, there are only twenty five (twenty six if you allow a shift of 0). However if we look at all the possible ways that we can rearrange twenty six letters we really have a lot more options.

- Assuming 26 letters in the alphabet, how many letters can you use to replace a? (For simplicity assume you could choose to replace a with itself if you were so inclined.)
- Once you have chosen a letter to use in place of a, how many choices for b?
- What about c?
- Going in order how many choices do you have for all the remaining orders?
- What happens if you multiply all these numbers of options together in order to get the total number of choices?

The number you should have come up with is 403 septillion 291 sextillion 461 quintillion 126 quadrillion 605 trillion 635 billion 584 million, or

403, 291, 461, 126, 605, 635, 584, 000, 000.

John Falconer in the seventeenth century described this as follows:

Schottus demonstrates, (though the calculation in his book be not exact) that a thousand million of men in as many years could not write down all those different transpositions of the alphabet, granting every one should complete forty pages a day, and every page contain forty several positions: For if one writer in one day write forty pages, everyone containing forty combinations, 40 multiplied by 40, gives 1,600, the number he completes in one day, which multiplied by 366, the number (and more) of days in a year; a writer in one year shall compass 585,600 distinct rows. Therefore in a thousand million of years he could write

585,600,000,000,000,

which being multiplied by 1,000,000,000, the number of writers supposed, the product will be

585,600,000,000,000,000,000,000,

which wants of the number of combinations no less than

348, 484, 017, 332, 394, 393, 600, 000.

[4, pages 5-6]

#### Comprehension Check:

- How many lines of cipher alphabets does Falconer say there could be on one page?
- How many pages a day does one person potentially write?
- How many men does Falconer suggest could be writing and for how long?
- Are these people able to in that time write out all the possible cipher alphabets?

**Checkpoint 1.1.9** Imagine that instead of men writing out the cipher alphabets it was a computer. Suppose that a computer can produce one new cipher alphabet every 0.1 seconds (so 10 alphabets a second), how many seconds would it take for the alphabet to write out all 403, 291, 461, 126, 605, 635, 584, 000, 000 alphabets? How many hours is that? How many years? (Assume, like Falconer, that there are 366 days in a year.)

**Answer**. 40,329,146,112,660,563,558,400,000 seconds, 11,202,540,586,850,156,544,000 hours, or 1,275,334,766,262,540,590 and 10/61 years

Today we would call the huge number we calculated above 26!, which is read "26 factorial," and certainly takes up a lot less room.

**Definition 1.1.10 Factorial.** Given a counting number n we define n-factorial as the number of ways in which we can arrange n objects, we calculate it by taking the product of all the consecutive positive integers from n down to 1, which we write:

$$n! = n(n-1)(n-2)\cdots 2 \cdot 1 \tag{1.1.1}$$

Note that for a variety of reasons 0! = 1.

Checkpoint 1.1.11 Use the definition above to calculate 4!, 7!, and 10!. Answer.

- 4!=24
- 7!=5040
- 10!=3628800

We can make a message harder to decipher simply by using a more inventive method of encipherment than just a shift. The last exercise in this section was enciphered with a slightly trickier cipher than the previous ciphers so I put the spaces and punctuation back in. Also, while it is a harder cipher it is not completely random, there is a pattern to how the key was generated; so in the table below the message try filling in the cipher alphabet below the plain alphabet as you work, and see if you can spot a pattern.

#### Checkpoint 1.1.12 Cipher Text:

"EACMPA XMT EADGL Z SFGLD, PAKGLR XMTPQAJC SFZS RGCCGBTJSGAQ ZLR RAJZXQ OTGSA GKNMQQGEJA SM CMPAQAA ZPA ZFAZR. GC XMT BMTJR QAA SFAK BJAZPJX, LZSTPZJJX XMT BMTJR RM Z DPAZS RAZJ SM DAS PGR MC SFAK ETS XMT BZL'S. XMT BZL MLJX QAA MLA SFGLD BJAZPJX ZLR SFZS GQ XMTP DMZJ. CMPK Z KALSZJ UGQGML MC SFZS ZLR BJGLD SM GS SFPMTDF SFGBI ZLR SFGL." - IZSFJAAL LMPPGQ  $\Diamond$ 

plain	a	b	с	d	е	f	g	h	i
CIPHER									
plain	j	k	1	m	n	0	р	q	r
CIPHER									
plain	s	t	u	v	w	x	У	z	
CIPHER									

Table 1.1.13 Monoalphabetic Substitution Table

Hint.

- What letters are on their own in the cipher? Which letters can be on their own in plain English?
- The cipher letter A is most common, what does that tell you? Likewise, S is second most common so what might that mean?
- The word QAA appears twice on its own and once as the end of another word, what three letter plaintext words might look like this?
- After you have spent some time looking at the message and trying to identify some of the ciphertext start to fill in the copy of substitution table and see if you notice any patterns.

**Answer**. "Before you begin a thing, remind yourself that difficulties and delays quite impossible to foresee are ahead. If you could see them clearly, naturally you could do a great deal to get rid of them but you can't. You can only see one thing clearly and that is your goal. Form a mental vision of that and cling to it through thick and thin." - Kathleen Norris"

#### **1.2 Arabic Numerical Ciphers**

Mathematics played no role in the ciphers we examined in the previous section. In fact math played no role in cryptology at all until **Arabic scholars**<sup>1</sup> performed basic data analysis on the Arabic language in the 9th century CE. They quickly realized that their observations could be used to break the basic ciphers we have been considering. However, that is a topic for the next chapter.

In this section we will introduce some other work done by Arabic scholars. [12, vol. 3] We are going to look at some methods of enciphering text by first changing the text to numbers, some of these methods today would be called **affine ciphers**.

They began by associating each letter of the alphabet with a number in a manner similar to the following:

1	2	3	4	5	6	7	8	9
a	b	с	d	е	f	g	h	i
10	20	30	40	50	60	70	80	90
j	k	1	m	n	0	р	q	r
100	200	300	400	500	600	700	800	900
s	t	u	v	W	х	у	Z	

 Table 1.2.1 A Numerical Alphabet

<sup>1</sup>When math historians, or historians in general, speak of Arabic scholars it is a reference to the language and not the ethnicity or location of the scholars. Once this is done there are then a variety of ways in which we may encipher our message. Carefully read through these passages from *ibn ad-Durayhim's* treatise written in the  $12^{th}$  century CE; try to get an idea of how his encipherments worked before going on.

#### "5. On the replacement of letters using the decimallyweighted numerical alphabet:

- By substituting decimal numerical alphabet for letters in four different ways: by writing the numbers in words as pronounced; or by finger-bending, using the fingers to communicate the message visually to a recipient; or by writing the numbers as numerals such as writing *(mhmd: forty, eight, forty, four)*; or by giving the cryptogram a semblance of a page of a financial register.
- By recovering the cryptogram numeral into a number of letters a method of encipherment which involves more sophistication. There are many combinations that can be used in this method; for example in (mhmd: jl, fb, jl, ca) or (kk, ga, kk, bb). One can even form delusive words such as (mhmd: lead, cad, deal, baa), or substitute two words for a letter, e.g. (ali: dig fad, cab ab), in which case a line is to be drawn over two words to denote that they represent one letter.
- By multiplying the number representing the letter by two, and so write (mhmd: q, jf, q, h) and (ali: ob, jh), etc; or multiply it by three, thus writing (mhmd: sk, kd, sk, jb) and (ali: rc, kg). Numbers can also be multiplied by four or five."<sup>2</sup> [12, vol. 3, pp. 69-70]

Comprehension Check: (Be sure to reference Table C.0.2 as you try to answer these questions.)

- In the first paragraph how did *mhmd* become forty, eight, forty, four?
- In the second paragraph above, how did the author derive jl from m or  $\overline{cab\ ab}$  from i ?
- When converting letters to *delusive words*, *m* was enciphered as *lead* and *deal*, how did this work? Can you find another word to encipher it as?
- What letter(s) would be easier to translate into a variety of words? Which letters would be harder to change into words?
- In the third paragraph why is the m enciphered as a single q in the first example, but requires two letters, sk in the second?

**Checkpoint 1.2.2** Demonstrate your understanding of the systems above by enciphering *"ibex"* using each of the methods described above.

- 1. By writing the letters as numerals.
- 2. By writing letters in combinations of two or more letters.

 $<sup>^{2}</sup>$ The examples here are very loosely based on the Arabic examples in the translation. The "mhmd" is Mohamed since Arabic is written without vowels, and for "ali" the a and l together are treated as a single letter.

3. By tripling the values of the letters.

**Hint**. For all of these the first step is to use Table C.0.2 to translate the letters to numbers. Then you will need to in some way manipulate those numbers and/ or translate them back to words and letters using the table. How specifically this happens depends on the cipher method, also for some methods there will be more than one answer.

#### Answer.

- 1. NINE TWO FIVE SIXHUNDRED
- 2. HA AA CB VT
- 3. KG DB JE ZXV

**Checkpoint 1.2.3** Encipher the word "tan" by doubling the values of the letters. In how many ways do you think you can encipher each letter if you are allowed to use up to four letters to represent each letter?

**Hint**. As an example *b* doubled is 4 which we could represent as a *D* or as AC or as CA or as BB (though the last one is kind of silly). Also, remember that you will need to use Table C.0.2.

Answer. There are multiple possible encipherments, a couple are  $V\ B\ S$  and another is QUK AA JJOK.

**Checkpoint 1.2.4** The following cipher text was enciphered using one of the above ciphers.

YU II J S JH ZT B J V V SK X S H J SQ T V B S H YU IG B SU GAJ T JF B SM QO FD S JH OM JD UT KS EC B ZX OO SLN V QS XVUS RJU SK EC J DB HJ H J YU JF B QKU UY JH O MK FJ B XZ YVU J MO SU KS Q SQ RPK MQ XV HJ O SK NP M D B F M NP Q B SQ M JF B VZT WXU B Q

#### Hint.

- The message is a quote from a famous Arabic poet and mathematician.
- There is at least one letter substitution which by necessity is always the same.
- Each pair or triple of letters represents only a single plaintext letter.
- The order of a pair or triple of letters doesn't change which letter it enciphers.
- As before you will need Table C.0.2.

**Answer**. "When I want to understand what is happening today or try to decide what will happen tomorrow, I look back." - Omar Khayyam

**Checkpoint 1.2.5** Enciphering by forming *delusive words* is potentially useful but also can also be very hard. See if you can convert the word "math" to delusive words.

**Hint**. Recall that before we could encipher "m" as either "LEAD" or "DEAL" this way. And, as with all of the ciphers here, you need to use Table C.0.2.

**Answer**. One possible solution is LEAD A SOM CAD.

**Reflection:** Looking back at the exercises you have completed try to comment on the following questions:

- Looking at the number of different ways you could encipher some letters, like *t*, what advantage might these new ciphers have over the simpler ones we looked at previously?
- Why might having a large number of different ways to encipher a single letter be a disadvantage? (Note: A cipher in which a single character or block of characters may be enciphered in multiple ways in the same message is called **homophonic**.)
- Which of the methods described above do you think has the potential to be the most useful and why?
- Do you think these methods are more or less secure than the methods we looked at in the previous section? Or, are they about the same?
- In what way do you think you might be able to tweak one of the above methods in order to make it more effective?

#### 1.3 How Shifty are You?

For each exercise where you need to encipher or decipher a message do enough work to demonstrate that you clearly understand how to do the job; say at encipher or decipher least 10 characters or write down the plain text alphabet and corresponding cipher text alphabet.

1. Encipher the following using a shift cipher with a key of 9:

"We've all heard that a million monkeys banging on a million typewriters will eventually reproduce the entire works of Shakespeare. Now, thanks to the internet, we know this is not true." - Professor Robert Silensky

#### Answer.

``FN'EN JUU QNJAM CQJC J VRUURXW VXWTNHB KJWPRWP XW J VRUURXW CHYNFARCNAB FRUU NENWCDJUUH ANYAXMDLN CQN NWCRAN FXATB XO BQJTNBYNJAN. WXF, CQJWTB CX CQN RWCNAWNC, FN TWXF CQRB RB WXC CADN.'' -YAXONBBXA AXKNAC BRUNWBTH

2. Encipher "zoologist" by converting it to numerals with the Numerical Alphabet Table (Table C.0.2), multiplying by 5, and finally converting those numerals back to letters.

Did you run into any particular problem enciphering this message? Is there only one unique way to do it?

**Answer**. There are in fact multiple answers to this, one of them would be:

STUVVWXYZ TRJ ST SN U LE ME VS VX

**3.** Encipher "dog" by converting it to numerals with the *Numerical Alphabet Table* (Table C.0.2), and then split those numeral up so as to form *delusive words*.

Did you run into any particular problem enciphering this message? Are there some letters that couldn't be changed? Is there only one unique way to do it?

Answer. BAA LEEK BAD

4. Another type of monoalphabetic substitution cipher is called **atabash** and first appeared in the bible. To encipher a message in atabash you replace a with Z, b with Y, c with X, and so on. Fill in the table below with the appropriate cipher alphabet for atabash. Now, try to use your table to encipher the following message with atabash.

plain	a	b	с	d	е	f	g	h	i
CIPHER									
plain	j	k	1	m	n	0	р	q	r
CIPHER									
plain	s	t	u	v	w	x	У	z	
CIPHER									

 Table 1.3.1 Monoalphabetic Substitution Table

"Mistakes are a part of being human. Appreciate your mistakes for what they are: precious life lessons that can only be learned the hard way. Unless it's a fatal mistake, which, at least, others can learn from." - Al Franken, "Oh, the Things I Know", 2002

#### Answer.

NRHGZPVH ZIV Z KZIG LU YVRMT SFNZM. ZKKIVXRZGV BLFI NRHGZPVH ULI DSZG GSVB ZIV: KIVXRLFH ORUV OVHHLMH GSZG XZM LMOB YV OVZIMVW GSV SZIW DZB. FMOVHH RG?H Z UZGZO NRHGZPV, DSRXS, ZG OVZHG, LGSVIH XZM OVZIM UILN. - ZO UIZMPVM, LS, GSV GSRMTH R PMLD, 2002

Decipher the message that was enciphered with a shift of 10.
 XYGDR OICRY GIYER YGNOD OBQOX DCDKU OYEDL VYYNC DKSXC KZBOD

DIFSY VOXDS WKQOD ROBOS DRSXU SPIYE FOQYD KDCRS BDGSD RKLVY

YNCDK SXKVV YFOBS DWKIL OVKEX NBISC XDIYE BLSQQ OCDZB YLVOW

WKILO IYECR YEVNQ ODBSN YPDRO LYNIL OPYBO IYENY DROGK CRTOB

BICOS XPOVN

Answer.

"Now they show you how detergents take out bloodstains, a pretty violent image there. I think if you've got a T-shirt with a bloodstain all over it, maybe laundry isn't your biggest problem. Maybe you should get rid of the body before you do the wash." - Jerry Seinfeld

Decipher this message that was enciphered using a shift of 25.
 NMDRG NTKCF TZQCZ FZHMR SOQDZ BGHMF SNXNT MFODN OKDRT BBDRR

HMSGD BTRSN LZQXE NQLZR SGDLZ HMZHL HMKHE DSGDL NRSHL ONQSZ MSLNS HUDEN QVNQJ HMRBG NNKZM CHMKH EDHRO KDZRT ODHMV NOJOK DZRTQ DHMHS RQDRT KSZMC SGDJM NVKDC FDNES GDUZK TDNES GDQDR

TKSSN SGDBN LLTMH SXZKA DQSDH MRSDH M

Answer.

"One should guard against preaching to young people success in the customary form as the main aim in life. The most important motive for work in school and in life is pleasure in work, pleasure in its result, and the knowledge of the value of the result to the community." - Albert Einstein

7. Decipher this message which was enciphered using atabash (see Exercise 1.3.4).

RMZXL NKOVG VOBIZ GRLMZ OHLXR VGBGS VYVHG LUFHD LFOWY VGVZX

SVIHZ MWGSV IVHGL UFHDL FOWSZ EVGLH VGGOV ULIHL NVGSR MTOVH

HYVXZ FHVKZ HHRMT XRERO RAZGR LMZOL MTUIL NLMVT VMVIZ GRLMG

LGSVM VCGLF TSGGL YVGSV SRTSV HGSLM LIZMW GSVSR TSVHG IVHKL

MHRYR ORGBZ MBLMV XLFOW SZEVO VVRZX LXXZ

#### Answer.

"In a completely rational society, the best of us would be teachers and the rest of us would have to settle for something less, because passing civilization along from one generation to the next ought to be the highest honor and the highest responsibility anyone could have." - Lee Iacocca

8. Another type of monoalphabetic substitution cipher is called a **keyword** cipher. The key for this cipher consists of two pieces, a *key word* and a *key letter*. You write the key word in the cipher alphabet starting underneath the key letter and then write the remaining letters of the alphabet in order after that. So, if the key word is *ZEBRA* with key letter f we start as follows:

plain	a	b	с	d	e	f	g	h	i
CIPHER						Z	Ε	В	R
plain	j	k	1	m	n	о	р	q	r
CIPHER	A		<u> </u>						
plain	s	t	u	v	w	x	У	z	
CIPHER									

Table 1.3.2 Keyword Cipher: Word=ZEBRA, Letter=f

Fill in the rest of the alphabet and then use it to decipher this message. MBYEH HXWBK RLMRU GLBHN DXVYP UKYHZ FUMBY FUMRW RUGLU GXUDD MBHLY PBHFU CYYFI MSIKH IBYWR YLMBY XUGEY KUDKY UXSYQ RLMLM BUMFU MBYFU MRWRU GLBUO YFUXY UWHOY GUGMP RMBMB YXYOR DMHXU KCYGM BYLIR KRMUG XWHGZ RGYFU GRGMB YVHGX LHZBY DDLMU NENLM RGY

#### Answer.

"The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that mathematicians have made a covenant with the devil to darken the spirit and confine man in the bonds of Hell." - St. Augustine

**9.** Setup a the blank monoalphabetic sunstitution table in the same way as Table 1.3.2 with a plaintext and ciphertext alphabet for a keyword cipher with key letter d and keyword *ARNOLD*. Then use it to encipher this quote:

"Strength does not come from winning. Your struggles develop your strengths. When you go through hardships and decide not to surrender, that is strength." - Arnold Schwarzenegger

 Table 1.3.3 Monoalphabetic Substitution Table

plain	a	b	с	d	e	f	g	h	i
CIPHER									
plain	j	k	1	m	n	0	р	q	r
CIPHER									
plain	s	t	u	v	w	x	У	z	
CIPHER									

Answer.

MPKRGOPL AHRM GHP ZHFR NKHF TDGGDGO. VHQK MPKQOOERM ARSREHI VHQK MPKRGOPLM. TLRG VHQ OH PLKHQOL LXKAMLDIM XGA ARZDAR GHP PH MQKKRGARK, PLXP DM MPKRGOPL. - XKGHEA MZLTXKWRGROORK

### Chapter 2

# Attacking the Alphabet

In this chapter look at the contributions of Arabic speaking scholars and others mostly to analysis:

- Arabic analysis
- Analysis in the seventeenth century, Falconer

#### 2.1 Arabic Analysis of the Alphabet

**Definition 2.1.1 Cryptanalysis. Cryptanalysis** is the process by which we try to determine the meaning of a message without the aid of a key. We will sometimes describe this as **decrypting** a message as opposed to **deciphering** a message which is what we do when we know the key.

The first place math was employed in the science of cryptology was in the analysis of languages. The following is from *al-Kindi's* treatise on cryptanalysis written around the year 873 CE. Try to read through it carefully and then consider the questions that follow it.

#### "Algorithms of Cryptanalysis

So we say, the enciphered letters are either in numerical proportions, that is poetry -because poetic meter, ipso facto, sets measures to the number of letters in each line-, or they are not. Non- poetry can be cryptanalyzed using either quantitative or qualitative expedients.

The *quantitative* expedients include determining the most frequently occurring letters in the language in which cryptograms are to be cryptanalyzed. If vowels functioned as the material from which any language is made, and non-vowels functioned as the shape of any language, and since many shapes can be made from the same material, then the number of vowels in any language would be greater than nonvowels. For instance, gold is the material of many shapes of finery and vessels; it may cover crowns, bangles, cups, etc.. The gold in these realizations is more than the shapes made of it. Similarly, the vowels which are the material of any kind of text are more than the non-vowels in any language. I mean by vowels the letters: (a), (y or i or e) and (o or u). Therefore the vowels in any language, inevitably, exceed in number the non-vowels in a text of that language. It happens that in certain languages some vowels are greater in number than some other vowels, while non-vowels may be frequent or scarce according to their usage in each language, such as the letter (s), of which frequency of occurrence is high in Latin.

Among the expedients we use in cryptanalyzing a cryptogram if the language is already known, is to acquire a fairly long plaintext in that language, and count the number of each of its letters. We mark the most frequent letter "first", the second most frequent "second", and the following one "third", and so forth until we have covered all its letters. Then we go back to the message we want to cryptanalyze, and classify the different symbols, searching for the most frequent symbol of the cryptogram and we regard it as being the same letter we have marked "first" -in the plaintext-; then we go to the second frequent letter and consider it as being the same letter we have termed "second", and the following one "third", and so on until we exhaust all the symbols used in this cryptogram sought for cryptanalysis.

It could happen sometimes that short cryptograms are encountered, too short to contain all the symbols of the alphabet, and where the order of letter frequency cannot be applied. Indeed the order of letter frequency can normally be applied in long texts, where the scarcity of letters in one part of the text is compensated for by their abundance in another part.

Consequently, if the cryptogram was short, then the correlation between the order of letter frequency in it and in that of the language would no longer be reliable, and thereupon you should use another, *qualitative* expedient in cryptanalyzing the letters. It is to detect in the language in which cryptograms are enciphered the associable letters and the dissociable ones. When you discern two of them using the letter order of frequency, you see whether they are associable in that language. If so, you seek each of them elsewhere in the cryptogram, comparing it with the preceding and following dissociable letters by educing from the order of frequency of letters, so as to see whether they are combinable or non-combinable. If you find that all these letters are combinable with that letter, you look for letters combinable with the second letter. If found really combinable, so they are the expected letters suggested by the combination and non-combination of letters, and also by their order of frequency. Those expected letters are correlated with words that make sense. The same procedure is repeated elsewhere in the ciphertext until the whole message is cryptanalyzed." [12, vol. 1, pp. 121-123]

Comprehension Check:

- What do you think the author means when he says "vowels function as the material of a language"?
- In what way then do the "non-vowels function as the shape"?
- He also says that there are more vowels than non-vowels, how many vowels are in this sentence you are reading right now? Were there more vowels? If not then how might his statement still be true?
- How does the gold in his analogy function like the vowels?
- Finally, how do the author's comments compare to your experiences in section Section 1.1?

What al-Kindi is describing above is what we now call **frequency analysis** which is the first step in cryptanalysis.

**Definition 2.1.2 Frequency Analysis.** Basic *Frequency Analysis* is the process of counting the characters in a text in order to determine how many of each character there are relative to the entire length of the text. This is typically the first step in the cryptanalysis, the process of breaking an unknown cipher or code.

**Checkpoint 2.1.3** Try following al-Kindi's directions in paragraph three above. Use the *n*-gram counter below to count the number of times each letter appears in the following paragraph and with what frequency, then plot the frequency of each character on the chart below.

"In the year 1878 I took my degree of Doctor of Medicine of the University of London, and proceeded to Netley to go through the course prescribed for surgeons in the army. Having completed my studies there, I was duly attached to the Fifth Northumberland Fusiliers as Assistant Surgeon. The regiment was stationed in India at the time, and before I could join it, the second Afghan war had broken out. On landing at Bombay, I learned that my corps had advanced through the passes, and was already deep in the enemy's country. I followed, however, with many other officers who were in the same situation as myself, and succeeded in reaching Candahar in safety, where I found my regiment, and at once entered upon my new duties." - A Study in Scarlet, Sir Arthur Conan Doyle [3]



Figure 2.1.4 Axes for Mapping Letter Frequencies

**Checkpoint 2.1.5** Below is the same paragraph as in Checkpoint 2.1.3 only now enciphered with a shift of three. As above, use the *n*-gram counter below to count the number of times each letter appears and with what frequency, then plot the frequency of each character on the chart below. LQWKH BHDUL WRRNP BGHJU HHRIG RFWRU RIPHG LFLQH RIWKH XQLYH UVLWB RIORQ GRQDQ GSURF HHGHG WRQHW OHBWR JRWKU RXJKW KHFRX UVHSU HVFUL EHGIR UVXUJ HRQVL QWKHD UPBKD YLQJF RPSOH WHGPB VWXGLHVWKHUHLZDVGXOBDWWDFKHGWRWKHILIWKQRUWKXPEHUODQGIXVLOLHUVDVDVVLVWDQWVXUJHRQWKHUHJLPHQWZDVVWDWLRQHGLQLQGLDDWWKHWLPHDQGEHIRUHLFRXOGMRLQLWWKHVHFRQGDIJKDQZDUKDGEURNHQRXWRQODQGLQJDWERPEDBLOHDUQHGWKDWPBFRUSVKDGDGYDQFHGWKURXJKWKHSDVVHVDQGZDVDOUHDGBGHHSLQWKHHQHPBVFRXQWUBLIROORZHGKRZHYHUZLWKPDQBRWKHURIILFHUVZKRZHUHLQWKHVDPHVLWXDWLRQDVPBVHOIDQGVXFFHHGHGLQUHDFKLQJFDQGDKDULQVDIHWBZKHUHLIRXQGPBUHJLPHQWDQGDWRQFHHQWHUHGXSRQPBQHZGXWLHVHVHVHVHV

A Study in Scarlet, Sir Arthur Conan Doyle [3]



Figure 2.1.6 Axes for Mapping Letter Frequencies

How did the plot change? In what ways did the plot not change?

**Checkpoint 2.1.7** Find a large sample of normal English text (at least 500 characters) and repeat what you did in Checkpoint 2.1.3; that is use the *n*-gram counter below to count the number of times each letter appears and with what frequency. Then plot the frequency of each character on the chart below.



Figure 2.1.8 Axes for Mapping Letter Frequencies

**Checkpoint 2.1.9** Take the English text you used in Checkpoint 2.1.7 and encipher it with a shift cipher. Analyze the ciphertext as you did in Checkpoint 2.1.5. How did the plot change? In what ways did the plot not change?



Figure 2.1.10 Axes for Mapping Letter Frequencies

**Checkpoint 2.1.11** Now use the n-gram counter to find the letter frequencies for the letters in this cipher text.

XLMWM EFPSG OSJVI PEXMZ IPCRS VQEPI RKPMW LXIBX LSTIJ YPPCA

LIRCS YEREP CDIXL MWCSY AMPPW IIXLE XIZIR ALIRX LMWMW IRGMT LIVIH XLIPI XXIVJ VIUYI RGMIW WXECX LIWEQ IEWPS RKEWX LIGMT LIVMW QSRSE PTLEF IXMGM RTEVX MGYPE VJSVE WLMJX GMTLI VXLIP IXXIV JVIUY IRGMI WIZIR QEMRX EMRXL IWEQI VIPEX MZITS WMXMS RXSIE GLSXL IVNYW XWPMH EPSRK PMOIX LIPIX XIVWX LMWQE OIWWY GLEGM TLIVZ IVCIE WCXSW TSXER HGVEG O

Plot the frequencies you found using Figure C.0.3. If you compare the shape of the chart you just made to the chart for normal English which you made previously in Checkpoint 2.1.3 do you notice any similarities? Can you use this to try and decrypt this message?

#### N-Gram Counter:

To use the n-gram counter copy and paste the text you wish to analyze into the input box, and select 1 for N since we are analyzing single letters.

Specify static image with <code>@preview</code> attribute, Or create and provide automatic screenshot as <code>images/n-gram\_sage\_in\_arab\_chapter - preview.pngviathembxscript</code>



Figure 2.1.12 N-Gram Analysis Tool

#### Substitution Cipher Tool:

To use the substitution cipher tool to encipher a message leave the plain text alone and enter the corresponding ciphertext in the box labeled cipher. For a simple shift cipher you can put the alphabet into the cipher box in the regular order and then use the shift drop down menu to select your desired shift.

Specify static image with <code>@preview</code> attribute, Or create and provide automatic screenshot as <code>images/substitution\_cipher\_tool\_in\_arab\_chapter - preview.pngviathembxscript</code>



Figure 2.1.13 Substitution Cipher Tool

**Checkpoint 2.1.14** Repeat what you did before in Checkpoint 2.1.7 with text from a variety of sources. Be sure to try both long and short pieces of text. Do you agree with the al-Kindi's statements about shorter pieces of text? Finally, make a table of your results for future reference.

#### 2.2 (\*)Alberti's Approach

Text based on the work of Leon Battista Alberti will go here.

#### 2.3 Falconer's Approach

In 1685 John Falconer, who we met briefly in Section 1.1, wrote the second ever published text in English on the subject of cryptology; it has the rather lengthy

title Cryptomenysis patefacta; or, the art of secret information disclosed without a key. containing plain and demonstrative rule, for decyphering. Beyond this work little is known about him for certain. A descendant of his writing in the nineteenth century claimed that John was the personal cipher keeper for King James II and followed the king into exile in France where he died. In slim support of this the book is dedicated to King James II and addressed to the king's secretary of state. However, no readily available documented evidence exists to independently corroborate his descendant's claims, and in fact there are some tax records for John and his wife, Mary Dalmahoy, in Edinburgh Scotland, from 1695, over six years after James fled to France. [13]

In his text John Falconer presents a wide variety of ciphers and, for most of them suggestions, for how we might attack them. In this section we will focus on his directions for how to break, or attempt to break, what we now call monoalphabetic substitution ciphers.

#### 2.3.1 Step 1:

Firs	st, Distinguish the Vowels from the Consonants.
1.	And first, the vowels generally discover themselves by their frequency; for because they are but few in number, and no word made up without some of them, they must frequently be used in any writing.
2.	Where you find any character or letter standing by it self, it must be a vowel.
3.	If you find any character doubled in the beginning of a word, in any language it is a vowel, as <i>Aaron, Eel, Jilt, Oogala, Vulture, etc.</i> , except for some English proper names, as Llandaff or Lloyd.
4.	In monosyllables of two letters you may distinguish it from the consonant joined with it by its frequency.
5.	In a word of three letters beginning and ending in the same letter the vowel is probably included.
6.	When you find a character doubled in the middle of a word of four letters, `tis probably the vowel $e$ or $o$ .
7.	In Polysyllables, where a character is double in the middle of the word, it is for the most part a consonant; and if so, the precedent letter is always a vowel, and very often the following.[4, pp. 8-9]

#### Comprehension Check:

- Do some of these sound familiar from what you read in Section 2.1?
- Which of these are new? Do any of those seem strange to you?
- In what ways does Falconer say that one, two, and three letter words can help us?
- According to Falconer when might we expect a double letter to be a vowel and when might it be a consonant?
- Try to find out why Falconer says "Jilt" and "Vulture" begin with a double vowel.

For the following *Checkpoints* find a large sample of normal English text (you can use the one you found for the chekpoints in Section 2.1, but something bigger may be better). You will also want to use the n-gram counting and word counting tools below.

**Checkpoint 2.3.1** In your text, what letters are most common? How many vowels are there? How many consonants are there?

**Checkpoint 2.3.2** What are the one, two, and three letter words in the text you found to analyze? Do they follow the pattern that Falconer described? Are any of them more common than others?

**Checkpoint 2.3.3** Look at your text for words with double letters, do they behave the way that Falconer says they should?

N-Gram Counter:

Specify static image with @preview attribute, Or create and provide automatic screenshot as images/interactive-3-preview.png via the mbx script

Figure 2.3.4 N-Gram Analysis Tool String Analyzer:

Specify static image with @preview attribute, Or create and provide automatic screenshot as images/interactive-4-preview.png via the mbx script

Figure 2.3.5 String Analyzer Tool *Word Counter:* 

Specify static image with @preview attribute, Or create and provide automatic screenshot as images/interactive-5-preview.png via the mbx script

Figure 2.3.6 Word Counter Tool







#### 2.3.2 Step 2:

Secondly, Distinguish the Vowels from Themselves
1. Compare their frequency, and $e$ , as we observed before, is generally the most used in the English tongue, next $o$ , then $a$ and $i$ ; but $u$ and $y$ are not so frequently used as some of the consonants.
2. It is remarkable that amongst the vowels, $e$ and $o$ are often doubled, the rest seldom or never.
3. $e$ is very often a terminal letter, and $y$ terminates words, but they are distinguishable, because there is no pro- portion to their frequency: $o$ is not often in the end of words, except in monosyllables.
4. <i>e</i> is the only vowel that can be doubled in the end of an English word, except <i>o</i> in <i>too</i> , etc.
<ol> <li>You may consider which of the vowels, in any language, can stand alone, as a, i, and sometimes o in English, a, e, o, in Latin or i the imperative of eo.[4, pp. 9-10]</li> </ol>
Comprehension Check:

- Do you agree with his comments about *e*, *o* and the other vowels? Why or why not?
- Can you think of examples of or exceptions to his third and fourth comments above?
- Can you think of or find an example of *o* standing alone in English?

**Checkpoint 2.3.7** Test out Falconer's observations about vowels using the same sample of text that you analyzed in exercise Checkpoint 2.3.1. Do his statements hold true or has the language changed since 1685?

#### 2.3.3 Step 3:



**Checkpoint 2.3.8** What consonnts does Falconer say are most common? In the text you have been analyzing which are most common?

Checkpoint 2.3.9 Consider statement two above and look at the same text you used previously, which consonants are commonly doubled in the middle and at the end of words?

**Checkpoint 2.3.10** In your sample of text which consonants do you commonly see at the end of words? Are there some that, when they do show up, are almost always at the end of a word? Are there any that are almost never at the end of a word?

**Checkpoint 2.3.11** Falconer's first three pieces of advice are similar to what we have looked at before; his fourth comment is something a little new. Using the same text which you analyzed before set the *n*-gram counter to count two letter combinations in your text (N = 2). Once you have the count take note of which combinations of letters appear most often and for each vowel which two or three consonants it is most often paired with.

#### 2.3.4 Step 4:

#### Additional Observations.

- 1. A word of three letters, beginning and ending with the same, may be supposed *did*
- 2. A word consisting of four characters, with the same letter in the beginning and end, is probably *that* or *hath*
- 3. A word consisting of five letters, when the second and last are the same, is commonly *which*, though it may be otherways, as in *known*, *serve*, etc. And you may judge of the truth of such suppositions by the frequency of the letters in the word supposed.

Next you may compare words one with another, as *on* and *no*, each being the other reversed; so *of* and *for*, the last being the first reversed with the addition of a letter; for and from will discover each other, etc.

You may also likewise observe some of the usual propositions and terminations of words, such as *com*, *con*, *ing*, *ed*, etc. Note that t and h are often joined in the beginning and end of English words, and sometimes in the middle.[4, pp. 11-12]

Comprehension Check:

- Can you think of more examples of three or four letter words beginning and ending with the same letter?
- What about five little words in which the second and last letters are the same?
- How many words like on and no or of and for can you think of?
- Compared with other suggestions we have looked at, how practical do these seem? Do they seem a little too subtle?

**Checkpoint 2.3.12** Looking one last time at the text which you have been using identify common prefixes and suffixes (what Falconer called *propositions and terminations*) within the text. Are you finding any of the examples which he gave?

To see how we can pull these ideas together take a look at Appendix E.

#### 2.4 Bringing it all Together

For exercises Exercise 2.4.1 to Exercise 2.4.4 you will review the ideas we have explored in this chapter. When you are done you should have generated for yourself a guide which you can use when you are attempting to analyze a piece of cipher text. To begin find a new piece of relatively typical English text which is at least 1000 characters long. Then, you can use the *n*-gram counter, word counter, and string counter at the bottom of the page to help you examine the text for the following information:

- **1.** letter frequencies
- 2. bi-gram (two letter combination) and tri-gram (three letter combination). frequencies
- **3.** Common words
- 4. prefixes and suffixes

Now, finally, put together all the information you have just gathered into a neat one page analysis guide. You should type this up neatly, in a reasonably sized font. If you want you can print out a copy of Figure C.0.4 and use it to help organize the information. It should include:

- A list of frequencies for all the individual letters in the alphabet.
- The frequencies for at least the top 25 bi-grams and tri-grams.
- A list of the top 25 most common words.
- A list of your top 25 most common prefixes and suffixes.
- And, any other types of information you think might be useful which you have looked at in this chapter.
- 5. Use the skills we have been practicing in class together with the cheat sheet you just put together in order to crack this cipher. (It may also help to take a look at Appendix E.) LU ZOO GSZG RH TLLW, HFYORNRGB RH HFKIVNV. HFXXVVWRMT RH GSV XLNRMT GLTVGSVI LU ZOO GSZG RH YVZFGRUFO. UFIGSVIZMXV RH GSV ZTIVVNVMG LU ZOO GSZG RH QFHG. KVIHVEVIZMXV RH GSV ULFMWZGRLM LU ZOO ZXGRLMH. - OZL GAF

**Hint**. Be sure to look at repeated common words. Also, as you uncover cipher letters try writing them down in Table C.0.1 and looking for a pattern.

**Answer**. "of all that is good, sublimity is supreme. succeeding is the coming together of all that is beautiful. furtherance is the agreement of all that is just. perseverance is the foundation of all actions." - lao tzu

**Solution**. When you look at the frequencies you see that G and V are most common, next most common are R and Z. So we know that one of the first two is probably e while the other is t, and one of the latter two is likely a, assuming the frequencies are fairly normal.

Then we see that GSV and GSZG appear frequently, and GS is the most common bi-gram (so that it is likely th). Putting this together we see that e is V, t is G, a is Z, and h is S. So that GSV and GSZG represent the and that.

We can also see that ZOO appears four times with Z replaced by a this looks like aOO, the likely candidate is that ZOO is all, so that O is l.

At this point we can start writing down the letters we have in the monoalphabetic substitution table (Table C.0.1). When you do that you will hopefully notice that the cipher letters are in reverse alphabetic order, so this was likely enciphered with atabash which we learned about in Exercise 1.3.4.

6. Use the skills we have been practicing in class together with the cheat sheet you just put together in order to crack this cipher. (It may also help to take a look at Appendix E.)

SNE ORGGRE VF VG GB QNER ZVTUGL GUVATF, GB JVA TYBEVBHF GEVHZCUF, RIRA GUBHTU PURPXRERQ OL SNVYHER ... GUNA GB ENAX JVGU GUBFR CBBE FCVEVGF JUB ARVGURE RAWBL ABE FHSSRE ZHPU, ORPNHFR GURL YVIR VA N TENL GJVYVTUG GUNG XABJF ABG IVPGBEL ABE QRSRNG. - GURBQBER EBBFRIRYG

**Hint**. Be patient, the letter frequencies are ... abnormal.

**Answer**. "far better is it to dare mighty things, to win glorious triumphs, even though checkered by failure ... than to rank with those poor spirits who neither enjoy nor suffer much, because they live in a gray twilight that knows not victory nor defeat." - theodore roosevelt

**Solution**. Looking at a basic frequency analysis we see that the most common single letters are G, R, B, E, V, U, A, F, N, H, the most common bi-gram is GU, and the most common tri-gram is GUR; from this we may conclude that G is t, U is h, and R is e.

Since the word spacing is preserved we also have the two letter words VF, VG, GB, and VA, and the one letter word N, these allow us to deduce the ciphertext-plaintext pairs N - a, B - o, V - i, and F - s.

When we start writing down the ciphertext we have worked out under the plaintext alphabet (use Table C.0.1) we can see that the letters we have discovered are in the correct order and with the correct spacing so that we appear to have a shift, trying this as a possible solution we see that we are correct and that a was shifted to N.

7. Use the skills we have been practicing in class together with the cheat sheet you just put together in order to crack this cipher. (It may also help to take a look at Appendix E.)

WKSTD QJUTJ OTKLE JGGJI DQUQU SFFYS CUDRL PSTKL PTKSI SKDIR

PSIEL DIQJF VDICS MPJHF LGJOT KDQQJ PTTKL CPSIR TKDIC DQTJH LSHFL TJPLS QJIHS EBWSP RTKST DQSVL PYUQL OUFSE EJGMF DQKGL ITSIR SVLPY LSQYJ ILHUT MLJMF LRJIJ TMPSE TDELD TGUEK DITKL LVLPY RSYSO OSDPQ JOFDO LDTDQ GJPLU QLOUF TJPLS QJIOJ PWSPR SIRQJ TKLJT KLPEJ GLQTJ HLILC FLETL RTKLP LSPLO DOTYW KJESI PLSQJ IQYIT KLTDE SFFYO JPJIL WKJES IPLSQ JISIS FYTDE SFFYG JQTML JMFLD OYJUR LQEPD HLSTP SDIJO LVLIT QTJTK LGWDF FTLFF YJUWK STTKL PLQUF TWJUF RHLTK LYESI MUTTK JQLLV LITQT JCLTK LTJMS QQTKL PLSPL OLWML JMFLK JWLVL PWKJD OYJUT JFRTK LGSPL QUFTW JUFRH LSHFL TJLVJ FVLOP JGTKL DPJWI DIILP EJIQE DJUQI LQQWK STTKL QTLMQ WLPLW KDEKF LRUMT JTKST PLQUF TTKDQ MJWLP DQWKS TDGLS IWKLI DTSFB JOPLS QJIDI CHSEB WSPRJ PSISF YTDES

FFYSQ TURYD IQESP FLTQK LPFJE BKJFG LQ

**Hint**. This is a quote from a famous fictional nineteenth century detective and makes use of a keyword cipher.

**Answer**. "... what is out of the common is usually a guide rather than a hindrance. In solving a problem of this sort, the grand thing is to be able to reason backwards. That is a very useful accomplishment, and a very easy one, but people do not practice it much. In the every-day affairs of life it is more useful to reason forwards, and so the other comes to be neglected. There are fifty who can reason synthetically for one who can reason analytically." - Sherlock Holmes in Study in Scarlet by Sir Arthur Conan Doyle

**Solution**. As always you need to start with a basic frequency analysis. From this you immediately get that the ten most common letters are L, T, J, S, Q, P, I, K, F, D, the most common bi-gram and tri-gram are TK and TKL, L appears frequently before and after other letters (in particular P), and finally TKLPL and TKLDP appear multiple times; from all of this we get the ciphertext-plaintext pairs L - e, T - t, H - k, P - r, and D - i.

Then we can start filling in bits and pieces of the message and looking for more clues, for example LVLPY becomes eVerY which we reasonably assume is every. Also, with the hint that it is a keyword cipher we can line up the ciphertext letters we have worked out underneath a copy of the plaintext alphabet (Table C.0.1) and look for patterns. Continuing in this way we can finally arrive at the solution and the key is KEYWORD: SHERLOCKED, key letter: a.

8. Consider the following comment from John Falconer:

I have not meddled here with any language but English; ... However, by a little practice of decyphering in one language, you may decipher an epistle in any, even tho the plain speech itself be a mystery to you, if you first observe the frequency of the letters, the terminal letters, what letters can be doubled in the beginning, middle, and end of words; and such general rules.[4, p. 12]

Find a large piece of typical text written in a language other than English; it needs to be native to that language not a translation from another language. Perform an analysis of this text and then create a cheat sheet just like you did for English (exercises Exercise 2.4.1 to Exercise 2.4.4). Finally, encipher a message written in this new language which your classmates can try to decipher using your cheat sheet.

**9.** A lot of the general advice that Falconer gave us was good, but his specifics didn't match what we expect to see when we read a more modern text. Find a typical piece of text or literature from seventeenth century England (or better yet Scotland) and analyze this text to create a cheat sheet like the one you did for modern English (exercises Exercise 2.4.1 to Exer-

cise 2.4.4). How do your findings match what Falconer told us to expect?

#### N-Gram Counter:

Specify static image with @preview attribute, Or create and provide automatic screenshot as images/interactive-6-preview.png via the mbx script

#### Figure 2.4.1 N-Gram Analysis Tool

Word Counter:

Specify static image with @preview attribute, Or create and provide automatic screenshot as images/interactive-7-preview.png via the mbx script

#### Figure 2.4.2 Word Counter Tool

*String Counter Tool:* You can use this tool to look for specific combinations of letters in you text rather than gathering gerneral information. This is helpful for looking for prefixes, suffixes, and double letters.

Specify static image with @preview attribute, Or create and provide automatic screenshot as images/interactive-8-preview.png via the mbx script

Figure 2.4.3 String Counter Tool







# Chapter 3

# Mixing Things Up

In this chapter look at real improvements to cryptology:

- Polyalphabetics (Alberti)
- Autokey Cipher (Vigenère)
- Keyed Columnar Ciphers and Codes (Falconer)
- Thomas Jefferson's contributions???

#### 3.1 Alberti's Great Idea

#### **Objectives**

- Alberti's Biography
- Alberti's Polyalphabetic
- Modern/Vigenère's Polyalphabetic

#### 3.1.1 Alberti's Biography

For now see his biography on the MacTutor History of Mathematics Site: Leon Battista Alberti

#### 3.1.2 Alberti's Polyalphabetic Cipher

**Definition 3.1.1 Polyalphabetic Substitution Cipher.** A polyalphabetic substitution cipher is a cipher in which a single plaintext letter maybe replaced by several different ciphertext letters, groups of letters, or symbols and every letter, group of letters, or symbol in the ciphertext may represent more than one plain text letter.

One of Leon Battista Alberti's greatest contributions to cryptology was not his cryptanalysis skills but an entirely new kind of cipher. Many efforts had been and would be made to shore up monoalphabetic substitution ciphers. Some of these efforts worked well enough but they were never sufficiently different to make them immune to the sorts of attacks we have studied before. Alberti made something truly different he invented a **polyalphabetic substitution cipher**. This was a cipher that did not just replace the plain text alphabet with a single separate alphabet but instead used multiple enciphering alphabets to create greater confusion and security.

The key to Alberti's new system was a device he called a "formula". This was a cipher disk consisting of a fixed outer disk and a mobile inner disk. In his basic description of his new cipher he would use the outer ring of upper case letters for his plain text and the inner mobile ring of lower case letters for his cipher text.



Figure 3.1.2 Alberti's Cipher Disk or "Formula"

"Say for example we mutually establish k as the index of the mobile circle. Writing, the formulae are positioned at will, say such k lies under the uppercase B [Figure 3.1.2] and the next letter corresponds to the letter that comes next. In writing to you, I will first put the uppercase B under which lies the index k in the formula; this is to signal you far away, wanting to read what I have written, that you should set up the twin formula in your keeping, positioning the mobile circle so that the B sits over the index k. Then all of the rest of the lowercase letters present in the coded text will take their meaning and sound from those of the fixed circle above them."

"After I have written three or four words I will mutate the position of the index in our formula, rotating the disk let's say, so that the index k falls below the upper case R. Then in this missive I write an uppercase R to indicate that k no longer refers to B, but to R, and the letters that will follow will assume new meanings." - Alberti [17, p. 181]

#### Comprehension Check:

- If you compare the inner and outer rings of the formula (cipher disk) what difference do you notice? Why do you think Alberti might have made these choices? (For reference I made an updated version in the appendix where I made similar choices, Figure C.0.6.)
- What step in the process above stops this from simply being a shift cipher?
- How does the recipient of a message using Alberti's method know how to set their disk and when they should change it?
- Is there an obvious weakness in the way that Alberti suggests using his system?

**Checkpoint 3.1.3** If you have not done so already make a copy of Alberti's formula [Figure 3.1.2] so that you can use it for enciphering and deciphering.

Now encipher "See my first message" using an index of c initially underneath the G, but shift it underneath V after the first two words.

#### Hint.

- Remember to line up the index c with the capital letters used of the key, here G and V.
- For Alberti, uppercase letters (the outer ring) are the plaintext and lowercase letters are the ciphertext.
- Because of what was and was not included on the disk you will need to make some choices.

#### Answer. G vbb ke V xsdba m&bbry&

**Checkpoint 3.1.4** Suppose that B lgiq R yam V ydme was enciphered using an index of k as directed by Alberti, what does it say?

#### Hint.

- First identify the three key letters used in this cipher.
- Remember to line up the index k with the capital letters used of the key, and to move the index when the key letter changes.
- For Alberti, uppercase letters (the outer ring) are the plaintext and lowercase letters are the ciphertext.

#### Answer. cats and dogs

**Checkpoint 3.1.5** Team up with a partner and agree on a lower case index. Now each of you should try enciphering a short message, only five or six words, be sure to adjust your formula after every couple of words. Then exchange messages with your partner and decipher each others messages.

Specify static image with @preview attribute, Or create and provide automatic screenshot as images/alberti\_disc - preview.pngviathembxscript



Figure 3.1.6 Alberti's Formula

#### 3.1.3 Vigenère's Cipher

In spite of the fact that Alberti invented the polyalphabetic cipher his is not the name associated with it, instead that honor goes to the Frenchmen **Blaise de Vigenère**. Vigenère made a careful study of the cryptographers who had come before him and in 1586 he published the culmination of his work in his *Traicté* des chiffress, ou secrètes manières d'escrire[16]. In the Traicté he described a wide variety of older ciphers and his improvements to them. Among these was his take on Alberti's polyalphabetic cipher.

One of the key advances that Vigenère made was to replace the disk with a table [Figure 3.1.7,  $[16, p. 50b (102)]^1$ ]. This made it easier to vary between alphabets without having to move inner and outer disks to different positions.

 $<sup>^1\</sup>mathrm{In}$  the page designation "p. 50b (102)" the 102 is the page in the pdf document while the 50b is the page in the manuscript.

"... set the capitals which run across the top [of the table] for the message to be conveyed & those that run perpendicularly down the left for the keys. I have put two rows of capitals here, one black and the other red, to show that alphabets of the text as, the keys, may be transposed and changed in as many ways as you want to keep knowledge of them from all others except ones correspondents. Therefore, to encipher the saying we used previously "au nom de *l'eternel*," with the key "*le jour obscur*" proceed in this way; from a in the alphabet across the top in red, come to the row with l and the box with b: u from e will be q: n from i, n: o from o, s: m from u, a: d from r, m: e from o, i: l from b, c: e from s, o: t from c, n: e from u, q: r from r, c: n from l, o: e from e, a: l from i, l. Altogether we get bgnsamiconcoal & so on for the remainder." [16, pp. 49b, 50, 50b (100-102)],  $[11, p. 110]^2$ 

 $<sup>^2 \</sup>rm Most$  of this translation is due to Mendelsohn [11], I have only filled in with my own at the end which he had not translated.

		O P Q R  S  T  V  X  A  B C D E  F  G  H  I  L  M  N
		EFGHILMNOPQRSTVXABCI
10	E	a  b  c  d  c  f  g  h  i  l  m  n  o  p  q  r  f  t  v  x
P	F	$b \mid c \mid d \mid e \mid f \mid g \mid h \mid i \mid l \mid m \mid n \mid o \mid p \mid q \mid r \mid l \mid t \mid v \mid x \mid a$
Q	G	$c \mid d \mid c \mid f \mid g \mid h \mid i \mid l \mid m \mid n \mid o \mid p \mid q \mid r \mid l \mid t \mid v \mid x \mid a \mid b$
R	H	$d \mid c \mid f \mid g \mid h \mid i \mid l \mid m \mid n \mid o \mid p \mid q \mid r \mid f \mid t \mid v \mid x \mid a \mid b \mid c$
S	I	e   f   g   h   i   l   m   n   o   p   q   r   f   t   v   x   a   b   c   d
T	L	$f \mid g \mid h \mid i \mid l \mid m \mid n \mid o \mid p \mid q \mid r \mid f \mid t \mid v \mid x \mid a \mid b \mid c \mid d \mid c$
V	M	$g   \frac{h}{h}   \underline{i}   \underline{l}   \underline{m}   \underline{n}   o   \underline{p}   q   \underline{r}   \underline{f}   \underline{t}   \underline{v}   \underline{x}   \underline{a}   \underline{b}   \underline{c}   \underline{d}   \underline{e}   \underline{f}$
X	N	$\begin{array}{c c c c c c c c c c c c c c c c c c c $
A	0	i $ l  m n o p q r f t v x a b c d e f g h$
B	P	$ \mathbf{l}   \mathbf{m}   \mathbf{n}   \mathbf{o}   \mathbf{p}   \mathbf{q}   \mathbf{t}   \mathbf{f}   \mathbf{t}   \mathbf{v}   \mathbf{x}   \mathbf{a}   \mathbf{b}   \mathbf{c}   \mathbf{d}   \mathbf{e}   \mathbf{f}   \mathbf{g}   \mathbf{h}   \mathbf{i} $
C	0	$  \mathbf{n}  \mathbf{n}   \mathbf{o}   \mathbf{p}   \mathbf{q}   \mathbf{r}   \mathbf{f}   \mathbf{t}   \mathbf{v}   \mathbf{x}   \mathbf{a}   \mathbf{b}   \mathbf{c}   \mathbf{d}   \mathbf{c}   \mathbf{f}   \mathbf{g}   \mathbf{h}   \mathbf{i}   1 $
D	R	n o p q r f t v x a b c d e f g h i M r
E.	S	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $
F	T	p   q   r   f   t   v   x   a   b   c   d   e   f   g   h   i   l   m   n   o
G	V	$\frac{\mathbf{q} \mathbf{r}}{\mathbf{r}} \mathbf{i} \mathbf{t} \mathbf{v} \mathbf{x} \mathbf{a} \mathbf{b} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{f} \mathbf{g} \mathbf{h} \mathbf{i} \mathbf{l} \mathbf{m} \mathbf{n} \mathbf{o} \mathbf{p}$
H	X	$\frac{\mathbf{r}}{\mathbf{l}} = \frac{\mathbf{r}}{\mathbf{l}} = \frac{\mathbf{r}}{\mathbf{l}$
I		
L	B	$\frac{\mathbf{r} \cdot \mathbf{v} \cdot \mathbf{x}}{ \mathbf{a}   \mathbf{b}   \mathbf{c}   \mathbf{d}   \mathbf{e}   \mathbf{f}   \mathbf{g}   \mathbf{n}   1   \mathbf{m}   \mathbf{n}   \mathbf{o}   \mathbf{p}   \mathbf{q}   \mathbf{r}   \mathbf{f} $
M	C	
IN	D	

Figure 3.1.7 Vigenère's Tableau

#### Comprehension Check:

- How did the *a* in the saying "*au nom de l'eternel*" get replaced with a *b* in the ciphertext *bqnsamiconcoal*?
- How did the *u* in the saying "*au nom de l'eternel*" get replaced with a *q* in the ciphertext *bqnsamiconcoal*?
- How did the first *e* in the saying "*au nom de l'eternel*" get replaced with a *i* in the ciphertext *bqnsamiconcoal*?
- How did the second *e* in the saying "*au nom de l'eternel*" get replaced with a *o* in the ciphertext *bqnsamiconcoal*?
- All four *e*'s in the plaintext were replaced different ciphertexts, why and how did this happen?

**Checkpoint 3.1.8** To further check your understanding try enciphering the message "vive la france" using the key aider and the black (outer) rows in Figure 3.1.7, since there are more letters in the message than the key you will need to restart at the beginning of the key when you get to the end.

**Hint**. Try writing the key above the message so that you can keep track of how they correspond, like this:
Key: aideraiderai Plaintext: vivelafrance Ciphertext: Q O

Also, recall that you look up the letters in the message along the top and the key along the side.

Answer. Ciphertext: QOTEASLQACVI

**Checkpoint 3.1.9** To further check your understanding try enciphering the message "escargot" using the key *ail*; in Figure 3.1.7 use the red (inner) labels for the message and the black (outer) column for the key, since there are more letters in the message than the key you will need to restart at the beginning of the key when you get to the end.

**Hint**. Try writing the key above the message so that you can keep track of how they correspond, like this:

Key: ailailai Plaintext: escargot Ciphertext:

Also, recall that you look up the letters in the message along the top and the key along the side.

Finally, enciphering (and deciphering) goes faster if you do all the characters enciphered with a particular key letter at one time; so first do everything enciphered with a, then i, and then l.

Answer. Ciphertext: IIREHXSL

**Checkpoint 3.1.10** Try deciphering the message NSDDTIMDQRHO using the key *vilo* and the black (outer) labels.

#### Hint.

- Recall that the plaintext letters run along the top and the key letters are along the left side.
- You need to look up the ciphertext letters in the interior of the table.
- Line up the ciphertext and the key in the same way you lined up the plaintext and key previously.

Key: vilovilovilo Ciphertext: NSDDTIMDQRHO Plaintext: tou

#### Answer. Plaintext: tour de france

**Checkpoint 3.1.11** Try deciphering the message DOLSAEXS using the key *manger* and the red (inner) labels.

Hint.

- Recall that the plaintext letters run along the top and the key letters are along the left side.
- You need to look up the ciphertext letters in the interior of the table.
- Line up the ciphertext and the key in the same way you lined up the plaintext and key previously.

#### Answer. Plaintext: baguette

Today when we use the Vigenère Cipher we use a table Figure C.0.8 which

is simpler, and makes the resulting cipher a little easier to crack. The video below explains using the more modern table and you will play with the modern version in the exercises.





YouTube: https://www.youtube.com/watch?v=5ISnCm4\_V-Y

Figure 3.1.12 Modern look at the Vigenere Cipher

## 3.2 Variation on a Theme

#### **Objectives**

- Agripa's Biography
- Agripa/Vigeners's Pigpen Cipher
- Modern Pigpen Cipher

#### 3.2.1 Biography of Heinrich Cornelius Agripa

Insert Bio Here When Available ...

## 3.2.2 Pigpen Cipher

In his treatise on cryptology [16, p. 275b (534)] Vigenère describes a cipher that he found in Agripa's *De Occulta Philosophia* [1, Vol. 3, p. CCLXXV (279)] in chapter 30 which discusses traditional ciphers used by cabals.

N E faist encore à oblier cefteinuention que touche Agrippa liu. 3. chap. 30. autresfois en trefgrande recommandation enuers les anciens Cabaliffes, depuis l'on en a faist listiere. Ce font quatre lignes fentrecroifantes à angles droists, deux d'icelles perpendiculaires, & deux trauerfieres, qui par ce moien viénent à eftablir neuf caracteres differends, qu'on accómode à autant de lettres, Si que diuerfifiez par vn point affisau milieu, des autres neuf qui en fot vuides, en refulteroat dix huir lettres de refte maniere.



MAIS vous les pouuez transpofer : & fi,gardant

Figure 3.2.1 Vigenere's Variant on the Pigpen Cipher



Figure 3.2.2 Agripa's Variant on the Pigpen Cipher

One may frame nine chambers by the intersection of four parallel lines intersectiong themselves at right angles as expressed in the figure: u l c t k bsja w n e хоf v m d урg & r i z q h Which being dissected into parts generates nine the particular figures: from the nine chambers. Characterize each letter in a chamber by the notation of one point to show the first letter in the chamber; two for the second letter; three for the third letter; so that the characters of Michael may be written in seven characters: ゴビロゴゴ Which may be written one after another drawn as three figures:



#### Comprehension Check:

- If you hadn't seen the first step in how the name "Michael" was enciphered above, then how might you misinterpret the second enciphering?
- Why in the final step do you think some of the symbols must be raised or lowered?
- In the final step above Agripa eliminates all the marks, why would this cause confusion? That is, if you follow his directions which letters will be confused with one another?
- In the translated grid the letters are placed into the grid right to left, this is also the case in Agripa's work Figure 3.2.2, try to find out why.

**Checkpoint 3.2.3** Use Agripa's cipher as translated above to encipher the quote: "Pigs are smarter than dogs, and both are smarter than Congress" - Elayne Boosler

**Answer**. Here are the first few words of the message:



Figure 3.2.4 Cipher Text:

**Checkpoint 3.2.5** Decipher the words enciphered here using Agripa's cipher as presented above.



Figure 3.2.6 Cipher Text:

Answer. Plaintext: "Pork 'n Beans"

The pigpen cipher as we use it today has evolved and standardized to use the key in Figure 3.2.7 which includes all twenty six letters of modern English. It has the advantages that it is quick to learn and it is easy to recreate the cipher key. Also, for each shape you never add more than a single dot so that there is less opportunity for confusion.

<sup>&</sup>lt;sup>1</sup>The translation is based, for the most part, on [2] with reference to the original text. The example was changed some in order to make the work more accessible.`



Figure 3.2.7 Modern key for the Pigpen Cipher

Checkpoint 3.2.8 Use the key in Figure 3.2.7 to decipher the message below.



Figure 3.2.9 Cipher Text

**Answer**. "I am fond of pigs. Dogs look up to us. Cats look down on us. Pigs treat us as equals." Winston S. Churchill

**Checkpoint 3.2.10** Try to encipher the first few words of this next message using the modern variation on the pigpen cipher (Figure 3.2.7).

"Never wrestle with pigs. You both get dirty and the pig likes it." George Bernard Shaw

#### Answer.



Figure 3.2.11 Cipher Text

#### Reflection:

- Do you think that the pigpen cipher offers as much security as the Vigenère cipher? Why or why not?
- In how many ways can you encipher each character using the pigpen cipher? What type of cipher does that make this?
- If you were confronted with a similar cipher which used symbols in place of letters how could you first rewrite your message so that you could then apply the cryptanalysis techniques we discussed previously?

## 3.3 An Automatic Hit

#### **Objectives**

- Vigenère's Biography
- Vigenère's Autokey Cipher

#### 3.3.1 Biography of Blaise de Vigenère

Insert biography when available

#### 3.3.2 The Autokey Cipher

Description of Alberti's autokey from Kahn ...

Below is a description of Vigenère's more secure variation on an autokey cipher. <sup>1</sup> Read carefully through the passage above, you will need to use Figure D.3.2 in order to follow along.

"Each letter may be enciphered by the preceding letter, thus: with the . . . text. . . . "Au nom de l'eternel" and the key D, we say, a [the first letter of the text] from D [the key letter] gives x; u [the second letter of the text] from A [the first letter of the text, which now becomes the key] gives i; n from U, a; o from N, h; m from O, g; d from M, u; e from D, p; l from E, t; e from L; m, t from E, l; e from T, s; r from E, h; n from R, i; e from N, x; l from E, t. By which method, taking D for the key, we would arrive at DXIAHGUPTMLSHIXT. The other method, which is more secret, is to encipher each letter of the clear text not by the letter [of that text] which precedes it, but by the letter by which the preceding letter is enciphered. Thus, a from D, which is the key, gives x; u from X [the first letter of the cipher text] h; n from H, e; o from E, e; m from E, c; d from C, o; e from O, u; l from U, m; ... etc." [16, pp. 49-49b] [11, p. 128]

<sup>&</sup>lt;sup>1</sup>This translation is mostly from Charles Mendelsohn in [11, p. 128], I have supplemented with material from the original description by Vigenère is in [16, pp.49-49b].

1	a	b	c	d	e	f	g	h	i	1
$B \mid$	m	n	0	p	q	r	S	t	u	x
C	a	Ь	C	d	e	f	g	h	i	1
D	x	m	n	0	P	q	r	S	t	u
E	a	b	c	d	e	f	g	h	i	1
F	u	x	m	n	0	P	<u>q</u>	r	S	t
G	a	b	1.C	d	e	f	g	h	i	1.
H	t	u	X	m	n	0	P	<b>P</b>	r	S
Ι	a	b	C	d	e	f	g	h	i	1
	S	tio	u	x	m	n	0	P	<u>q</u>	r
M	a	b	C	d	e	f	g	h	i>	1
N	r	S	t	u	X	m	n	0	<u>p</u>	<u>P</u>
0	a	b	C	d	e	f	g	h	i	
P	<b>q</b>	r	S	t	u	x	m	n	0	P
2	a	b	C	d	e	f	g	h	i	
R	p	q	r	S	t	u	x	m	n	0
S	a	6	C	d	e	f	g	h	i	
		1				And the same in such	Section and	-	000	63
T	0	.P	<b>q</b>	r	S	t	u	X	111	<u>n</u>
	<mark>0</mark>    a	<u>.</u> Р   b	<b>q</b> c	r	s e	t f	u g.	h	i	

Figure 3.3.1 Table used for Vigenère's Autokey Cipher

**Comprehension Check:** It will be easier to follow what is happening in this passage if we write out the message as follows:

Text for the first cipher method:

Plaintext:a u n o m d e l e t e r n e lKey Letter:D A U N O M D E L E T E R N ECiphertext:D X I A H G U P T M L S H I X T

- Vigenère uses a *D* as the key to encipher the first letter, in the first method described where does he get the subsequent letters for the key?
- The letter *D* enciphers *a* as *X*, but the *a* is not in row *D* of Figure D.3.2 so how do we know that the *a* should be *X* ?
- At one point Vigenère uses a key letter of U but there is no row U on his table, looking at how the U enciphers the n, what row is he using in place of U?
- The *a* was enciphered as X, but the first letter of the ciphertext is D, also the ciphertext is one character longer than the plaintext, what is the explanation for both of these? Where did the leading D come from?

Text for the second cipher method:

Plaintext: a u n o m d e l e t e r n e l Key Letter: D X H E E C O U ... Ciphertext: D X H E E C O U M ...

- In the second method described he again starts with a key of *D* but then where does he get the the next key letter *X*? What about the key letter *H*? Try to complete the cipher using the second method.
- Vigenère believes his second method is more secure, looking at the *key* for the second method why do you think he believes this?
- If you compare the *key* for his second method to the cipher text he generates why is Vigenère wrong about his second method being more secure?

**Checkpoint 3.3.2** Check your understanding by enciphering the message "automatic confusion" using Vigenère's first and second autokey methods, use Q for the initial key letter.

Answer. Method 1: Plaintext: a u t o m a t i c c o n f u s i o n Key Letter: Q A U T O M A T I C C O N F U S I O Ciphertext: Q P I G A G R H M U N D H M A F M G H

Method 2:

Plaintext: a u t o m a t i c c o n f u s i o n Key Letter: Q P E L G D X G R N T A B R F I Q L Ciphertext: Q P E L G D X G R R R L F P E I Q L F

**Checkpoint 3.3.3** The message CHFABH CXNXM GFMRR MDDMR was enciphered using Vigenère's first autokey method, check your understanding of the method by deciphering it.

#### Answer.

Ciphertext: C H F A B H C X N X M G F M R R M D D M R Key Letter: C S T O R M T H E B A S T I L L E N U L Plaintext: s t o r m t h e b a s t i l l e n u l l

**Checkpoint 3.3.4** The message QMPTM GPOHG SOOHL QXUGC was enciphered using Vigenère's second autokey method, check your understanding of the method by deciphering it.

#### Answer.

```
        Ciphertext:
        Q
        M
        P
        T
        M
        G
        P
        O
        H
        G
        S
        O
        H
        L
        Q
        X
        U
        G
        C

        Key
        Letter:
        Q
        M
        P
        T
        M
        G
        P
        O
        H
        G
        S
        O
        H
        L
        Q
        X
        U
        G

        Plaintext:
        h
        i
        n
        g
        i
        n
        s
        i
        g
        h
        t
        x
        U
        G
```

**Reflection:** If you go back and look carefully at the preceding couple of exercises you will notice that deciphering a message enciphered using the first autokey method is equivalent to second method and vice versa. In what way does this weaken the cipher?

## 3.4 Stirring Things Up

#### **Objectives**

- Falconer's Discussion of Permutations
- Falconer's Columnar Transposition

#### 3.4.1 Key Permutations

#### Sect. 2: Of Secret Writing by Altering the Places of the Letters, Where the Powers Remain the Same

Part 1: Of the Combinations of Three or More Letters

The first remarkable improvement I find of this kind of cryptography by altering the places of letters, is by the regular combinations of three, four, or more letters: I had it of a gentleman, who (I am fully satisfied) would put it to no bad use; but since it may fall of bad hands, I have his leave to provide against its harm. And that we may proceed regularly therein it is necessary, first to inquire, "how many several ways any given number of letters may be combined?" (i.e.) How many differnt positions they can regularly admit of. And for that end I have hereunto subjoined the following table:

Letters		Several ways	1
I	nia	I	
2	, .	2	
3	7	6	
4	led	24	
5	0it	120	
6		720	1
7	3	5040	
8	pe	40320	
1. 9	Y	3628.30	
IO	Ma	3628800	
II		39916800	
1 12	-	479001600	
&c.		a	

Construction of the Table.

- 1. At the left-hand of your table stands a rank of figures expressing the number of letters to be combined, encreasing in an arithmetical proportion from unity, or 1 and by common excess of 1, or unity.
- 2. To the right hand of these stands the number of *Combinations*,<sup>1</sup> or several ways they can be combined.
- 3. As for 1, being it has but one positions, I set 1 opposite to it in the rank of combinations.
- 4. I multiply 1, in the column of combinations, by two, in the column of letters, and set the product, viz. 2, opposite to it, which shows how often two letters or things can be combined, viz. two times.
- 5. I multiply the 2 in the row of combinations, by 3 in the column of letters; and over against it I set 6 for the product, which shows that three letters have sixe regular positions or combinations.
- 6. ...

Demonstration.

- 1. It is manifest, that one letter or thing has but one position, and two letters have twice the position of 1, viz. once before and once after it. e.g. *AB*, *BA*.
- 2. From the combination of two letters we find that of 3, for the new Letter added is three times applicable to the former positions, viz. in the beginning, middle, and end, e.g. the letter C being joyned with AB, the first combination may be CAB, and the second CBA. Again put C in the middle, and it yields other differences, viz. ACB, BCA. And when C is put in the lat place, you have a fifth and sixth difference, viz. ABC, BAC.
- 3. From the combination of three letters arises that of four letters; viz. ABCD: for D can be four several times applied to each of the former six differences; e.g. in the first, second, third, and fourth or last place.
- 4. From the twenty four combinations arising from four letters, you have that of 5; e.g. if the letter E is added, it is five times applicable to each of the former twenty four regular positions: and so for any other number in infinitum.

I have said *regular* positions, because any number of letters or things may be *irregularly* varied in their positions very much above this order; for example, AB is capable of these irregular variations, AA, AB, BA, BB; and at this rate, three letters have no less than 27 positions in all, viz. six regular (as above) and twenty one irregular positions. ... - John Falconer [4, pp. 37-40 (65-68)]

#### Comprehension Check:

- How many ways does Falconer state that we can rearrange the distinct letters ABCDE?
- How does Falconer arrive at that number? What does he mean when we says "if the letter E is added, it is five times applicable to each of the former twenty four regular positions"?
- Based on what Falconer says above how many ways should we be able to arrange the distinct letters ABCDEF? How would he explain that?
- When Falconer speaks of irregular variations in the last paragraph which combinations of A and B is he considering that he hadn't before?
- Why does he say that there would be 27 positions, how does he get 27 positions from 3 letters?
- How many positions would there be if you had 4 letters, or 5?

What Falconer is discussing here are the factorials we learned about earlier. (Definition 1.1.10) Factorials are related to two other ideas permutations and combinations, which we will explore later when we discuss more complicated ciphers. For now, let's see how Falconer uses these combinations of letters.

<sup>&</sup>lt;sup>1</sup>Today we call these permutations.

#### 3.4.2 Transposition

A New Method How to Write Secretly by the Art of Combinations

- 1. To write by the method proposed, a certain number of letters are combined to lock and unlock the epistle. The differences of writing down the positions [of the letters] ... may be varied to a vast number; ...
- 2. The order of rows is agreed upon in parting.
- 3. The number of letters combined, which is the key, may be expressed in the epistle by some mathematical figure, as  $\triangle$  for three letters,  $\Box$  for 4, etc. or by some private mark.
- 4. They [the individuals communicating] frame a rectangular table of as many columns as there are letters combined.
- 5. The letters so combined are placed in their natural order along the top of the table.
- 6. Having determined of how many lines the table shall consist, the order of combinations agreed upon is set down in a row in the first column towards the left hand; as you may see in the following table.
- 7. The table being thus prepared for writing, they observe the order of the combinations, and write according to its direction.
- 8. When they have placed one letter of every column of all the lines, they begin a new, and so go on until the writing be finished.
- 9. And lastly, they take the letters out of the table according to their partitions, as so many barbarous words, upon paper apart and send it to the confidant. John Falconer [4, pp. 68-72 (40-43)]

#### Comprehension Check:

Let's see if you can follow Falconer's directions. Below I set up Table 3.4.1 according to his description in steps (5) and (6) and used it to encipher the pangram "the quick brown fox jumps over the lazy sleeping dog."

- Looking at Table 3.4.2 why is the *the* from the beginning of the sentence written backwards? (Be sure to look carefully at the letters to the left of the row when answering.)
- In the second row of the same table why is the *qui* from *quick* written in the order it is written? (Again, be sure to look carefully at the letters to the left of the row when answering.)
- In the last row we put down the *n* from *brown* and the *fo* from *fox*, why are they in the order they are in and looking at the next table (Table 3.4.3) where do we put the *x* from *fox* and why?
- How do we finish writing the rest of the message into the boxes in the table?
- Looking at the final message why is there a little triangle at the start of the message and why were the blocks of letters written in the order they were written?

• In what ways is this different from other ciphers we have looked at? (Hint: In this cipher what does cipher text *E* represent, or cipher text *F*?)

Table 3.4.1 Falconer's Transpo-sition Table Initial Setup

Table3.4.2Falconer'sTranspositionpositionTableAfter the FirstPass

		A	В	C	10				
1	CBA						А	В	C
2	CAB				1	CBA	Е	Η	Г
3	ACB				2	CAB	U	Ι	Q
4	BCA				3	ACB	С	В	K
5	BAC				4	BCA	W	R	C
		I			$\overline{5}$	BAC	F	Ν	C

Table 3.4.3 Falconer's Transpo-sition Table Fully Filled

		А	В	С
1	CBA	EUS	HJY	TXZ
2	CAB	UPE	ISE	QML
3	ACB	COP	BEN	KVI
4	BCA	WHO	RRG	OTD
5	BAC	FL	NEG	OA

Final Message:

" $\triangle$  EUS HJY TXZ UPE ISE QML COP BEN KVI WHO RRG OTD FL NEG OA"

**Checkpoint 3.4.4** Decipher the following message which used the same key as the one represented in Table 3.4.1:

 $\bigtriangleup$  WU OO NR MD A IN L T L N U R D E A

Answer. Now I'm all turned around.

**Solution**. First put the blocks of text into the table row by row in alphabetical order:

Table 3.4.5 Falconer's Transposition Practice

		А	В	С
1	CBA	WU	00	NR
2	CAB	MD	А	IN
3	ACB	L	Т	L
4	BCA	Ν	U	R
5	BAC	D	Е	Α

Now read the message out character by character according to the combinations at the left:

 ${\it nowimall turned} around$ 

Finally, we get the original message.

Now I'm all turned around.

**Checkpoint 3.4.6** Encipher the following message using the same key as the one represented in Table 3.4.1:

"As long as the world is turning and spinning, we're gonna be dizzy and we're gonna make mistakes." - Mel Brooks

**Answer**.  $\triangle$ LSNDNE SINENM ADIBOS NUNZMB GRGZAR OTIIAL ANWYKO TNRNMK SIEAEO WNOET HGEDIS EAGWS RSNEK ODNRA LPAGE

Solution. First we break up the message into blocks of three to make it easier

to work with.

```
ASL ONG AST HEW ORL
DIS TUR NIN GAN DSP
INN ING WER EGO NNA
BED IZZ YAN DWE REG
ONN AMA KEM IST AKE
SME LBR OOK S
```

Then copy each character into the table using the combination keys at the side to tell us which column to put each character in.

 Table 3.4.7 Falconer's Transposition Practice

		А	В	С
1	CBA	LSNDNE	SINENM	ADIBOS
2	CAB	NUNZMB	GRGZAR	OTIIAL
3	ACB	ANWYKO	TNRNMK	SIEAEO
4	BCA	WNOET	HGEDIS	EAGWS
5	BAC	RSNEK	ODNRA	LPAGE

Finally, we read the message out of the table from left to right and from top to bottom.

 $\triangle {\rm LSNDNE}$  SINENM ADIBOS NUNZMB GRGZAR OTIIAL ANWYKO TNRNMK SIEAEO WNOET HGEDIS EAGWS RSNEK ODNRA LPAGE

**Checkpoint 3.4.8** Encipher the following message using the three letters as before, but with the row keys ACB - CAB - BAC - ABC, Table C.0.11 is a blank table you can copy if you need to:

"A person who never made a mistake never tried anything new." - Albert Einstein

# **Answer**. $\triangle$ AVADAI ERENB PEKALN SAETR ODVHT RMNYE WARNI NEEIE HMTGN OIRNS NSIET ETEWE

**Solution**. First we break up the message into blocks of three to make it easier to work with.

APE RSO NWH ONE VER MAD EAM IST AKE NEV ERT RIE DAN YTH ING NEW ALB ERT EIN STE IN

Then copy each character into the table using the combination keys at the side to tell us which column to put each character in.

		А	В	$\mathbf{C}$
1	ACB	AVADAI	ERENB	PEKALN
2	CAB	SAETR	ODVHT	RMNYE
3	BAC	WARNI	NEEIE	HMTGN
4	ABC	OIRNS	NSIET	ETEWE

Finally, we read the message out of the table from left to right and from top to bottom.

 $\bigtriangleup AVADAI ERENB PEKALN SAETR ODVHT RMNYE WARNI NEEIE HMTGN OIRNS NSIET ETEWE$ 

## 3.5 Presidential Secrets

## **Objectives**

- Jefferson's Biography
- Jefferson's Letter with Vigenère's Cipher
- Jefferson's Wheel Cipher

## 3.5.1 Thomas Jefferson's Biography

A brief biography of Thomas Jefferson.

## 3.5.2 Thomas Jefferson and Vigenère's Cipher

Among Thomas Jefferson's papers in the library of congress there are atleast 63 distinct items which discuss ciphers. In one of these, a letter from Jefferson to Meriwether Lewis (Thomas Jefferson to Meriwether Lewis, April 20, 1803), Jefferson suggests that they use Vigenère's cipher for their correspondence.



then copy out the cyphered line thus, uvyvqb&mgtsfresssnjemcuqitm numbers are thus, 18 is by 1798 is thus bubq the method is this. look for t in the  $1^{st}$  vertical column & a in the  $1^{st}$  horizontal one gives u . . . V . . . n . . . h . . . t ...у ...е . . . i . . . v . . . m ...р . . . q . . . а . . . 0 . . . b . . . n

#### Comprehension Check:

- Look carefully at Jefferson's work, can you find his error?
- How did Jefferson turn 18 into bv?
- How did Jefferson turn 1798 into bubq?

**Checkpoint 3.5.1** Below is the rest of the quote from Thomas Blacklock that Jefferson started enciphering. Try to complete the enciphering.

The man, whose mind on virtue bent, Pursues some greatly good intent, With undiverted aim, Serene, beholds the angry crowd; Nor can their clamors, fierce and loud, His stubborn honor tame.

#### Answer.

UVYVQB &MGTSF RCSSSN JEMCUQ ISLQHK AJTWXG NS&&UP

XQQHBH MYBXJF UJBBXI RIAWSK BUSENE TSKNCT FJ&PZX

AIWIFF HERLGC & IFPEW JCHLJA SQEJBC VXYJSK LUPRID

PHXQYG WYMCPH &CWSSG SGUVU

**Solution**. Start by writing out the plaintext with the key text above it, just as Jefferson did:

antipod esan tipodes anti podesa ... pursues some greatly good intent ...

Next, using the key letter to tell you the column and the plaintext to tell you the row, look up the cipher letters inside the table:

antipod esan tipodes anti podesa ... pursues some greatly good intent ... QHKAJTW XG ...

**Checkpoint 3.5.2** Use the key word happy to enchipher Jefferson's birthday of April 13, 1743.

Answer.

IQGYJ IDQWB K

**Solution**. As before write out the message with the key lined up above it. Then you can look up the key letter along the top of Jefferson's table and the message letters or numbers along the sides.

happy happy h april 13174 3 IQGYJ IDQWB K

#### 3.5.3 Thomas Jefferson's Wheel Cipher

Here is a brief description of Thomas Jefferson's wheel cipher, and here is his description Jefferson's Wheel Cipher, that we examine below.

From a cylinder of white wood of about 2 inches in diameter and 6 to 8 long bore through the center a hole sufficient to receive an iron spindle or axis of 1/3 or 1/2 inch diameter. Divide the perimeter into 26 equal parts (for the 26 letters of the alphabet) and with a sharp point draw parallel lines through all the points of the division from one end to the other of the cylinder, and trace those lines with ink to make them plain. Then cut the cylinder end wise into pieces of about 1/6 of an inch thick, they all resemble backgammon men with plane sides. Number each of them as they are cut off, on one side, that they may be arranged in any order you please. On the perimeter of each & between the black lines, put all the letters of the alphabet, not in their established order, but jumbled & without order so that no two shall be alike. Now string them in their numerical order on an iron axis, one end of which has a head and the other a nut or crew the use of which is to hold them firm in any given position when you chose it. They are now ready for use, your correspondent having a similar cylinder similarly arranged.

Suppose I have to cypher this phrase "Your favor of the  $22^{nd}$  is received."

I turn the  $1^{st}$  wheel till the letter y presents itself

turn the  $2^{nd}$  and place it's **o** by the side of the **y** of the  $1^{st}$ 

turn the  $3^{rd}$  and place it's u by the side of the o of the  $2^{nd}$ 

turn the  $4^{th}$  and place it's r by the side of the u of the  $3^{rd}$ 

turn the  $5^{th}$  and place it's f by the side of the r of the  $4^{th}$ 

turn the  $6^{th}$  and place it's a by the side of the f of the  $5^{th}$ 

and so on till I have got all the words of the phrase arranged in one line, fix them with the screw. You will observe that the cylinder then presents 25 other lines of letters, not in any regual sense, but jumbled & without order or meaning. Copy one of them in the letter to your correspondent. Then he receiving it, he takes his cylinder and arranges the wheels so as to present the same jumbled letters in the same order in one line. He then fixes them with the screw and examines the other 25 lines and finds one of them presenting him these words "Your favor of the  $22^{nd}$  is received" which he understands. As the others will be jumbled & have no meaning he cannot mistake the true one intended.

#### Comprehension Check

• If the cylinder we start with is 8 inches long, how many discs do we get following Jefferson's directions?

- Using factorials (see Definition 1.1.10), in how many ways could you rearrange the number of discs you found in the previous problem?
- In his description Jefferson says that we should write the alphabet around the edge of each disk "not in their established order, but jumbled & without order so that no two shall be alike", how many ways can we rearrange the letters of the alphabet so that they are not in their usual order?
- Jefferson was unfortunately wrong when he said "As the others will be jumbled & have no meaning he cannot mistake the true one intended." Why did he believe this was the case, what did he mean by this? Can you construct a way to arrange letters on a disk so that you could end up with more than one message?

While his cipher was not used while Jefferson was in office a variation of it was employed from about 1920 to 1945 in the form of the M-94 Cipher Cylinder.

**Checkpoint 3.5.3** Use the Jefferson Wheel simulator below to encipher the message "Jefferson".

Answer. Because after lining up the letters you want you can choose to write down any other row, there are multiple different as nwers. One posibility is: MMIHODVWR9RN

**Checkpoint 3.5.4** Decipher the message LY2VAALYQVCV below using the simulator.

Answer.

attack at dawn

Specify static image with @preview attribute, Or create and provide automatic screenshot as images/jefferson $_wheel_interact - preview.pngviathembxscrip$ 



Figure 3.5.5 Jefferson's Cipher Wheel

## 3.6 How Mixed Up Do You Feel?

1. Encipher this message using a *Pigpen Cipher*. Figure C.0.10

- "You can put wings on a pig, but you don't make it an eagle."
- William J. Clinton

#### Answer.



Figure 3.6.1 "You can put wings on a pig, ..."

2. Decipher this message written using a *Pigpen Cipher*. Figure C.0.10



Figure 3.6.2 Cipher Text

#### Answer.

That's All Folks

**3.** Encipher the following message with Alberti's polyalphabetic cipher (use the simulator below); use and index letter of s and key letters C, G, and X (switch key letter after every other word).

"Men can do all things if they will." - Leon Battista Alberti

#### Answer.

Cdmb sxb G&h tmm Xxkpg& ke Clm nnhff

**Solution**. Start by setting the index to s and the key letter to C. Then you can start translating the plaintext to ciphertext. Remember that you need to write the key letter down each time you change it, the plaintext for Alberti are the uppercase letters around the outside of his formula, and sometimes you need to leave letters out or make substitutions since not all the letters are available. In the end you should have something like: Cdmb sxb G&h tmm Xxkpg& ke Clm nnhff.

4. Encipher the following message using the modern version of the Vigenere Square Figure C.0.8 (you can check your final answer using the simulator below, but you should be able to do this by hand) and a keyword of *FRANCE*.

"Hope is a good breakfast, but it is a bad supper." - Francis Bacon

Answer.

MFPRKW FXOBFF WVAXHE XKBHVM YZSNDE IJUCRI W

**Solution**. Write down the plaintext with the key above it and then, using the key letter to indicate the row and plain letter the column, look up the ciphertext in the table.

france france france france france f hopeis agoodb reakfa stbuti tisaba dsuppe r MFPRKW FXOBFF WVAXHE XKBHVM YZSNDE IJUCRI W

5. Use the modern Vigenere Square Figure C.0.8 to decipher the following message which was enciphered using a key word of *STATE*. (Again, you can check your final answer using the simulator below, but you should be able to do this by hand.)

LAEKI SKEGS KXCKI LLTHW MVCXW KBTBW LAEKI KNLMS XIRXT SKAMM GGHTV VPOKO SGDEI SKNBR YYRHQ XTIEY JXCHP AGPHA WEL

**Answer**. "There are no secrets to success, it is the result of prparation, hardwork, and learning from failure." - Colin Powell

**Solution**. Write out the ciphertext with the key word written above it as before. Now look use each key letter to tell which row to look in and

slide your finger along to the cipher letter; the column header is your plaintext letter.

state state state state state state state state ... LAEKI SKEGS KXCKI LLTHW MVCXW KBTBW LAEKI KNLMS ... there are ...

6. Encipher this quote with an *Autokey Cipher* with a key letter of *F*, use Vigenere's original table for this one. Figure C.0.9

"Obstacles are those frightful things you see when you take your eyes off your goal." - Henry Ford

Answer.

ERGFO OUMIO FTLXF CSPHN OQATA M ...

**Solution**. write out the message with the key above, like Vigenere's polyalphabetic cipher, but now the key is essentially the message its self. Above the first letter is our key letter, then after that you write down the message. Thus lok up the key letter on the side of the table and find the plain letter in the corresponding row. This last step is a little tricky because the letters are in pairs, so if the key letter is S and we need a plain t the cipher letter will be F.

FOBST ACLES ARETH OSEFR IGHTF U ... obsta clesa retho sefri ghtfu l ... ERGFO OUMIO FTLXF CSPHN OQATA M ...

7. Encipher this quote with an Autokey Cipher with a key letter of J, use the modern Vigenere Square for this one. Figure C.0.8

"Success is not final, failure is not fatal: it is the courage to continue that counts." - Winston Churchill

Answer.

KMWEG WKAAF BHYNV NLQFI TFLVM AFBHY FTTLT BBALA LGQIL ...

**Solution**. As before write out the message, place the key letter above the first plaintext letter, and write out the rest of your message after that as a key. Then, using the modern Vigenere table, the key letters in the top row tell you which row to look in and the plain letters gives you the column.

SSUCC ESSIS NOTFI NALFA ILURE ISNOT FATAL ITIST HECOU ... succe ssisn otfin alfai lurei snotf atali tisth ecour ... KMWEG WKAAF BHYNV NLQFI TFLVM AFBHY FTTLT BBALA LGQIL ...

8. Use the modern Vigenere Square Figure C.0.8 to decipher the following message which was enciphered using an Autokey Cipher and key letter of X.

ARBHN LAALA LVVEN LRAPV TZALC ACNGW RAMTM ZXFSR VVT

**Answer**. "Donuts, is there anything they can't do?" - Matt Groening **Solution**. To decipher this you need to decipher the first character with the X, look in row X for the A and then there is a d at the top of that column. Now repeate the process with the D you just found and the R, this is how you get the o. Keep repeating this process until you find the message.

Xdonu tsist herea nythi ngthe ycant domat tgroe ning ARBHN LAALA LVVEN LRAPV TZALC ACNGW RAMTM ZXFSR VVT donut sisth erean ythin gthey cantd omatt groen ing

**9.** Encipher the following message using Falconers Columnar Transposition with a key of *{ACB,BCA,CAB,ABC}*. Table C.0.11

"Were I not a king, I would be a university man." - King James I

Answer.

WGI RWE EIV NLI EOR IUS TBY AEM ODT KAA IUN NN

**Solution**. Create a table with three three columns and four rows. Label the rows with the keys and the columns with A, B, and C. Write down the message character by character with the keys at the side telling you which columns to write the characters in. Finally, write down the blocks of text from left to right and top to bottom.

Cipher Table:

		Α	В	С
ACB	:	WGI	RWE	EIV
BCA	:	NLI	EOR	IUS
CAB	:	TBY	AEM	ODT
ABC	:	KAA	IUN	NN

Final cipher text:

WGI RWE EIV NLI EOR IUS TBY AEM ODT KAA IUN NN

**10.** Encipher the following message using Falconers Columnar Transposition with a key of *{CBA,ABC,BCA,CAB,ACB,BAC}*. Table C.0.11

"I have loved justice and hated iniquity: therefore I die in exile." - Pope Gregory VII

Answer.

AHF HDE INR VAO ETR LEE ENI ODI VID JQI UUN DIE SIE IYI TTX EHE CTL AE

Solution. Cipher Table:

ABCCBA:AHFHDEINRABC:VAOETRLEEBCA:ENIODIVIDCAB:JQIUUNDIEACB:SIEIYITTXBAC:EHECTLAE

Completed Message:

AHF HDE INR VAO ETR LEE ENI ODI VID JQI UUN DIE SIE IYI TTX EHE CTL AE

11. Decipher the following message using Falconers Columnar Transposition with a key of  $\{ABC, BCA, BCA, ACB\}$ . Table C.0.11 ISUIIP TTRNLE IADOLA OTTSM SREUIR NSSRAE NOYVA TOI ES IHNLH TLBEK EOTWS HDUSE **Answer**. "It is not in the stars to hold our destiny, but in ourselves." - William Shakespeare

**Solution**. Set up a table with columns labeled A, B, C, and rows labeled with the keys. Write each block of text into the table from right to left and top to bottom. Finally, read the characters out of the table one at a time based in the order given by the keys.

ABCABC:ISUIIPTTRNLEIADOLABCA:OTTSMSREUIRNSSRAEBCA:NOYVATOIESIHNLHACB:TLBEKEOTWSHDUSE

Completed Message:

"It is not in the stars to hold our destiny, but in ourselves." - William Shakespeare

**12.** Encipher declaration using Jefferson's wheel cipher.

Answer.

WHIQCGXXWTH

13. Encipher independence using Jefferson's wheel cipher.

Answer.

EBWVEUJUNPZG

14. Decipher ZI2UHM60XHII, which was enciphered with Jefferson's wheel cipher.

Answer.

moving on up to

Substitution Cipher Cell:

```
import textwrap
@interact
def_{p} =
   input_box('abcdefghijklmnopqrstuvwxyz',label='Plain',
   type=str,width=50,height=1),
            с =
                input_box('ZYXWVUTSRQPONMLKJIHGFEDCBA',label='Cipher',
                type=str,width=50,height=1),
            shift=[0..25],
            mode=selector(['encipher', 'decipher'],
                buttons=True),
            spaces = selector(['yes', 'no'], buttons=True),
            m=input_box('sage', label="Message", height=5,
                width=50, type=str)):
    P = str(p.encode('ascii', 'replace').decode()).upper()
    C = str(c.encode('ascii', 'replace').decode()).upper()
   C = C[shift:]+C[:shift]
    Message =
        str(m.encode('ascii', 'replace').decode()).upper()
    print("\nPlain alphabet: \t", str(P).lower())
    print("Cipher alphabet:\t", str(C))
    if len(C)!=len(P):
        print("Key lengths do not match.")
    else:
        if mode == 'encipher':
```

```
inText = P
    outText = C
else:
    inText = C.lower()
    outText = P.lower()
    Message = Message.lower()
output = ""
for char in Message:
    try:
        position = inText.index(char)
        output += outText[position]
    except:
        if spaces == 'yes': output += char
        else: pass
print("\nHere is your output:\n")
if spaces == 'yes':
    print(textwrap.fill(output, 50))
else:
    for i in range(0, len(output), 5):
        print(output[i:i+5],)
        if (i+5)%50 == 0: print("\n")
```

Alberti's Cipher Formula:

```
# constants:
plain_alphabet = "ABCDEFGILMNOPQRSTVXZ1234"
cipher_alphabet = "acegklnprtvz&xysomqihfdb"
U = [plain_alphabet[i] for i in range(len(plain_alphabet))]
L = [cipher_alphabet[i] for i in range(len(cipher_alphabet))]
# interactive:
@interact
def _(Key=("Key Letter",slider(U)),
      index_letter=("Index Letter",slider(L))):
    alpha_len = min(len(plain_alphabet), len(cipher_alphabet))
    angle = 2*pi/alpha_len
    #print Key, index_letter
    text_plot_sum = text("",(0,0))
    radius=1
    index1 = cipher_alphabet.index(index_letter)
    index2 = plain_alphabet.index(Key)
    index = index1-index2
    #print index
    for i in range(alpha_len):
        if cipher_alphabet[i]==index_letter:
            text_plot_sum +=
                text(cipher_alphabet[i],(radius*cos(-angle*i),radius*sin(-angle*i)),rota
            text_plot_sum +=
                text(plain_alphabet[(i-index)%alpha_len],(1.25*radius*cos(-angle*i),1.25
        else:
            text_plot_sum +=
                text(cipher_alphabet[i],(radius*cos(-angle*i),radius*sin(-angle*i)),rota
            text_plot_sum +=
                text(plain_alphabet[(i-index)%alpha_len],(1.25*radius*cos(-angle*i),1.25
    text_plot_sum.show(axes=False,aspect_ratio=1,figsize=[4,4])
```

Vigenere Cipher Cell:

```
import re
import textwrap
@interact
def _(m=input_box('sage', label="Enter your message",
   height=3, width=50, type=str),
        key=input_box('sage', label="Enter your key",
            height=1, width=20, type=str),
        mode = selector(['encipher', 'decipher'],
           buttons=True),
        spaces = selector(['yes', 'no'], buttons=True)):
    plain_alpha = "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
    clean_message =
        str(m.encode('ascii', 'replace').decode()).upper()
    if spaces == 'no':
        clean_message = re.sub('[^A-Z]', '', clean_message)
    cipher_key =
        re.sub('[^A-Z]','', str(key.encode('ascii', 'replace').decode()).upper())
    key_list = [plain_alpha.index(ch) for ch in cipher_key]
    if mode == 'decipher':
        key_list = [-1*k for k in key_list]
    cipher_text = ""
    key_counter = 0
    for ch in clean_message:
        try:
            tmp_pos = plain_alpha.index(ch)
            cipher_pos = (tmp_pos+key_list[key_counter])%26
            cipher_text += plain_alpha[cipher_pos]
            key_counter = (key_counter+1)%len(key_list)
        except:
            if spaces == 'yes':
                cipher_text += ch
    print("\nHere is your output:\n")
    if spaces == 'yes':
        print(textwrap.fill(cipher_text, 42))
    else:
        for i in range(0,len(cipher_text),6):
            print(cipher_text[i:i+6],)
            if (i+6)%42 == 0: print("\n")
```

Falconer's Tabular Cipher:

```
key = keys[0]
while "" in key: key.remove("")
message_table = [["" for x in range(chars)] for y in
    range(len(key))]
for i in range(0,len(text),chars):
    row = (i/chars)%len(key)
    for j in range(chars):
        trv:
            col = columns.index(key[row][j])
        except:
            col = chars-1 #pass
        try:
            message_table[row][col] += str(text[i+j])
        except:
            pass
out_message = ""
print("Chracters in text: ",len(text))
print("Cipher Table:")
for k in range(len(key)):
    print("\t", str(key[k][0: chars]),":\t","\t".join(message_table[k]))
    for i in range(chars):
        out_message += str(message_table[k][i])+" "
print("Completed Message:")
#for i in xrange(0,len(out_message),50):
#
    print("\t",out_message[i:min(i+50,len(out_message))].strip())
print(textwrap.fill(out_message, 50))
```

Jefferson's Wheel Cipher:

```
# wheels:
disk_0 = list("A0LOVTZSMHNJRUIEWCDQPKBYGXF")
disk_1 = list("AXZTKYMHPESDURFLWIVOGNBQ1JC")
disk_2 = list("AJDWQIYCFXMUSGLBZTH2RVENKPO")
disk_3 = list("AEVDQULIBSJ3HNPFWMYKGXCZTRO")
disk_4 = list("AHUNFRILTG4MDOQPEXSYJKVWBCZ")
disk_5 = list("AEUMDGZRJBQTXC5SLYFPNIVOHKW")
disk_6 = list("A6LOVTZSMHNJRUIEWCDQPKBYGXF")
disk_7 = list("AXZTKYMHPESDURFLWIVOGNBQ7JC")
disk_8 = list("AJDWQIYCFXMUSGLBZTH8RVENKPO")
disk_9 = list("AEVDQULIBSJ9HNPFWMYKGXCZTRO")
disk_A = list("AHUNFRILTGdMDOQPEXSYJKVWBCZ")
disk_B = list("AEGZRJBQTXCoSLYFPNIVOHKUMDW")
# cylinder:
cylinder =
    [disk_0, disk_1, disk_2, disk_3, disk_4, disk_5, disk_6, disk_7, disk_8, disk_9, disk_A, disk_B
# interactive:
@interact(layout={'top': [['wheel0',
    'wheel1', 'wheel2', 'wheel3'],['wheel4',
    'wheel5', 'wheel6', 'wheel7'],['wheel8', 'wheel9', 'wheelA', 'wheelB']]})
def _(
        wheel0=selector(disk_0,label="0"),
        wheel1=selector(disk_1,label="1"),
        wheel2=selector(disk_2,label="2"),
        wheel3=selector(disk_3,label="3"),
        wheel4=selector(disk_4,label="4"),
```

```
wheel5=selector(disk_5,label="5"),
    wheel6=selector(disk_6,label="6"),
    wheel7=selector(disk_7,label="7"),
    wheel8=selector(disk_8,label="8"),
    wheel9=selector(disk_9,label="9"),
    wheelA=selector(disk_A,label="10"),
    wheelB=selector(disk_B,label="11")
):
args=locals()
text_plot_sum = text("",(0,0))
column_index = 3
for alphabet, wheel in zip(cylinder, sorted(args)):
    character = args[wheel]
    temp_wheel =
        alphabet[alphabet.index(character):]+alphabet[:alphabet.index(character)]
    row_index=0
    for i in range(-3,4):
        column_head = (column_index-3)/2
        text_plot_sum +=
            text(str(column_head),(column_index,1.5),fontsize=16,color="blue")
        if temp_wheel[i]==character:
            text_plot_sum +=
                text(temp_wheel[i],(column_index,-row_index),fontsize=16,color="red"
        else:
            text_plot_sum +=
                text(temp_wheel[i],(column_index,-row_index),fontsize=16,color="gree
        row_index+=2
    column_index+=2
text_plot_sum.show(axes=False,aspect_ratio=1,figsize=[5,5])
```

## Chapter 4

# Triumphs of Logic and Statistics

In this chapter discuss the emerging roll of mathematics in analysis:

- Falconer's Attacks
- Babbage's attack
- Friedman and Statistics
- Sukhotin's Algorithm

## 4.1 A Simple Solution

#### **Objectives**

- Factoring Integers
- Falconer's Solution to the Columnar Transposition

## 4.1.1 The Power of Prime Factors

The strength of Falconer's keyed columnar transposition cipher is the almost astronomical number of combinations of keys that can be used. As we have seen previously with just three characters we have six row keys which, depending on how many of them we use and how we arrange them, can give us one thousand nine hundred fifty six subtly different ciphers. If we increase the number of characters to four or five then that number goes up to

1,686,553,615,927,922,354,187,744

and

 $18, 183, 954, 211, 052, 603, 322, 452, 474, 095, 140, 831, \dots$   $738, 614, 446, 765, 249, 926, 715, 439, 301, 461, 074, 500, \dots$   $732, 116, 312, 180, 273, 095, 148, 765, 061, 059, 468, 326, \dots$   $378, 636, 312, 510, 693, 233, 993, 926, 141, 650, 787, 502, \dots$  $931, 879, 726, 557, 669, 253, 713, 958, 681, 131, 266, 045, \dots$  respectively. This would seem to offer security in the extreme, but as we will see, Falconer himself gives us two relatively simple ways to crack this cipher. We begin with the more mathematical approach.

... take the number of partitions of the seeming words in the epistle, and find out their several divisors, which may be performed by the following rules.

How to find out the equal divisors of any number.

1. Divide the number given by some **prime number** (i.e.) such a number that cannot be divided, but by it self, or unity; and the *quotient* by some or other prime number, and the *last quotient again by a prime* number; and so go on until the *last quotient of all be one*; and thus you shall find a certain number of prime divisors.

2. Make a rectangular table that shall consist of as many columns as you have prime divisors, which you must place one after another at the tops of the columns; ...

By multiplying the first prime divisor towards the left hand of the table by the second, and writing the product under the second.

Next, by the third prime divisor, multiplying all the figures in the table towards the left hand, setting the several products in the third column: and so forth, throughout all the prime divisors; ...

Example to find out all the divisors of 450

The first line contains the first divided, and the respective quotients; the lowest line is the several prime divisors.

Now 450, the number given, being divided by 2, a prime divisor, the quotient is 225, which being divided by 3, you have 75 for a new quotient; and again divided by 3, you have 25 for another quotient. This last divided by 5, gives 5, which being a prime number, you have 1, or unity in the last quotient of all: so that your prime divisors are, 2, 3, 3, 5, 5, all which set down in the tops of the columns [of a new table], and multiplying them according to the [second] rule given, the operation will stand thus.

3	3	5	5
6	9	10	25
	18	15	50
		30	75
		45	150
		90	225
			450
	36	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

All the divisors of 450, are 2, 3, 5, 6, 9, 10, 15, 18, 25, 30, 45, 50, 75, 90, 150, 225; and one of them (supposing the epistle to have consisted of 450 seeming words) should have been the number of letters combined for the key: ...

- John Falconer [4, pp. 44-47 (72-75)]

#### Comprehension Check:

• What are *prime numbers* and what are the *prime divisors* of a number?

- How did Falconer decide to start dividing by 2?
- Have you ever learned a rule that may have told you that 225 and 75 were divisible by 3?
- Looking at the second table in the above quote, where do we get the numbers in the top row?
- In the second column of the second table there is a 6 under the 3, how did Falconer come up with that?
- In the third column how did he get the 9 and the 18?
- What about the other columns, where did those numbers come from?
- Falconer tells us to multiply each prime in the second table by all the figures to its left, why don't we get duplicate numbers then? What rule did he not explicitly state?
- In the last paragraph Falconer tells us that if 450 were the number of blocks of text in the enciphered message then one of the factors of 450 is the number of key letters in the message. Looking back at how Falconer's transposition cipher works why is this the case?

**Checkpoint 4.1.1** Test your understanding of what Falconer has described by factoring 210 into prime factors and then writing out all the different divisors

#### Answer Table 4.1.2 Prime Factorization Table 4.1.3 All Possible Factors

210	105	35	7	1		2	3	5	7
2	3	5	7				6	10	14
								15	21
								30	42
									35
									70
									105
									210

**Solution**. First make a table of the primes factors by writing down the 210 and under that a 2 (since it is even), then next to the 210 write 105=210/2. Next, with a little work, we see that 105 is divisible by 3, so write 3 under the 105 and 35 next to 105, since 35=105/3. Continue this process to fill in Table F.0.9 with the rest of the prime factors.

Now make a table with the prime factors written across the top. Starting with the second column, multiply each number in the table by all the numbers to its left by which it has not already been multiplied. So, we multiply 3 by 2 to get 6, then 5 by 2 to get 10 and so on. Try to finish filling in Table F.0.10.

#### Table 4.1.4 Prime Factorization Table 4.1.5 All Possible Factors

210	105	35				2	3	5	7	
2	3						6	10	14	
									21	
									210	

**Checkpoint 4.1.6** Test your understanding of what Falconer has described by factoring 980 into prime factors and then writing out all the different divisors **Answer**.

#### Table 4.1.7 Prime Factorization

980	490	245	49	7	1
2	2	5	7	7	

#### Table 4.1.8 All Possible Factors

2	2	5	7	7
	4	10	14	49
		20	28	98
			35	196
			70	245
			140	490
			•	980

**Checkpoint 4.1.9** Look at the text enciphered below using Falconer's transposition cipher. Using the techniques described above, how many key letters may have been used to encipher this message?

```
NILOEOTHIETTSGTN HCITRTSRNIAEOIOC TATEUOOOETURHVFS
LNEURRHYMSIHHIHQ EELHEFETTMRDUNRO NEAMTDPLHYETLOAS
TTEPEEYTPEOESTTA THMOIITESHTEUPUE ETDSEBASETFEONOU
OEKEBDAFASNHDSRT NSECLSEREWHHEICM EMTWKRIEBRNTAYTK
TUEEFUWEREIAPTHA IYWAEUHBIFTWDRUD WHSLSNTOPLFNTEMN
GRVEDCGTEPTDATIM RMSIIYLDEEECRBOE AEEBEINOTPNLEOAR
SEOTLWLAIHTOTEOW RRMHLHINTAEULNFO DHLTAAAROREANLOH
ETAUHHOEFEMEIETT HOTYTPNTRTSTGFBI AURSUPTHOHRHTWEN
GEIATHLHNIIDVSRS FSRSNHFTNOUTIDHR EIUASTSREHGFLETI
DDSEISTOTTIIELAE DEAREEAEENFOELEH NBETSIHENAEDHIDE
```

Hint.

- Start by counting the number of blocks of letters.
- Use what we have learned to factor the number of blocks into its prime factors, and find all the possible divisors.
- Look back at how Falconer's transposition works (Subsection 3.4.2), what can the factors tell us about the enciphering process?

**Answer**. There are thirty blocks of text. Factoring that, we get the prime factors 2,3, and 5. Putting these together, the complete list of factors of 30 is 2, 3, 5, 6, 10, 15, and 30. Since Falconer's cipher requires writing letters into a table, we can conclude that the table's dimensions were likely 3 by 10, 5 by 6, 6 by 5, or 10 by 3. (It is unlikely that the table was 1 by 30 or 2 by 15.) Using this information we can conclude that there were likely 3, 5, or 6 key letters and we could try to pick apart the message.

#### 4.1.2 But Really There's an Easier Way (sort of)

After walking us through a careful exposition of how to find all the factors of a number and encouraging us to use this in order to find out the number of letters used for our key, Falconer lets us know that there is effectively an easier way to attack his own cipher. "or rather for dispatch, take out the seeming words, and write them down in [columns] beginning at the first, and then proceed to the second, third, fourth, fifth, etc., until you have gone through them" - John Falconer [4, p.47 (75)]

Let's follow Falconer's advice using the message we used when we introduced his cipher in Section 3.4:

" $\bigtriangleup$  EUS HJY TXZ UPE ISE QML COP BEN KVI WHO RRG OTD FL NEG OA"

Which transforms to the following when we rewrite it following Falconer's directions:

#### Table 4.1.10 Transpose Table for a Falconer Cipher

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Е	Η	Т	U	Ι	Q	С	В	Κ	W	R	0	F	Ν	0
2	U	J	Х	Р	$\mathbf{S}$	Μ	Ο	Е	V	Η	R	Т	$\mathbf{L}$	Ε	Α
3	S	Y	Ζ	Е	Е	$\mathbf{L}$	Р	Ν	Ι	Ο	G	D	•	$\mathbf{G}$	•

"1. Search in the several lines for some of the particles [(words or ngrams)] of that language you shall suppose the epistle to have been writ in. If in English, make suppositions, e.g. for such little words as *the, that, for, of, to, and, etc.* and the like, without some of which no man can well express business of any moment."

"2. Having supposed in any of the lines; for some one of those mentioned, or the like particles, you may prove the truth of your supposition, by taking out the opposite letters of all the lines: And if they do not make words, or syllables, or produce such letters as can probably follow one another in that order, your first supposition is false, and you must suppose anew."

"3. Having by fresh suppositions found some useful word: And the letters of the other lines (in the same order) agreeing, the words or syllables arising from them, will direct you to some new [column] that goes before or after in the true order: And thus you may proceed till you have found out the whole writing, which by this time will be no great difficulty." - John Falconer [4, pp.48-49 (76-77)]

#### Comprehension Check:

Taking Falconer's advice from step one look at each line of Table 4.1.10 we created above as you consider the following.

- Are there three letters in line one which we would expect to go together assuming this is written in English?
- There is a Q in row one column six, what needs to come after that? What column is it in?
- Looking now at line two what three letters do we again see that should go together?
- In line three are there three letters you could put together to get a common English ending? Are there other letters you could arrange with them in order to get an entire word?

• Finally, columns thirteen and fourteen only have two characters in them each, where do you think they belong if we were to rearrange the lines?

Use the observations you just made in order to rearrange the columns in the table, you should make a copy of this table to help you.

Table 4.1.11 Blank Transpose Table for a Falconer Cipher

Col's								
1	 							
2	 							
3	 							

Test your understanding by working on the following exercises of increasing size, be patient they are finicky and time consuming. As a hint, all of the quotes contain the word *WINTER*.

Checkpoint 4.1.12 Try to use this new method to decipher this quote. IRPL WPDL FSNE INIH NIEE RBY OFY TNRY CEB EGC

MAS SBH ERS CEE AHS

#### Hint.

- Transpose each block, you will end up with a table of four lines and fifteen columns like Table 4.1.10.
- Can you find common or even not so common words or combinations in any lines?
- When you rearrange the columns do you see more words?
- Stay organized, if you don't keep things lined up this will never work

**Answer**. "If winter comes, can spring be far behind" By Percy Bysshe Shelly **Solution**. Begin by writing each block of text vertically and looking for scrambled words. From the hint we know to look for the word WINTER. Spotting the letters to spell WINTER in the top row we rearrange the columns and then start to look for other words.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 I W F I N R O T C E M S E C A R P S N I B F N E G A B R E H P D N I E Y Y R B C S H A E S L L E H E Y

You should notice that columns 3 and 4 can spell IF in the first row and if we put them at the beginning we also get the word SPRING in the second row.

2 1 5 8 10 6 3 4 7 9 11 12 13 14 15 W I N T E R F I O C M E S C A P R I N G B S N F E A B R E H ....

Reassuringly we see that this also gives us the name PERCY in the third row and SHELLY wrapping around from row 3 to 4. Finally, you may notice that swapping columns 9 and 7, and columns 12 and 13 will give us the word COMES in the first row. 4 3 2 1 5 8 10 6 7 9 11 12 13 14 15 I F W I N T E R O C M S E C A N S P R I N G B F E A B R E H I N D P E R C Y Y B S H S E S H E L L E Y

After make those last two swaps we can now read off an entire message.

4 3 2 1 5 8 10 6 9 7 11 13 12 14 15 I F W I N T E R C O M E S C A N S P R I N G B E F A R B E H I N D P E R C Y B Y S S H E S H E L L E Y

"If winter comes, can spring be far behind" By Percy Bysshe Shelly

**Checkpoint 4.1.13** Try to use this new method to decipher this quote. UIA ARM LDU GVN EWC RIE TSA HEF INV HRT STI TEC NMU EFO SRR UOH AE TTG TH HHO

#### Hint.

- Transpose each block, you will end up with a table of four lines and fifteen columns like Table 4.1.10.
- Can you find common or even not so common words or combinations in any lines?
- When you rearrange the columns do you see more words?
- Stay organized, if you don't keep things lined up this will never work

**Answer**. "Laughter is the sun that drives winter from the human face." Victor Hugo

**Checkpoint 4.1.14** Try to use this new method to decipher this quote. WYTWIA TAODLH IDHLES ASAHGD NETIAH ENHTNA OHDNTC SWNEHE OTEIDR HSNUNS FHWSWL TEISIE MHORIK OUDMTD SNBMEI ESLERC CECTHS AIWINE HSOHE RNSNTN

#### Hint.

- Transpose each block, you will end up with a table of four lines and fifteen columns like Table 4.1.10.
- Can you find common or even not so common words or combinations in any lines?
- When you rearrange the columns do you see more words?
- Stay organized, if you don't keep things lined up this will never work

**Answer**. "It was one of those March days, when the sun shines hot and the wind blows cold. When it is summer in the light and winter in the shade." Charles Dickens

Part of what helped you in the previous three examples was knowing that the word WINTER was in each quote, this is called a **crib**.

**Definition 4.1.15 Cryptologic Crib.** In cryptology a **crib** is a piece of information about a cipher text, such as a known piece of the plain text, which

helps you decrypt the cipher.

A few closing observations:

- Look carefully at how the blocks of characters (columns in the tables) are grouped after they are deciphered, i.e. did you ever get column one next to column thirteen or fourteen?
- Why do you think this happened? How might it be related to the number of key letters?
- Given your answers to the previous questions, how can we make use of Falconer's strategies for finding divisors and the number of key letters?

#### 4.1.3 Mathematical Interlude

Prime numbers and factors are significant in mathematics and play a central role not just here but in modern cryptographic systems used by computers. Therefore, a clear definition of primes and divisibility are crucial.

**Definition 4.1.16 Divisibility.** We say that one whole number a is **divisible** by another whole number b if there exists a unique whole number q called the quotient such that a = qb.

**Definition 4.1.17 Prime Number and Factor.** A number is **prime** if it is divisible only by one and its self. A **prime factor** of a number is a prime number which divides evenly, without remainder, into that number.  $\diamond$ 

## 4.2 A Seventeenth Century Idea

#### **Objectives**

• Falconer's Attack

## 4.2.1 Falconer's Attack on Polyalphabetics

"There is an invention of secrecy much insisted on (though none of the swiftest) by the author of the *Secret and Swift Messenger*, and others, beyond any yet mentioned, for intricacy, wherein each particular line, word, or letter, is written by a new alphabet: but the cited author himself acknowledges it too tedious for a current correspondence;" - John Falconer [4, p.17 (45)]

So begins John Falconer's discussion of using the polyalphabetic cipher. Falconer agrees with other cryptographers of his time that the cipher, while secure, is to slow and cumbersome and so not worth the trouble of using. However, for the sake of giving a complete account of the current state of the science of cryptography in his time, Falconer goes on to discuss in detail how to encipher and decipher messages with this type of cipher. What is interesting and more significant is that he makes a solid attempt at cracking this cipher, possibly the first real attack since its invention [9, p.155].

For his first example of how to attack a polyalphabetic cipher Falconer assumes each line is enciphered by a separate cipher alphabet. [4, pp.20-23 (48-51)]

I. Example in the Lines

Y pb vdgrts id ztte ixt Hdafytgh am forced to keep the Soldiers idcb wofr rihm obr rihm rxfh upon hard duty and hard diet: dfaawi fd ze espi gtww cpfzwe egupply us, or they will revolt to cqn Nwuxg bynnmrtg. qibc. the enemy speedily. Hast.

"1. When there is only one alphabet used for a line, the writing might be discovered an in plain cipher, if you make a new operation for each line. But there may be other ways to decypher any such writing: for,"

"2. If you find out but one letter in a line, (and that may certainly be done by a few suppositions) it will of itself give an alphabet for the whole line, as you may perceive by the *counter-table*, which follows: ... you need only to search for i in the upper line of it, and try in what line Y is opposite to it; and those two lines give you an alphabet."

"Having found one alphabet for the first line, you have likewise by this means the first letter of the key. e.g. In the fifteenth line of the table, Y standing against i, and P beginning that line (as you may perceive) P must be the first letter of the key;"

"... you may proceed to find the alphabet of the second, third, or any other line, as you did the first;"

For reference, here is a copy of the Vigenere table (what Falconer calls the *counter-table*) as Falconer was using it:

1 Ao pb cd ef qhik1 m nqrstuwxy2 a2 Bh i1 m n0 pqtuw $\boldsymbol{u}$ h cd f qkrsra 3 Cqh i k1 mn0 pqtuwrcd rp4 Dh 1 quwra kmn0 r+ U d q $\frac{10}{10}$ ry 5 Ei k1 mn0 pr† u2 h a s e 6 Fmnu $\frac{1}{2}$ xu7. k1 0 pa t a h s 7 G1 mn0 quw ru 7. ab ptqk cpb 8 Hmn0 qr+ u 11  $\overline{r}$ u7. ach k 1 d qh c9Imn0 puwr $\boldsymbol{u}$ 7. a d 1 rs t 10 Km0 pa rt и w xu zahcd a se k1 uwxyzab ce11 Lpqrstd f qh 1 tuwyzab 12 Mqxcd ef qh km13 Nt uwxyzabcd ef ghikmn1 14 Owzabceghi0 qt uxydf k1 m15 Pxb ceghimw ad k1 p16 Qxyab cdghik1 mzef q17 Rxyzab cd ef ghik1 mn0 18 Sxyzabcdef gh ik1 m n0 19 Tų zabcd egh ik1 m n0 g20 Vzah cdeh ik1 m n0 pf 21 Wab cd ef ghik l m n opqghik l m n o p22 Xyb cd ef qr23 Yzbcdefghi k l m n o p q rstuwxyc d e f g h i k l m n o p q r s24 Zabtu w

Figure 4.2.1 Falconer's Counter-Table

#### Comprehension Check:

- Looking at Falconer's cipher text, why would you assume that the Y represented *i*? What other letter might it represent?
- How did He decide that line 15, which starts with *P*, was the one that was used to encipher the first line of the cipher text?

- Verify that the fifteenth line, the one labeled *P*, is the correct one to use for the first line by deciphering the rest of the line.
- Looking at the next three lines of the cipher text try, without looking at the accompanying plain text, to decide which row of the *counter-table* was used to encipher them. What can you look at to try and help you decide?
- Finally, what is the keyword of phrase that was used to encipher this message?

In the next section of his text Falconer tackles the problem of deciphering a message in which we switch alphabet with each new word. It is here that he makes a significant observation that can help us find the length of the keyword.

"1. Having found an alphabet for the first, second, or indeed any word near the beginning of the epistle, go through all the immediate following words, until you find another that is decyphered by the same alphabet."

"2. From the last found word count the like number, and you have a new word decypherable by the found-alphabet: and thus you may go on until you have once gone through the whole writing, marking the whole series with some particular mark: And then,"

"3. Begin the epistle again at some word immediately before or after that which was first found, and count forwards as before, until you come to the end of the epistle. [Repeat this process until each word is marked as part of a series.]" - John Falconer [4, pp.24-25 (52-53)]

*Comprehension Check:* Refer to this cipher text when answering the comprehension check questions.

	1	2	3	4	5	6	7	8
1 -	МΧ	EIB	WKG	LOCD	SK	GWZRF,		
2 -	BM	ZEW	CQN	YQTUW	YP	YNQIX,		
3 -	WG	0SL	XLH	IPN	QH	FSCNYW,		
4 -	NY	INF	MAX	EKH	ХО	HQQNLUKPGUU,		
5 -	SD	AEX	GUR	XHGUA	RI	KNTRNO,		
6 -	LW	FKC	YMI	RCBPU	GY	MQFUHGYOMXB,		
7 -	RC	YCU	DRO	XIEXSR	BS	DBZAM,		
8 -	МΧ	EIB	WKG	COKCYX	SK	QNEXARFF,		
9 -	BM	ZEW	CQN	URTLPI	YP	MSTI,		
10 -	WG	0SL	XLH	ERWCNA	QH	NOCZKSB,		
11 -	AI	UNQ	XNXKQMABFY	FHIRUH	DB,			
12 -	YG	TKN	RSYMNRL	ORSBER	NL,			
13 -	ZH	ENAN	CNN	QYSXQ	HNWIGY	GB	AXNXF,	
14 -	ΖH	ENAN	CNN	QYSXQ	HNNIGY	GUR	GMAXK	ZDB

- Start by trying to find some repeated pieces of text. Falconer seems to imply that these are enciphered using the same alphabet from his counter-table; thinking about how we use the Vigenère cipher normally why could we end up getting theses repetitions?
- How far apart are the repeated strings? What factors do they have in common?
- Follow Falconer's advice and break the message into groups of strings which were enciphered (hopefully) with the same shift.

- Now use what we learned about frequency analysis to figure out the words in each group. Keep in mind that each shift corresponds to a row in Falconer's counter-table and that the row markers should spell out a keyword.
- What does our message say? What was the keyword?
- How did finding the length of the keyword help reduce this problem to one we had already solved?

**Reflection:.** Exactly one hundred years after Vigenère published his work Falconer lead us in the right direction to crack the polyalphabetic substitution cipher. First, he pointed out that whether we encipher each new line, word, or letter by a different alphabet if we can group elements of the message which were enciphered the same way together then we can treat each group as a monoalphabetic substitution cipher. Additionally, if we can identify the length of the key then we can use that to figure out how to group the strings or characters together. However, Falconer was still very much tied down to the use of word spacings, without them his descriptions and attacks would not work. It would be another century and a half before Charles Babbage and Friedrich Kasiski would recognize how to expand these ideas into a general plan of attack.

## 4.3 Nineteenth Century Revelations

#### Objectives

• Babbage/Kasiski's Attack

#### 4.3.1 Kasiski's Attack

#### 4.3.1.1 Initial Observations

What follows are excerpts from Friedrich Kasiski's 1863 text on cryptography *Die Geheimschriften und die Dechiffrir-Kunst*, which was translated into English and privately published by R. W. Pettengill in 1954 as *Secret Writings and the Art of Deciphering*[10]. For brevity we will only examine a few of Kasiski's key points and we will use a long English rather than a short German quote in order to make his points. While these adjustments have been made to aid in understanding, whenever possible we will remain faithful to the original.

For this section we will use the following example quote for our analysis. It is enciphered using a Vigenère cipher with the keyword BERLIN. You may find it useful to redo the enciphering for yourself.

#### Example 4.3.1 Sample Vigenère Enciphering.
...

Plain Text: 1	Cipher Text:			
	123456	123456	123456	123456
	Key: BERLIN	BERLIN	BERLIN	BERLIN
	(1) UICWBU	FKVYME	BPIPUN	SULPBU
Tell the general, Remarque,	(2) BXKSMN	SQPSIF	NSSTTV	AIUEWN
that the army has mobilized	(3) UXRNSG	IIIFAF	JEEDWA	ULVPIF
to attack the Russians on the	(4) UIIYNE	PRKLVQ	ULVQZR	OGYEPR
eastern front, and the French,	(5) FRXWQF	IEEOBU	FEDPZV	DEEDWA
the English, and the Americans	(6) ULVHMF	UIIYNE	PRKHPV	DLZDKH
on the western front which	(7) SVVYBY	ZECWYH	JIK	
is currently all quiet.				

VI. The Actual Art of Decipherment [Cryptanalysis]. A. General Remarks.

72. The actual art of deciphering consists in deciphering any unknown cipher text without previous knowledge of the meaning of the ciphers and without the key.

It is impossible to discover the key from a few words of cipher text ..... Consequently we must have available at least four to six rows of cipher text if we are to be able to recover the key.

75. The influence of the key finds expression in the fact that at intervals of as many ciphers as the key contains letters repetition of single ciphers or of several ciphers can occur, which can only be due to the fact that, when the same letters of the key were united with a repetition of the plain text to form ciphers. If we have a relatively long cipher text which is enciphered with the key "Berlin," and if we divide it off into rows of six ciphers and number the ciphers of each row from one to six, we shall find that among the ciphers numbered 1 of each row the cipher U occurs most frequently, among the ciphers numbered 2 I is most frequent, the most frequent among the ciphers numbered 3 is V, among the ciphers numbered 4 Y, among the ciphers numbered 5 B, and among the ciphers numbered 6 F. This is because the ciphers U, I, V, Y, B and F have [likely] arisen by combination of the letters b, e, r, l, i, n of the key with the letter e of the text; since according to previous observations the letter e occurs most frequently in the text, naturally the above ciphers must occur most frequently under the numbers mentioned.

<sup>&</sup>lt;sup>1</sup>Kasiski's original quote was "Die Armee wird mobil gemacht" also enciphered with BERLIN.

Next most frequent in this cipher text are the ciphers S in column one, E in column two, K in column three, P in column four, W in column five and V in column six because these ciphers S, E, K, P, W and V result from a combination of the letters b, e, r, l, i, n of the key with the letter t of the text and t is the next most common letter letter after e.  $[10, pp.29-30 \ (34-35)]$ 

## Comprehension Check:

- In section (75) Kasiski writes about letters of the *key* uniting with repetitions of *plain text*, how does this compare to Falconer's observations in Section 4.2?
- We know that the Vigenère cipher can encipher many different plain text letters as the same cipher text letter, so why is it reasonable for Kasiski to believe that all the U's in column 1 should represent e or that all the P's in column 5 might represent t? (Answer in terms of how we know the Vigenère cipher works.)
- Break the plain text message in Example 4.3.1 into blocks of six then use the key BERLIN and the modern Vigenère table Figure C.0.8 to encipher it, you should get the same cipher text as that which is in the figure.
- Is Kasiski right about his guesses, that is do the U and I in the first and second columns represent e, or do the E and K in the second and third columns represent t? How do your answers to this question relate to Kasiski's comments in section (72)?

Further observations by Kasiski:

76. ... where repetitions of two or more ciphers are found in the cipher text one can conclude that these are due to the fact that, in enciphering, repetitions in the plain text were repeatedly combined with the same letters or the key.

If we encipher with the key "BERLIN" as above, we shall frequently find in the cipher text in columns 1 and 2 the ciphers UL, in 2 and 3 XY, in 3 and 4 KS, in 4 and 5 EP, and in 5 and 6 BU because these ciphers UL, XY, KS, EP and BU have arisen by combining the letters BE, ER, RL, LI and IN in the key with the [repetitions of] th in the text. [10, pp.30-31 (35-36)]

**Comprehension Check:** (For these it will be helpful for you to look at the work you did in the above questions.)

- Look carefully at the cipher text in Example 4.3.1, how often do you find the bi-grams UL, XY, KS, EP and BU? Do they always line up with the cipher key so that they give th when you decipher them?
- For this observation Kasiski enciphered th using each of the pairs of letters BE, ER, RL, LI and IN from BERLIN, but he didn't consider enciphering it with NB (i.e. wrapping around from column 6 back to 1). Encipher the bi-gram th using the rows NB in the Vigenère table Figure C.0.8, what cipher bi-gram do you get? How often to you find that bi-gram in the cipher text in Example 4.3.1? (Remember that you need to look from columns 6 to 1.)

• Can you find other repeated bi-grams? Can you find longer repeated n-grams?

## 4.3.1.2 Determining the Key

Kasiski's observations above, and Falconer's previously, give us a way to attack Vigenere's cipher if we know the key length. But first we need to find the key length.

78. ... In a simple cipher text repetitions are more frequent and are merely accidental, that is to say, rarely occur at regular intervals, whereas in the composite cipher text [i.e. Vigenère cipher] these repetitions occur less often but usually at intervals such that the number of letters between them is divisible by a number which gives the number of letters in the key.

80. ... We now try to determine the number or letters in the key. For this purpose we look for all repetitions of two or more ciphers in the enciphered text, then count the interval from one another of like repetitions; write these out beneath the enciphered text with numbers indicating the intervals, and endeavor to break this number up into its factors.

For instance, we find 16 letters between two repeated ciphers UL, we add two ciphers to the number 16 and get 18 as the interval for the repetition UL.

We write  $UL = 18 = 2 \cdot 9 = 3 \cdot 6$ .

According to section (76) we can infer from this that the key contains either two, nine, three, or six letters.

Since the number of letters in the key is still uncertain and ... the repetition of the ciphers UL might be purely accidental, we write out one by one all the repetitions of several ciphers, noting the number representing their intervals, and break these numbers down into their factors.

Those factors which are found most frequent indicate the number or letters in the key. [10, pp.31-32 (36-37)]

## Comprehension Check:

- Why did Kasiski add two to the number of characters between the two repeated copies of UL?
- The three letter cipher ULV also appears more than once, after counting the number of characters from one to the next how many letters should you add to that count before factoring it? What about for the repeated cipher UIIY?
- Go through the entire cipher text in Example 4.3.1, look for all the repeated ciphers which are at least two characters long, then for each one write down the distance between them and the factors of those distances. What number(s) pop up most often? Why does this make sense?

Finally we find the key: In what follows Kasiski makes use of what he calls *the key table* which is a rearranged Vigenère table with plain text letters on the left, cipher letters along the top and the possible key letters on the interior. So it tells us that if we have a cipher letter J which we believe represents a plain

text letter i then looking in the table we can see the corresponding key letter was a B.

1																										
	A	B	C	D	E	F	${G}$	H	I	J	K	L	${\cal M}$	N	O	P	${\cal Q}$	R	S	T	U	V	W	X	$\boldsymbol{Y}$	Z
12.22%~e	W	X	Y	Z	A	B	C	D	E	F	G	H	Ι	J	K	L	M	N	O	P	Q	R	S	T	U	V
9.28% t	H	Ι	J	K	L	${\cal M}$	N	O	P	Q	R	S	T	U	V	W	X	$\boldsymbol{Y}$	Z	A	B	C	D	E	${F}$	G
8.06% a	A	B	C	D	E	F	G	H	Ι	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
7.62% o	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	${G}$	H	Ι	J	K	L
7.10% i	S	T	U	V	W	X	Y	Z	A	B	C	D	E	${F}$	G	H	I	J	K	L	M	N	O	P	Q	R
6.82% n	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	${F}$	${G}$	${\cal H}$	I	J	K	L	M
6.45% s	Ι	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	${F}$	${G}$	H
5.91% r	J	K	L	${\cal M}$	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	Ι
5.76% h	T	U	V	W	X	Y	Z	A	B	C	D	E	${F}$	G	H	Ι	J	K	L	${\cal M}$	N	O	P	Q	R	S
4.19% l	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	${F}$	${G}$	${\cal H}$	I	J	K	L	${\cal M}$	N	0
3.93%  d	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W

Figure 4.3.2 Kasiski Key Table

• Check that you understand the *key table* by trying to find the key for this message based on the given plain text and cipher text.

plain: winter is at a close CIPHER: OXEBRX AH RB N IDDJM

- Why do you think that Kasiski ordered the plain text characters the way he did?
- Why do you think that he didn't include all the possible plain text letters?

Finding the key:

85. ... If we have determined according to section 80, the number of letters in the key, we divide the cipher text into rows containing as many ciphers as the key contains letters and number the several columns. In theory those ciphers which occur most frequently in a given column should signify the letter e in the text;

86. After we have divided the cipher text into rows according to the number of letters in the key and have numbered the several columns, we take out all ciphers from column 1 and note the number of occurrences, then we do the same for columns, 2, 3, etc. [See Example 4.3.1]

If we have found in this way the cipher U, let us say, occurs seven times, cipher S three times and cipher F three times in column 1, we copy from the key table letters beneath the ciphers U, S and F on a separate sheet of paper as follows:

7 U - Q B U G M H C D N J 3 S - O Z S E K F A B L H 3 F - B M F R X S N O Y U

Among these letters must be found the first letter of the key. The letter B occurs as second letter under cipher U, as eighth under cipher S, and as first under cipher F. Hence B is combined seven times with t to form U, three times with r to form S, and three times with e to form F. From this we must conclude that B is the first letter of the key. The letter M also has a more remote probability of being the first letter of the key since it occurs as fifth under U, and as second under F ... In

the same way we just found the first letter of the key, we now look for the other letters. [10, pp.34-35 (39-40)]

### Comprehension Check:

• In the columns labeled 2 in the example we have been working with three most common ciphers are I, E, and L appearing 6, 5, and 4 times respectively. From this Kasiski would have us write down the following from the columns of the key table:

6 I - E P I U A V Q R B X F 5 E - A L E Q W R M N X T B 4 L - H S L X D Y T U E A I

Which two possible key letters appear in all three rows?

• In the columns labeled 3 in the example we have been working with three most common ciphers are V, K, and I appearing 5, 4, and 4 times respectively. Follow Kasiski's lead and write down the corresponding columns from the key table:

Which possible key letters appear in all three rows?

- In the columns labeled 4 in the example we have been working with three most common ciphers are Y, P, and W appearing 4, 4, and 3 times respectively. Repeat the steps from the previous two questions to try and find the fourth key letter.
- In the columns labeled 5 in the example we have been working with three most common ciphers are B, W, and M appearing 4, 3, and 3 times respectively. Repeat the steps from the previous two questions to try and find the fourth key letter.
- In the columns labeled 6 in the example we have been working with three most common ciphers are F, V, and U appearing 5, 3, and 3 times respectively. Repeat the steps from the previous two questions to try and find the fourth key letter.
- Put together all the possible key letters we get for all the columns of the message, do they (or at least can they) spell "BERLIN" as we expect them to?

Try to practice what we have been looking at with these exercises. You can also look at the sample Vigenere analysis in the appendix (Section E.3).

**Checkpoint 4.3.3** The following text was enciphered using a key of CIPHER. Follow Kasiski's steps to see if you could have found this for yourself.

	(1)	UQCJIR	TIBPWJ	UQCNYC	CZIYEE	UNDYQR	VQDUME	VWPJSE
	(2)	HMHZSI	QNIOIF	TLTYFR	KATTIR	WFLHWE	QTDUKV	TBWLWR
	(3)	OMBHRL	RBDALR	VXTYMF	FBWLTC	CKTDLZ	EPPYED	KAWHHY
	(4)	GTSPRK	JMLVVK	JGVVZV	TVDYWV	UBXTEK	KWCDEJ	VPPASW
	(5)	CXGLPR	VMLOSD	JMGLWG	GKILHR	PLPMVZ	GVSASN	JWBOIF
	(6)	YMSHHV	DBDMKI	CBXAYU	GJJARF	YPTMIC	VPXTWV	NNPUME
	(7)	HMGPSI	CVSALR	VIGHQZ	UEPZLZ	UUPZXV	TPTOMD	UMAMPZ
	(8)	IPILHR	NICAII	PAJTQF	PMSHXL	TVZLCR	PLHHMU	TMIBVE
	(9)	KVVASR	TIBPWZ	CUPACF	WZDYHV	TABVRJ	GQVUIL	TIGHQZ
(	10)	UUTYIC	AVDKHV	FPXZLV	CLPZQL	EPPZXF	UINCII	AODVHR
(	11)	PLHPKE	GLIVLZ	OEXALY	KAWHRU	VWALEU	VPTDEP	

## Hint.

- 1. Look for repeated strings.
- 2. For each pair of repeated strings write them down and next to them write down the number of characters from the start of one to the start of the next and the factors of that number.
- 3. Assuming that the most common factor is 6, which we know it should be, use the subset cell to find the most common letters in each column.
- 4. Use these letters and the key table (Figure 4.3.2) to try and find the key, which should be CIPHER.

#### Answer.

"Since Aramis's singular transformation into a confessor of the order, Baisemeaux was no longer the same man. Up to that period, the place which Aramis had held in the worthy governor's estimation was that of a prelate whom he respected and a friend to whom he owed a debt of gratitude; but now he felt himself an inferior, and that Aramis was his master. He himself lighted a lantern, summoned a turnkey, and said, returning to Aramis, "I am at your orders, monseigneur." Aramis merely nodded his head, as much as to say, "Very good"; and signed to him with his hand to lead the way." The Man in the Iron Mask, Alexandre Dumas

**Checkpoint 4.3.4** Follow Kasiski's steps to find the key for this message. Then use the Vigenère cipher cell to decipher it.

(1)	IERSDO	EDPRHZ	AMWXBU	OEFAYX	IE0P00	EWWUOG	UNYRQJ
(2)	OPFIGX	BFHUQP	SMMFTG	TEZYZJ	SYUKWK	BIIROE	FDILFT
(3)	MVWEXY	WFFHGE	OYHFFO	FYBYWA	STZKBZ	GGASSI	BFSIQR
(4)	QDPZBT	TESCSV	RUUMKO	TWKUKT	ZOSZBT	TAVCME	HSARZS
(5)	USOPWW	ZIBYSV	DOFWIJ	YUMDLR	ZFKAFH	WCQCJR	DPDKBV
(6)	OGFHWP	MVHBRR	SYCCOF	FOLRMW	WEETKE	JASPFT	ZOZVDY
(7)	KWSCVV	UNFINO	PVYAQW	JSOYHJ	QLDDPR	HGTECS	VXGPML
(8)	DSVXVV	YWSCNI	CZZEUO	AJWGKH	WCBZZY	RUJDPV	FXZEOD
(9)	PRHYAU	ACFZJZ	GSLOFG	SEUEFM	MRBVYP	WBQFIF	PEKSZV
(10)	TBDAHB	QMOGQC	GXDVFF	MTAYVN	WGTOFO	EYCZFH	WZWJGR
(11)	ESAYVF	TFGCZK	AVQEQT	HVITSQ	ANSVMM	SYIILR	BYSUUG
(12)	ZOAKDB	IEJCWW	HUQKAX	OUCZNU	LKAKCF	MYAXOV	LNOTDI
(13)	EYOGFH	WUQEUF	IIKRER	GQMRLK	OEOARO	MXLYWZ	EEDPKF
(14)	ACXELO	TPOGML	GCAKVR	YUKUMK	SRDHSN	VFRBGB	LCMZHU
(15)	QRMZWE	HUQRWK	AFBJTI	URPRRH	DGWNBY	SHZFGB	BLBNFE
(16)	HRQCWC	BELYZV	JRMLZS	ATVNDA	UDMIOA	PBABBY	DUULAZ
(17)	XVPHDI	WNNFFR	HEJLME	SNFHSW	IJYBRS	LOMCSK	ULWNBF
(18)	OPAUFD	ZPKUQR	WDPVAR	ZSWOUV	RYUTLV	MDCEQT	ZKVJZN
(19)	HEKYNK	VRQLWW	MEHFBH	AVQGDR	PEHBQM	SQQVWX	WWHUQS
(20)	GMQVHL	AFVKZK	OTZAFG	PFVNPL	GKLVRU	UMOSBY	VBZOJC

## Hint.

- 1. Look for repeated strings.
- 2. For each pair of repeated strings write them down and next to them write down the number of characters from the start of one to the start of the next and the factors of that number.
- 3. Use the subset cell to find the most common letters in each "column" as if you had split up the cipher according to the length you found in the previous step.
- 4. Use these letters and the key table (Figure 4.3.2) to try and find the key.

## Answer. The keyword is "ironmask"

The message is:

"And from that moment, D'Artagnan, accommodating his action to the pace of the horse, like a true centaur, gave up his thoughts to nothing - that is to say, to everything. He asked himself why the king had sent for him back; why the Iron Mask had thrown the silver plate at the feet of Raoul. As to the first subject, the reply was negative; he knew right well that the king's calling him was from necessity. He still further knew that Louis XIV. must experience an imperious desire for a private conversation with one whom the possession of such a secret placed on a level with the highest powers of the kingdom. But as to saying exactly what the king's wish was, D'Artagnan found himself completely at a loss. The musketeer had no doubts, either, upon the reason which had urged the unfortunate Philippe to reveal his character and birth. Philippe, buried forever beneath a mask of steel, exiled to a country where the men seemed little more than slaves of the elements; Philippe, deprived even of the society of D'Artagnan, who had loaded him with honors ..." The Man in the Iron Mask, Alexandre Dumas



YouTube: https://www.youtube.com/watch?v=TxClRjnRNJw

Figure 4.3.5 Modern look at Kasiski

Sage Cell for counting instances of specific strings:

```
import re
@interact
def string_count_simple(
                    text=input_box('Place the text that you
                        would like to analyze here !!! ',
                    label="Enter your message", height=3,
                        width=50, type=str),
                    key=input_box('sage', label="String to
                        search", height=1, width=20,
                        type=str)):
    message = re.sub('[^A-Z]','
       ',str(text.encode('ascii', 'replace').decode()).upper())+"
    message = re.sub('\\s+', ' ', message)
    print("This is what your cleaned up message looks
       like:\n\n",message[0:min(60,len(message))],"...\n\nis
       this what you intended?\n")
    count = 0
    K = re.sub('[^A-Z]', '
        ',str(key.encode('ascii', 'replace').decode()).upper())
    N = len(K)
    c=""
    for i in range(len(message)-N+1):
        c = message[i:i+N]
        if c == K:
            count += 1
            print("Instance", str(count), "\t", message[max(i-2,0):min(i+N+2,len(message)))
    print("\nYour string appears ",str(count)," times.")
```

Sage cell for calculating subsets of letters in a message:

```
tmp_message = sets[i]
count = \{\}
for i in range(len(tmp_message)):
    c = tmp_message[i:i+1]
    if c in count: count[c] += 1
    else: count[c] = 1
print("The length of this subset is
   {}".format(len(tmp_message)))
count = sorted([(value, key) for key, value in
   count.items()])
print("Character\tCount\t\tPercent")
count.reverse()
IC = 0
for c in count[0:min(30,len(count))]:
    if len(c[1])>6:
        tab="\t"
    else:
        tab="\t\t"
    print(c[1],tab,c[0],"\t\t %.4G" %
        (c[0]/len(tmp_message)*100.0))
    IC += c[0]*(c[0]-1.0)/len(tmp_message)^2
print("And, the index of coincidence for this set is
   %.4G\n" % (IC))
```

Sage cell for enciphering and deciphering the Vigenère Cipher:

```
import re
import textwrap
@interact
def _(m=input_box('sage', label="Enter your message",
    height=3, width=50, type=str),
        key=input_box('sage', label="Enter your key",
            height=1, width=20, type=str),
        mode = selector(['encipher', 'decipher'],
           buttons=True),
        spaces = selector(['yes', 'no'], buttons=True)):
    plain_alpha = "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
    clean_message =
        str(m.encode('ascii', 'replace').decode()).upper()
    if spaces == 'no':
        clean_message = re.sub('[^A-Z]', '', clean_message)
    cipher_key =
        re.sub('[^A-Z]','',str(key.encode('ascii','replace').decode()).upper())
    key_list = [plain_alpha.index(ch) for ch in cipher_key]
    if mode == 'decipher':
        key_list = [-1*k for k in key_list]
    cipher_text = ""
    key_counter = 0
    for ch in clean_message:
        try:
            tmp_pos = plain_alpha.index(ch)
            cipher_pos = (tmp_pos+key_list[key_counter])%26
            cipher_text += plain_alpha[cipher_pos]
            key_counter = (key_counter+1)%len(key_list)
        except:
            if spaces == 'yes':
                cipher_text += ch
```

```
print("\nHere is your output:\n")
if spaces == 'yes':
    print(textwrap.fill(cipher_text, 42))
else:
    for i in range(0,len(cipher_text),6):
        print(cipher_text[i:i+6],)
        if (i+6)%42 == 0: print("\n")
```

## 4.3.2 (\*)Babbage's Contributions

Insert Information on Babbage's Cryptologic Work ...

## 4.4 Do You Feel Logical?

1. Encipher this message by hand using Falconer's transposition cipher and a key of {ACB, BCA, CBA, BAC}

"A quick movement of the enemy will jeopardize six gunboats."

#### Answer.

ANWZS UOLS QTIE KHEG IFLI CTJX VNAB OEPN MEOU MMDA EERO EYIT

**Solution**. Set up a cipher table with the rows labled by the set of keys and the columns by the letters A, B, and C. Then place the characters from the message into the table one at a time following the key for each row.

Cipher Table:

		А	В	С
ACB	:	ANWZS	UOLS	QTIE
BCA	:	KHEG	IFLI	CTJX
CBA	:	VNAB	OEPN	MEOU
BAC	:	MMDA	EERO	EYIT

Completed Message:

ANWZS UOLS QTIE KHEG IFLI CTJX VNAB OEPN MEOU MMDA EERO EYIT

2. Decrypt this transposition cipher with crib WEATHER: HIPYEN TAUTCI WSDWPF ETTEEA ARONRN ENPMNC RTEPTH RERHR EMTCA PPHHI OREOL FAIEL TETCA RNRNF OSIAN MYWRR RTNIU CEDNN ASIAO RLSFA IITHU TDENA HASOD HYAEQ RNTUA FETRE TTENR RITDO YHNDT IMOER

Answer.

weather report for march thirty first rain temps nearly steady in the mid to upper thirties winds e at ten to twenty mph chance of rain one hundred percent rain fall around a quarter of an inch

3. Decrypt this transposition cipher with crib AFGHANISTAN: HATSNNI OLHHHIV WLAOINE AYSUMA OPNADA RGTLCF ERRDRG YIEHEH ENHLYS SGFYOT UPGRIN HITDTI IYRAHN DHWVAI CAHEVP AMOHUA IIIEEE DWHVEC RDCIBR ONIGEE

## Answer.

how are you he said cordially gripping my hand with a strength for which I should hardly have given him credit you have been in afghanistan i perceive

4. Decrypt this transposition cipher: IQAY ASHE PECV FIIR NUJA UCND LKX OYD ZLE ITA MWG BAR

#### Answer.

painful zombies quickly watch a jinxed graveyard

5. Encipher this message by hand with a Vigenère and key of DUNGEON "The wizard quickly jinxed the gnomes before they vaporized."

#### Answer.

## WBRCMN NUXDAM QXOSWO RLRGNU KKBBPY FHITBU YGNIMI DJBXMN RG

**6**. Decrypt this message enciphered with a Vigenère cipher by using the techniques you learned from Falconer and Kasiski. HS OOE BGHE RCI LG JO SOF KCFCXRSF, KYR KXDDGMESF DSS TYZAU KE BQ. DHC-VGPBVI ZBG L, MOMOC GVBPSV, YQ KJSNV JO SOF CACMOY OV YFF OOPHKXR. HJOJ QQXDWUDPR QP L QQEAZG YQ OOWOCTDLPNO MSF-BZCOC LBF K DWPOWS NKCUG KTFA CTHVSYU-TYZA, ERPSTPFZNI QITXTGJOO, OPN TZNEXWPKESF LJ HYY MFOKO KKXOCYC. DC FODWTKMZG SY SXOCM YKJ KGBP HJO LDCBEAGXEG, CXO GQ WZRGBLHG NTR VRP HGBXG UOPA YRPB FSGWFOO PGDHSGX FG, VRLH VRP PCBROKX HOU MZBEVFRGN FDOX EVG CACV, KYR YO LH OXNS GXESTOO WPDZ DOCDSUCTCP. DSOV FPFA OGSPSYU K WZJGN XM VRTBIC CCWXO TTYX HJO SCVOW, OPN ZB VRP TQVWCYSYU OYCBKXR GJOCZQMV VQVXSU PZZNYHSF WP KKDS GGFPFCV MCZOD OPN ACTDXOPDPOWC. QCT K OOA YC HYY HS YOCS DEDWNI PARVZMGN TB WXAOEUTBI KYR NKJWPQ ZIV YFF RBZDGBEM VY EVG LPGV KOJCXEOIO. EVCD OCPO, HS IBLRWKWZA LPUCX EC UOEHNO OCYX LBF DZ OEMZAOYOOVO ZITCPZXOD HQ YFF POH GWBCCWXOWPQD.

#### Answer. Keyword "lock".

we met next day as he had arranged and inspected the rooms at no b baker street of which he had spoken at our meeting they consisted of a couple of comfortable bedrooms and a single large airy sitting room cheerfully furnished and illuminated by two broad windows so desirable in every way were the apartments and so moderate did the terms seem when divided between us that the bargain was concluded upon the spot and we at once entered into possession that very evening i moved my things round from the hotel and on the following morning sherlock holmes followed me with several boxes and portmanteaus for a day or two we were busily employed in unpacking and laying out our property to the best advantage that done we gradually began to settle down and to accommodate ourselves to our new surroundings

Decrypt this message enciphered with a Vigenère cipher by using the 7. techniques you learned from Falconer and Kasiski. NUTZSK BLYBAU JWUONX BWZFXG OCTUCS BWEXCZ IUSSQO MFOBGR FAXBHJ IYCBMU GUTBHI JYTUBA HOKOIZ GUSJFE BHJIUJ PHIFVK FHCFUR UBECOZ BMKSCK TILNCY GIXUOT FMNBXX FXADYJ ICSUIC BHZUIG WIOENN FGUSNO GCIBNO PHIPHY FKAFHZ VJUOBO TXOTUY UYXTBK MYLUHK XIXMYG OMZIYI JNEPZN JMLPLK GUZIYX TUTENU PEAQBO TLKTCJ FHIFUZ TORMCB BHYJMR BHJOYG SWNBLR FMZPHY POZIWG SIRJHG UBOTCY MUTECY BPKSSY JHMVFG SITFCZ DITTCY UMUGFO UNRFYR TYZIUT UBKTYG TUTEUT ECYBVU VNZILK FGOMYY MITHCZ TVXFUJ UBGUHU QIOONK YWKFXY BKABLZ FLUGUS JFKJNO TMKQUX BNKEZX PGZIYS BCTMUT EVEBMI BLIFFE QYXDYV UCHMYI SYKLIU ACTHCZ TQGZNN SIAHBG XCREYX OYYTIL SYKEMG OXYMCS FULBPU SCZFLK TIXUIL UBKNUX TBNFHZ IYBFAK UUZJIT BMSJAN UVKTOV QIYFXO TMIBHZ PLGUFK BMZEQG SZOTBT PNXFYY PZGOSS BATJNA EYGSYZ PVKTYK OHKBLZ IYCFMZ FLTFRZ SYSJNE XBKSYL PLZNIA MNXJYY UUTEMG OXCIYX FUXFMU NYSJMK SUHMYL SUSFVA JFJJHM TNKOUT UYJEOX JHMTOS NYXCSZ IYLVAO UCBFML SISDBG SFKTNU OXATNG OXLFPK SGGZVK GIAOXO OXKFXZ IYHSCY UFEQUR NYZUIH VNZIYC IIRFCY MUTEQO UBZIYK YWKQNO PHUGNN JMCFMZ FLTQIO ONGOXG MCTFIL IUXEQN JNKCYG DBUONN FMKBWU BMZJMI PPKSYJ XCZIUJ FHYFOT EYXHLU XNNPZZ IYYXYK UGESNR FMUNOI IJXJTK EVEUBK IIXUCI VFZVLO TNYPZK OARBHJ UBKTBX VVNFLK PZZFHG UNGJHY UBKIYO HBZPZL JZZFYT PLZXYT USLFYZ BHJGIX NMGOUR NIYUCS QYTFNX BVRFWU QJODYH VLZIYT JHMUBK BCXXCZ ICZTZX BAXBHI F

Answer. Keyword "bug".

many years ago i contracted an intimacy with a mr william legrand he was of an ancient huguenot family and had once been wealthy but a series of misfortunes had reduced him to want to avoid the mortification consequent upon his disasters he left new orleans the city of his forefathers and took up his residence at sullivans island near charleston south carolina this island is a very singular one it consists of little else than the sea sand and is about three miles long its breadth at no point exceeds a quarter of a mile it is separated from the main land by a scarcely perceptible creek oozing its way through a wilderness of reeds and slime a favorite resort of the marsh hen the vegetation as might be supposed is scant or at least dwarfish no trees of any magnitude are to be seen near the western extremity where fort moultrie stands and where are some miserable frame buildings tenanted during summer by the fugitives from charleston dust and fever may be found indeed the bristly palmetto but the whole island with the exception of this western point and a line of hard white beach on the seacoast is covered with a dense undergrowth of the sweet myrtle so much prized by the horticulturists of england the shrub here often attains the height of fifteen or twenty feet and forms an almost impenetrable coppice burthening the air with its fragrance

8. Decrypt this message enciphered with a Vigenère cipher by using the techniques you learned from Falconer and Kasiski. REZIAK VVISWB KSHYAL VISJMM CLTSHM QCKVHO OIDZBE AROHPF ROHOOO CMOEDP TTMKHO GUFVZT TPVVZZ RTPLZX CEODVM MXTUDV LTJNYT BYEOTT TZVEYI ZVWWXC EETKII OWDXBZ RBWAJT UFZTTT NZVNTE IMGXJT TTIKXV IZBMEX JFYNEZ WCEEGM XEMDUC ODEMJA GGZXRO GALYEQ ENTMEW ORMCOV XCEDTN FVZIRX PRHIOF UQIWOA FIMDTO EPIPVQ ZTTDLN LDCTHP VLVDTT ZJIGFE JOXINT QSBYIM EPJKZR BTTTJR GJNUPV TMKHQG BFMOSX DEVWOE XTUVRO SRDZDE IYTDCI WDLMQW LVZDMI BYMNWA GSRRYF UCICPT WTTVZL VDDTLL GZDFWM SEXOZX IEJDVQ HGDFJL EIWKLM EQXNVP OTTPBZ LVDMRK FQKLUH PVHVLX IPRXRA EEWJWD BXTQEX CAFLIP ACEZXP RHVRDX DVHVTF WQJVZS GABRRY HMSBVW OEPXBJ EXCGGI TCDNID ZBMIGU UMCXDN MEWJMO IACBFI SPQGQD IITIXB YQTNQL SESRLQ SOVSIT TTWCHI UYQMIG DPTTZW MMSFXE ISOEAJ BDCHEF WWUSAR OSCTXD OZPARW JRFDNR HYEZSC DXJTTT XRTZRI WQTLDH MSXIIK ADTLWS MMMGRF VTTTTV ZQVDQP SVCOOO XXYIMA ZSWEIO OPTKZT CEDOGK LDSFXU VXCEZX OYXRAE LMCPJN MCLKLZ GDTGFJ ZADAGD SMNUCO NENBQV QERDNS IWJXZA XXVSCO HQTLXI NORIPV FGIZSA ZAVSZD BJPZEB NPFAZV QGQNEN TADULG CEJROK IYTAIP ZRFORH TVIKFA GBYINO XKOEKJ FFWMGV JBXTUJ IZMQSI CQJSFL QKLDNY NOIENP QMKZXZ DFDIJX VTQLPZ GCAXBW JXARUV PKIIEP BMSCDT EXVKII SUIGZK JTDTIU CHYODX PSATTT VLQWED RQGLZR MCLDCI EIAGGV ZPMGMU OZYIXB YEIERU WIXRHU RPKSJK YTICPH YDTAFP PTUDVZ AZNFDV JXZAPX TPAMIF XVXMOS BGWGIM LQIBVV PNPTZV EXHODU SMIAFX WEADTT DCKSIC QAWFOD NSQITO AODXSE IRTTPB VZZNEW WLPYSA BMFJOH QAMKXZ REQMDM NPXPKV HDNFWM BITTTT KYEICQ DNIIXO SCQJMI GFWMIM BHFDVV WROGAL SIGADV MCCDNO GMRWZD NNAVID NSPKFR NIPTZR FGEZJU SIMORA MKXZRE IWXIOH QGBYII ISAIEG ZDAKMI XCEIWW CIVNPU WLRYTT PBDEIY AUBYIN YYOWCW HAPTCG PZTFTZ JADTTH CTLVBM HQJXJW AGSFRO HQGMJX RAEDVC CGANDC IEAEII MEXVTU KMVJAO DIARRY ITPLTS MRQRBV HOHQZM PXJASG MVQZNF LQKLNO YTWWXC EODUSM IAFXWE WDNFWM TMKHOG

#### Answer. Keyword "vampire".

i went straight to my own room and commenced to work afresh on the biliteral cipher more than ever had i the conviction upon me that the reading of the secret writing would be the first step to the attainment of my wishes regarding marjory it would have been strange therefore if i had not first attempted the method which she had herself suggested the reducing the baconian cipher to its lowest elements for many hours i laboured at this work and finally when i had reduced the baconian five symbols to three i felt that i had accomplished all that was possible in that way when i had arrived at this result and had tested its accuracy in working i felt in a position to experiment with my new knowledge on the old number cipher

first i wrote out my method of reduction as a sort of addendum to the paper which i had prepared for marjory then i made a key to cipher and one to decipher by this time the night was well on and the grey of early morning was beginning to steal in by the edges of the blinds i was not sleepy however i was too much excited to think of sleep for the solving of the problem seemed almost within my grasp excited to a state which almost frightened me by its intensity i got ready my copy of the number cipher and my newly prepared key with an effort which took me all my resolution i went on steadily writing its proper letter under each combination without once looking back for i knew that even should some of the letters be misplaced in the key the chance of recognising the right ones would be largely increased by seeing a considerable number of letters together then i glanced over the whole and found that many of the symbols made up letters with such a basis to work on the rest was only labour a few tentative efforts and i had corrected the key to agreement with some of the combinations in the cipher

These will be helpful in completing these problems. Transposition Tool Cell

```
@interact
def _(message=input_box('EUS HJY TXZ UPE ISE QML COP BEN KVI
    WHO RRG OTD FL NEG
    OA', label="Message", type=str, width=50, height=3),
        key_order_str=input_box('1,2,3,4,5,6,7,8,9,10,11,12,13,14,15',label="Column
            Order",type=str,width=50,height=2)):
    message_blocks = message.split()
    key_order = list(map(int,key_order_str.split(',')))
    print(message_blocks)
    print(key_order)
    for i in range(len(max(message_blocks, key=len))+1):
        row=''+str(i)
        for j in key_order:
            row += ' '
            if i == 0:
                row += str(j)
            else:
                if len(str(j))>1:
                    row += ' '
                try:
                    row += message_blocks[j-1][i-1]
                except:
                    row += ' '
        print(row)
```

Sage cell for calculating subsets of letters in a message:

```
@interact
def subset_count(
    text=input_box('Place the text that you
        would like to analyze here!!!',
        label="Enter your message", height=3,
        width=50, type=str),
        Subsets=[1..10]):
```

```
message =
    AlphabeticStrings().encoding(str(text.encode('ascii', 'replace').decode()))
S = Subsets
sets = [""]*S
for i in range(0,len(message)):
    sets[i%S] += str(message[i])
for i in range(S):
    tmp_message = sets[i]
    count = \{\}
    for i in range(len(tmp_message)):
        c = tmp_message[i:i+1]
        if c in count: count[c] += 1
        else: count[c] = 1
    print("The length of this subset is
        {}".format(len(tmp_message)))
    count = sorted([(value, key) for key, value in
        count.items()])
    print("Character\tCount\t\tPercent")
    count.reverse()
    IC = 0
    for c in count[0:min(30,len(count))]:
        if len(c[1])>6:
            tab="∖t"
        else:
            tab="\t\t"
        print(c[1],tab,c[0],"\t\t %.4G" %
            (c[0]/len(tmp_message)*100.0))
        IC += c[0]*(c[0]-1.0)/len(tmp_message)^2
    print("And, the index of coincidence for this set is
        %.4G\n" % (IC))
```

Kasiski's Key Table: Figure 4.3.2 Sage cell for enciphering and deciphering the Vigenère Cipher:

```
import re
import textwrap
@interact
def _(m=input_box('sage', label="Enter your message",
    height=3, width=50, type=str),
        key=input_box('sage', label="Enter your key",
            height=1, width=20, type=str),
        mode = selector(['encipher', 'decipher'],
            buttons=True),
        spaces = selector(['yes', 'no'], buttons=True)):
    plain_alpha = "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
    clean_message =
        str(m.encode('ascii', 'replace').decode()).upper()
    if spaces == 'no':
        clean_message = re.sub('[^A-Z]', '', clean_message)
    cipher_key =
        re.sub('[^A-Z]','', str(key.encode('ascii', 'replace').decode()).upper())
    key_list = [plain_alpha.index(ch) for ch in cipher_key]
    if mode == 'decipher':
        key_list = [-1*k for k in key_list]
    cipher_text = ""
    key_counter = 0
```

```
for ch in clean_message:
   try:
        tmp_pos = plain_alpha.index(ch)
        cipher_pos = (tmp_pos+key_list[key_counter])%26
        cipher_text += plain_alpha[cipher_pos]
        key_counter = (key_counter+1)%len(key_list)
    except:
        if spaces == 'yes':
            cipher_text += ch
print("\nHere is your output:\n")
if spaces == 'yes':
    print(textwrap.fill(cipher_text, 42))
else:
    for i in range(0,len(cipher_text),6):
        print(cipher_text[i:i+6],)
        if (i+6)%42 == 0: print("\n")
```

# Chapter 5

# (\*) An Industrial Revolution

## Objectives

- Telegraph Codes and Ciphers?
- Enigma
- Lorenz?
- Purple?
- Also included Stinson episode, use Friedman article
- Turing Papers?

## 5.1 World War I

## **Objectives**

- ADFGVX
- George Zimmermann
- American Cipher Chamber

# 5.2 Between the Wars

## **Objectives**

- Vernam One-Time Pad and Perfect Secrecy
- Henry L. Stinson, "Gentleman's Mail"
- Rise of Enigma
- Polish Cipher Bureau

# 5.3 World War II

## **Objectives**

- Bletchley Park and Enigma
- Lorenz Cipher, Colossus, Electronic Computing
- Japanese Ciphers

# 5.4 (\*)A Statistical Approach

## **Objectives**

- Friedman's Use of Statistics
- Sukhotin's Algorithm



YouTube: https://www.youtube.com/watch?v=raN0806R4yc

Figure 5.4.1 Incidence (or Index) of Coincidence

# Chapter 6

# Mathematics to the Rescue

## **Objectives**

- Modular Arithmetic
- Affine
- Matrices
- Hill?

## 6.1 Affine Ciphers

## **Objectives**

- Biography of Lester Hill
- Modulus
- Affine Cipher

## 6.1.1 Lester S. Hill

Insert bio here ...

## 6.1.2 Moduli

The cipher we will focus on here, Hill's Cipher, is an early example of a cipher based purely in the mathematics of number theory and algebra; the areas of mathematics which now dominate all of modern cryptography. Number theory has a long and rich history with many fundamental results dating all the way back to Euclid in 300 BCE, and with results found across the globe in different cultures. Number theory as we understand and use it today is due in large part to Carl Friedrich Gauss and his text *Disquisitiones Arithmeticae* published in 1801 (when Gauss was 24). Algebra (or more properly linear and abstract algebra) as it is going to be used here is much younger tracing its roots back only a couple hundred years to the early nineteenth century; here too much is owed to Gauss.

As with previous topics we will begin by looking at an original source text and trying to understand what it is saying. However, given the importance of

this material to the rest of what we will be discussing in subsequent chapters, we will look at the material from a more modern perspective.

## Cryptography in an Algebraic Alphabet.

By Lester S. Hill, Hunter College

1. The Bi-Operational Alphabet

Let  $a_0, a_1, \ldots, a_{25}$  denote any permutation of the letters of the English alphabet; and let us associate the letter  $a_i$  with the integer *i*. We define operations of modular addition and multiplication (modulo 26) over the alphabet as follows:

$$a_i + a_j = a_r,$$
  
$$a_i a_j = a_t,$$

where r is the remainder obtained upon dividing the integer i + j by the integer 26 and t is the reaminder obtained on dividing ij by 26. The integers i and j may be the same or different.

It is easy to verify the following salient propositions concerning the bi-operational alphabet thus set up:

(1) If  $\alpha$ ,  $\beta$ ,  $\gamma$  are letters of the alphabet,

- $\alpha + \beta = \beta + \alpha$  and  $\alpha\beta = \beta\alpha$  [commutative law]
- $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$  and  $\alpha(\beta\gamma) = (\alpha\beta)\gamma$  [associative law]
- $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$  [distributive law]

(2) There is exactly one "zero" letter, namely  $a_0$ , characterized by the fact that the equation  $\alpha + a_0 = \alpha$  is satisfied whatever the letter denoted by *alpha*.

(3) Given any letter  $\alpha$ , we can find exactly one letter  $\beta$ , dependent on  $\alpha$ , such that  $\alpha + \beta = a_0$ . We call  $\beta$  the "negative" of  $\alpha$ , and we write:  $\beta = -\alpha$ .

(4) Given any letters  $\alpha$ ,  $\beta$  we can find exactly on letter  $\gamma$  such that  $\alpha + \gamma = \beta$  [i.e.  $\gamma = \beta - \alpha$  is unique].

(5) Distinguishing the twelve letters,

 $a_1, a_3, a_5, a_7, a_9, a_{11}, a_{15}, a_{17}, a_{19}, a_{21}, a_{23}, a_{25},$ 

with subscripts prime to 26, as "primary" letters, we make the assertion, easily proved: If  $\alpha$  is any primary letter and  $\beta$  is any letter, there is exactly one letter  $\gamma$  for which  $\alpha \gamma = \beta$ .

(6) In any algebraic sum of terms, we may clearly omit terms of which the letter  $a_0$  is a factor; and we need not write the letter  $a_1$ explicitly as a factor in any product.

2. An Illustration

Let the letters of the alphabet be associated with the integers as follows:or, in another convenient formulation:

0	1	2	3	4	5	6	7	8	9	10	11	12
k	р	с	0	h	a	r	n	g	$\mathbf{Z}$	e	у	s
13	14	15	16	17	18	19	20	21	22	23	24	25
m	W	f	1	v	i	q	d	u	x	b	$\mathbf{t}$	j

It will be seen that

$$c + x = t$$
,  $j + w = m$ ,  $f + y = k$ ,  $-f = y$ ,  $-y = f$ , etc.

an = z, hm = k, cr = s, etc.

The zero letter is k, and the unit letter is p. The primary letters are: a b f j n o p q u v y z.

Since this particular alphabet will be used several times, in illustration of further developments, we append the following table of negatives and reciprocals:

Letter :  $\mathbf{a}$ b c d е f g h i Neg. : 1 i х g u 0 t. r у Rec. : u n V The solution to the equation  $z + \alpha = t$  is  $\alpha = t - z$  or  $\alpha = t + (-z) = z$ 

t+v=f.The system of linear equations:  $o \alpha + u \beta = x$ ,  $n \alpha + i \beta = q$  has solution  $\alpha = u, \beta = o$ , which may be obtained by the familiar method of elimination or by formula. [5, pp.306-308]

**Reading Questions:** 

- Hill starts by describing how we will add and multiply with the alphabet, looking at his description why in his illustration does i + w which should be 25 + 14 = 39 (see Hill's Correspondence ??) come out to be m which is 13?
- In his illustration he also says hm which should be 4 times 13, or 52, is k which is 0, why is this the case?
- Along the same lines, why does f + y equal k and why does an (a times n) equal z?
- Thinking about your previous answers, what are the values of the following: j + z, nf, au + j, and bv + jw.

In this section of text Hill has introduced us to the idea of modular arithmetic and modular equivalence, in particular the idea of equivalence modulo 26. This is a concept which will be central to most everything else we do so we need to spend a little more time trying to precisely understand modular equivalence.

**Definition 6.1.1 Modular Equivalence.** If n is a positive integer then we say that two other integers a and b are equivalent modulo n if and only if they have the same remainder when divided by n, or equivalently if and only if a - b is divisible by n, when this is the case we write

$$a \equiv b \pmod{n}$$
.

The number n is called the *modulus*.

**Example 6.1.2** Suppose that n = 14, then  $36 \equiv 8 \pmod{n}$  because 36 = 36 $2 \cdot 14 + 8$  and  $8 = 0 \cdot (14) + 8$  so we get the same remainder when we divide by n = 14. Alternately, we can observe that 36 - 8 = 28 and  $28 = 2 \cdot (14)$  is divisible by n = 14.

Using the same value for n we get that  $3 \cdot 5 \equiv 1 \pmod{n}$  because 15 = $1 \cdot (14) + 1$ , so the remainder when  $3 \cdot 5$  is divided by n is 1.

Test your understanding by filling in the rest of this multiplication table:

k 1 m

k

е m n 0 р q q b j n  $\mathbf{Z}$ 

У р

f

 $\Diamond$ 

$\times_{14}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
2	0	2	4	6	8	10	12	0	2	4				
3	0	3	6	9	12	1	4	7	10	13	2	5	8	11
4	0	4	8	12		6		0						
5	0	5	10	1	6	11								
6	0	6	12	4	10	0								
7	0	7	0	7										
8	0													
9	0													
10	0													
11	0													
12	0													
13	0													

- What is strange or different about the row for 7? Why do you think all the remainders come out this way?
- What is the difference between the even and odd rows (excluding row 7)?

Finally, fill in this addition table for addition modulo 14.

## Table 6.1.4 Addition Modulo 14

$+_{14}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1													
2	2													
3	3													
4	4													
5	5													
6	6													
7	7													
8	8													
9	9													
10	10													
11	11													
12	12													
13	13													

**Definition 6.1.5 Additive Identity.** We call 0 the *additive identity* because for all a and all possible moduli n we get

$$a+0 \equiv a \pmod{n}.$$

 $\diamond$ 

**Definition 6.1.6 Additive Inverse.** We say that a and b are *additive inverses* modulo n if

$$a+b \equiv 0 \pmod{n},$$

and we write b = -a.

 $\Diamond$ 

**Definition 6.1.7 Multiplicative Identity.** We call 1 the *multiplicative identity* because for all a and all possible moduli n we get

$$a \cdot 1 \equiv a \pmod{n}$$
.

 $\Diamond$ 

 $\Diamond$ 

**Definition 6.1.8 Multiplicative Inverse.** We say that a and b are *multiplicative inverses* modulo n if

$$a \cdot b \equiv 1 \pmod{n}$$
,

and we write  $b = a^{-1}$ .

**Checkpoint 6.1.9** Look back at Example 6.1.2 and write down the pairs of additive and multiplicative inverses. Do all the numbers modulo 14 have additive inverses? Do all of them have multiplicative inverses?

**Checkpoint 6.1.10** Write down another multiplication and addition table as you did in Example 6.1.2 but with a modulus of n = 10, so when you multiply and add you will always divide by 10 afterwards and write down the remainder. After you write down the tables write down the pairs of multiplicative and additive inverses. Do all the numbers modulo 10 have additive inverses? Do all of them have multiplicative inverses?

Reflection Questions: Look back at what Hill had to say and at the examples you have worked through when you used moduli of n = 14 and n = 10 as you think about the following questions.

- No matter which modulus you use, do all the numbers have additive inverses, i.e. numbers you can add to them in order to get 0?
- No matter which modulus you use, do all the numbers have multiplicative inverses, i.e. numbers you can multiply them by in order to get 1?
- If you look at the numbers which do have multiplicative inverses how do they relate to those which Hill described as prime to 26?

**Definition 6.1.11 Relatively Prime.** We say that two integers are *relatively* prime if the largest positive integers which divided them both, their greatest common divisor, is 1. For example the greatest common divisor of 7 and 36 is 1 so they are relatively prime, however the greatest common divisor of 30 and 36 is 6 so they are not relatively prime.  $\Diamond$ 

**Checkpoint 6.1.12** Which numbers, other than 7, that are less than 36 are *relatively prime* to 36?

**Checkpoint 6.1.13** Which numbers less than 14 are *relatively prime* to 14? How do these compare to the list of numbers which have *multiplicative inverses*?

**Checkpoint 6.1.14** Which numbers less than 10 are *relatively prime* to 10? How do these compare to the list of numbers which have *multiplicative inverses*?

**Checkpoint 6.1.15** Which numbers less than 26 are *relatively prime* to 26? How do these compare to the list of numbers which have *multiplicative inverses*? (You will want to use Figure C.0.13.)

## 6.1.3 Affine Cipher

**Definition 6.1.16 Affine Cipher.** An *affine cipher* is a cipher with a two part key, a multiplier m and a shift s and calculations are carried out using modular arithmetic; typically the modulus is n = 26. Characters of the plain text are enciphered with the formula

$$CIPHER \equiv m(plain) + s \pmod{26},$$

and characters of the cipher text are deciphered with the formula

$$plain \equiv m^{-1}(CIPHER - s) \pmod{26},$$

or

$$plain \equiv m^{-1}CIPHER - m^{-1}s \pmod{26}.$$

Note that the multiplier m must be relatively prime to the modulus so that it has a multiplicative inverse.  $\diamond$ 

**Checkpoint 6.1.17** Let's encipher the message "hello world" with an affine cipher and a key of m = 5 and s = 16; assume that we match up the alphabet with the integers from 0 to 25 in the usual way so that a is 0, b is 1, c is 2, etc.. In this way the letter h is replaced by the number 7 and when we encipher it we get

$$5 \cdot 7 + 16 \equiv 25 \pmod{26}$$

and 25 is Z, so plain h becomes cipher Z. Next e is replaced by 4 and we get

$$5 \cdot 4 + 16 \equiv 10 \pmod{26}$$

and 10 is K, so plain e becomes cipher K. The plain l corresponds to 11 and

$$5 \cdot 11 + 16 \equiv 19 \pmod{26},$$

which is T, that is plain l is replaced by cipher T. Try to encipher the rest of the message on your own, you will want to use Figure C.0.13 to help you with the multiplication modulo 26.

#### Answer. ZKTTI WIXTF

**Checkpoint 6.1.18** Now let's decipher the message AJINF CVCSI JCAKU which was enciphered using an affine cipher and a key of m = 11 and s = 4. Note that  $m^{-1} \equiv 19 \pmod{26}$  and  $-s \equiv 22 \pmod{26}$ . Take the A and replace it by 0 and then using the formula above we get

$$19(0+22) \equiv 2 \pmod{26}$$

so we replace cipher A with plain text c. The J is replaced by 9 and

$$19(9+22) \equiv 17 \pmod{26}$$

therefore cipher J becomes plain r. To use the other formula for deciphering we need  $m^{-1}s \equiv 2 \pmod{26}$ . Then converting the cipher I to 8 we get

$$19(8) + 2 \equiv 24 \pmod{26}$$

which is plain  $\boldsymbol{y}$  or with the next letter  $\boldsymbol{N}$  we get

$$19(13) + 2 \equiv 15 \pmod{26}$$

which is **p**. Try to decipher the remaining characters in the message on your own.

Answer. cryptology rocks

**Checkpoint 6.1.19** Encipher the message "a fine affine cipher" using the key m = 17 and s = 12.

Answer. MTSZC MTTSZ CUSHB CP

**Checkpoint 6.1.20** Decipher the message RXGTM CHUHJ CFWM which was enciphered using the key m = 3 and s = 7.

#### Answer. more than a shift

To *decrypt*, as opposed to just decipher, an affine cipher you can use the techniques we learned in Chapter 2 since they are a type of monoalphabetic substitution cipher. However, we can also take advantage of the fact that it is an affine cipher.

DZIUI UDZYH ILUDO HHIBY GITZY LSYUU OQYDZ ODEYE IJJPY GLWTD IDEOU YBGIT ZYLYP EIDZO AYWKH BIBYH KLDZY SMJDI TJIYL OBPHK MLDYY BHKLD ZYUZI HDUIB GYIDI UOSKB KOJTZ OXYDI GEYGO BODO GAIDE IDZXO UIGHL YCMYB GWOBO JWUIU XMDEY GOBOJ UKMUY DZODO BOJWU IUDKL YGKVY LDZYK LIQIB OJAYW UKDZO DEYGO BPYGI TZYLD ZYSYU UOQY

Analyzing this we get that the most common characters are Y, D, I, O and U; the most common bigrams are DZ, ZY, YG, and OB; the most common trigrams are DZY, OBO, LDZ, and DZO. Therefore it is reasonable to assume that DZY is the, Y is e, and D is t. So when this was enciphered we have to of had

$$24 \equiv m \cdot 4 + s \pmod{26}$$
$$3 \equiv m \cdot 19 + s \pmod{26}$$

Subtracting the second expression from the first we get

$$21 \equiv m \cdot -15 \pmod{26}$$

or

$$21 \equiv m \cdot 11 \pmod{26}.$$

Looking at the multiplication table modulo 26 we can see that m = 9 since  $9 \cdot 11 \equiv 21 \pmod{26}$ . Substituting m = 9 into the first equation above we get

$$24 \equiv 9 \cdot 4 + s \pmod{26}$$

which simplifies to

$$24 - 10 \equiv s \pmod{26}$$

so that s = 14. We can then get the inverse keys  $m^{-1} \equiv 3 \pmod{26}$  and  $-m^{-1}s \equiv 10 \pmod{26}$ . Using these with the affine cipher cell we get the deciphered message:

thisi sthef irsta ffine ciphe rmess ageth atwew illde crypt itwas encip hered witha keyof ninef orthe multi plier andfo urtee nfort heshi ftsin ceiti samon oalph abeti cweca natta ckitw ithba sicfr equen cyana lysis butwe canal souse thata nalys istor ecove rtheo rigin alkey sotha tweca ndeci phert hemes sage

Or, in a more readable form

"this is the first affine cipher message that we will decrypt ..."

**Checkpoint 6.1.21** Try to decrypt this message which was enciphered using an affine cipher.

IPAGS WWANP YMFZC TAEWI PZFGC TZANW QWCNN YCMAY ECGGA CZAHV QWQIE PFACN ZFGWI KYPMC DCGZB YBACP HPIHH YPMIL ANCPI LADTI NWQHC QGEIN KFCHV AAPCP AXFCS GZYPM IPAWQ EYTAF CHCDN ACHQM IPASB GZCYN GCPHZ FAGIS PHITZ FADIO KYPMI TZFAF CDDHI INGIW AZYWA VATIN AZIDH WAZFC ZZFAG ANLCP ZGFCH CDGIN AZYNA HYFCH NYGAP TNIWW QGACZ CPHEC GKPIO KYPMI SZZFA CGFAG ITWQB YBAEF APYGS HHAPD QFACN HZFAO DCPMI TZFAV ADDYD IIKAH CZZFA ODIOK YZECG CUSCN ZANZI ZEADL AZFYG OISDH PIZVA CLYGY ZINCZ GIDCZ ACPFI SNCBC ZYAPZ ALYHA PZDQC PHBIG GYVDQ CPCDD PYMFZ GYZZY PMEYZ FCENQ TCOAY EAPZI SZYPZ IZFAF CDDCP HIBAP AHZFA HIINZ IWQCG ZIPYG FWAPZ YZECG GFAND IOKFI DWAGE FIGZI IHSBI PWQGZ AB

## Hint.

- First use frequency analysis to identify at least two of the letters in the message.
- With your two letters set up two equations like this:

 $CIPHER \equiv m(plain) + s \pmod{26}$ .

- Subtract the second equation from the first and try to find m.
- Substitute your value for m into the first equation and use it to find s.
- Now that you have the key you should be able to decipher the message as you had previously.

**Answer**. The message begins with "One summer night, a few months after my ..."

Sage N-Gram Cell:

```
import re
@interact
def ngram_count_simple(
                     text=input_box('Place the text that you
                        would like to analyze here !!! ',
                     label="Enter your message", height=5,
                         width=50, type=str), N=[1..6],
                     mode=selector(['All
                         Characters', 'Alpha-Numeric', 'Alpha
                         Only'], buttons=True)):
    message = text.upper()
    if mode == 'Alpha Only':
        message = re.sub('[^A-Z]','',message)
    elif mode == 'Alpha - Numeric':
        message = re.sub('[\\W]|[_]','',message)
    message = re.sub('\\s+','',message)
    count = \{\}
    for i in range(len(message)-N+1):
        c = message[i:i+N]
        if c in count: count[c] += 1
        else: count[c] = 1
```

```
print("The length of your message is {}
   characters".format(len(message)))
count = sorted([(value, key) for key, value in
   count.items()])
print("\tRank\tN-Gram\tCount\tPercent")
count.reverse()
rank = 1
for c in count[0:min(30,len(count))]:
    if len(c[1])>6:
        tab=" "
    else:
        tab="\t"
    print(str("\t"+str(rank)+"."),tab,c[1],tab,c[0],"\t
       %.4G" % (c[0]/len(message)*100.0))
    rank += 1
if len(count)==26 and N==1 and mode=='Alpha Only':
   print("Pangram!!!")
```

Affine Cipher Cell:

You can use this Sage Cell to encipher and decipher messages that used an *affine cipher*. To decipher you will need to use the second formula listed in Definition 6.1.16. Also, be sure you understand how to encipher and decipher by hand.

```
import re
@interact
def _(me=input_box('sage', label="Enter your message",
   height=3, width=50, type=str),m=[1..25], s=[0..25]):
    S = AffineCryptosystem(AlphabeticStrings())
    clean_text =
       S.encoding(str(me.encode('ascii', 'replace').decode()))
    try:
        e = S(m, s)
        cipher_text = e(clean_text)
        print("Your affine enciphering text is\n")
        for i in range(0,len(cipher_text),5):
            print(cipher_text[i:i+5])
            if (i+5)%50 == 0: print("\n")
    except:
        print("error, likely in your key")
        cipher_text = clean_text
```

## 6.2 Hill's Cipher

## Objectives

- Matrices
- Hill's Cipher

## 6.2.1 Matrices

**Definition 6.2.1 Matrix.** A *matrix* is a rectangular array of numbers such as

$$\begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{pmatrix}$$

which, in this setting, we will use to represent a transformation for enciphering and deciphering.  $\diamondsuit$ 

The cipher we will look at in this section, *Hill's Cipher* will work much like an affine cipher but will use matrices for the multiplier and shift and not just numbers. All the matrices we will use will be either square like this

$$\begin{pmatrix} 1 & 7 \\ 0 & 5 \end{pmatrix}$$

so that it has the same number of rows and columns or will be a special type of matrix called a **vector** which has only one column like this

$$\begin{pmatrix} 18 \\ 7 \end{pmatrix}.$$

Finally, all operations will be done using arithmetic modulo 26.

We can add and multiply matrices with one another or with vectors as long as the dimensions match.

Two  $2 \times 2$  square matrices can be added like so

$$\begin{pmatrix} 23 & 3\\ 0 & 15 \end{pmatrix} + \begin{pmatrix} 9 & 2\\ 5 & 19 \end{pmatrix} = \begin{pmatrix} 23+9 & 3+2\\ 0+5 & 15+19 \end{pmatrix}$$

where the corresponding entries are added together. Then completing the addition and reducing modulo 26 we get

$$\begin{pmatrix} 32 & 5\\ 5 & 34 \end{pmatrix} \equiv \begin{pmatrix} 6 & 5\\ 5 & 8 \end{pmatrix} \pmod{26}.$$

Likewise we can add two vectors

$$\binom{17}{12} + \binom{14}{8} \equiv \binom{5}{20} \pmod{26}.$$

But, we cannot add a matrix to a vector since they are not the same size.

In order to multiply a matrix with a vector or a matrix with another matrix we need to multiply rows by columns like so

$$\begin{pmatrix} 8 & 1 \\ 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 10 \end{pmatrix} = \begin{pmatrix} 8 \cdot 1 + 1 \cdot 10 \\ 5 \cdot 1 + 3 \cdot 10 \end{pmatrix}.$$

Breaking that down a little more, the first entry on top is

$$\begin{pmatrix} 8 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 10 \end{pmatrix} = \begin{pmatrix} 8 \cdot 1 + 1 \cdot 10 \end{pmatrix}$$

. .

and the second entry on bottom is

$$\begin{pmatrix} 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \cdot 1 + 3 \cdot 10 \end{pmatrix}.$$

When we simplify the result modulo 26 we get

$$\binom{8\cdot 1+1\cdot 10}{5\cdot 1+3\cdot 10} \equiv \binom{18}{9} \pmod{26}.$$

Similarly we can multiply two matrices together

$$\begin{pmatrix} 3 & 8 \\ 5 & 15 \end{pmatrix} \cdot \begin{pmatrix} 1 & 7 \\ 13 & 16 \end{pmatrix} = \begin{pmatrix} 3 \cdot 1 + 8 \cdot 13 & 3 \cdot 7 + 8 \cdot 16 \\ 5 \cdot 1 + 15 \cdot 13 & 5 \cdot 7 + 15 \cdot 16 \end{pmatrix}$$
$$\equiv \begin{pmatrix} 3 & 19 \\ 18 & 15 \end{pmatrix} \pmod{26}.$$

Before moving on to see how Hill used these ideas to create a cipher practice your arithmetic with matrices and vectors, you will want to use Figure C.0.13 so that the multiplication is not so tedious.

**Checkpoint 6.2.2** Carry out the arithmetic operations and simplify the results modulo 26.

$$\binom{17}{5} + \binom{13}{9}$$

Answer.

$$\begin{pmatrix} 4\\14 \end{pmatrix} \pmod{26}$$

**Checkpoint 6.2.3** Carry out the arithmetic operations and simplify the results modulo 26.

$$\begin{pmatrix} 3 & 11 \\ 17 & 5 \end{pmatrix} + \begin{pmatrix} 9 & 25 \\ 1 & 4 \end{pmatrix}$$

Answer.

$$\begin{pmatrix} 12 & 10 \\ 18 & 9 \end{pmatrix} \pmod{26}$$

**Checkpoint 6.2.4** Carry out the arithmetic operations and simplify the results modulo 26.

$$\begin{pmatrix} 4 & 19 \\ 15 & 22 \end{pmatrix} \cdot \begin{pmatrix} 19 \\ 4 \end{pmatrix}$$

Answer.

$$\binom{22}{9} \pmod{26}$$

**Checkpoint 6.2.5** Carry out the arithmetic operations and simplify the results modulo 26.

$$\begin{pmatrix} 21 & 17 \\ 0 & 11 \end{pmatrix} \cdot \begin{pmatrix} 14 \\ 5 \end{pmatrix}$$

Answer.

$$\binom{15}{3} \pmod{26}$$

**Checkpoint 6.2.6** Carry out the arithmetic operations and simplify the results modulo 26.

$$\begin{pmatrix} 16 & 3 \\ 7 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 17 \end{pmatrix} + \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

**Hint**. Order of operations is the same for matrices and vectors as it is for numbers, so first multiply and then add.

Answer.

$$\begin{pmatrix} 13\\ 0 \end{pmatrix} \pmod{26}$$

## 6.2.2 Hill's Cipher

## 6.2.2.1 Enciphering with Matrices

Let's see how we can encipher hill cipher using a key matrix

$$m = \begin{pmatrix} 7 & 12 \\ 3 & 3 \end{pmatrix}.$$

Since it is a  $2\times 2$  matrix we will first break the message into groups of two letters

(hi) (ll) (ci) (ph) (er)

then to encipher each block we will convert each of these to a vector using  $a = 0, b = 1, c = 2, \ldots etc.$  and multiply it by the matrix m. That is (hi) becomes

$$m \begin{pmatrix} h \\ i \end{pmatrix} = \begin{pmatrix} 7 & 12 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$
$$\equiv \begin{pmatrix} 15 \\ 19 \end{pmatrix} \pmod{26}$$
$$\equiv \begin{pmatrix} P \\ T \end{pmatrix}.$$

Similarly we can encipher the (ll):

$$m \begin{pmatrix} l \\ l \end{pmatrix} = \begin{pmatrix} 7 & 12 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 11 \\ 11 \end{pmatrix}$$
$$\equiv \begin{pmatrix} 1 \\ 14 \end{pmatrix} \pmod{26}$$
$$\equiv \begin{pmatrix} B \\ O \end{pmatrix}.$$

And for (ci) we get:

$$m \begin{pmatrix} c \\ i \end{pmatrix} = \begin{pmatrix} 7 & 12 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$
$$\equiv \begin{pmatrix} 6 \\ 4 \end{pmatrix} \pmod{26}$$
$$\equiv \begin{pmatrix} G \\ E \end{pmatrix}.$$

Checkpoint 6.2.7 Finish enciphering hill cipher by enciphering the (ph) and the (er) just as we enciphered the other bi-grams. Hint.

- 1. Change each pair of letters into a pair of numbers, this is a vector.
- 2. Multiply the vector by the matrix m.
- 3. Reduce the result modulo 26 and change back to letters, this is your cipher text.

Answer. (ph) $\equiv$ HO and (er) $\equiv$ YL

Checkpoint 6.2.8 Try to encipher the message linear madness using the matrix

$$m = \begin{pmatrix} 15 & 0\\ 17 & 7 \end{pmatrix},$$

note that since the message has odd length you will need to tack on an extra null letter, like a z.

## Hint.

- 1. Change each pair of consecutive letters into a pair of numbers, this is a vector.
- 2. Multiply the vector by the matrix m.
- 3. Reduce the result modulo 26 and change back to letters, this is your cipher text.

 $\mathbf{Answer.}$  Your enciphered message should be something like <code>JJNPA PYWTM</code> <code>IMKN</code>.

**Checkpoint 6.2.9** For a variation to Hill's Cipher you can both multiply by a matrix and add a shift, this make Hill's behave very much like an affine cipher. Try to encipher the word shifty by first multiplying by the matrix

$$m = \begin{pmatrix} 23 & 14\\ 7 & 9 \end{pmatrix},$$

and then adding the vector

$$s = \begin{pmatrix} 10 \\ 3 \end{pmatrix}$$
.

Answer. CKEADO

### 6.2.2.2 Matrix Inverses and Deciphering

We will be able to decipher messages which used Hill's cipher in exactly the same way we enciphered them once we can find **inverse matrices**.

**Definition 6.2.10 Inverse Matrix.** Given a matrix m the *inverse matrix*  $m^{-1}$  is the unique matrix such that

$$m \cdot m^{-1} = m^{-1} \cdot m = I$$

where I is the *identity matrix* (i.e.  $I \cdot m = m \cdot I = m$  for all matrices m).

Consider the matrix

$$m = \begin{pmatrix} 8 & 1\\ 5 & 3 \end{pmatrix},$$

the inverse for this will be

$$m^{-1} = \begin{pmatrix} 7 & 15\\ 23 & 10 \end{pmatrix},$$

and if we multiply them together we get

$$m \cdot m^{-1} = \begin{pmatrix} 8 & 1 \\ 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 7 & 15 \\ 23 & 10 \end{pmatrix} = \begin{pmatrix} 8 \cdot 7 + 1 \cdot 23 & 8 \cdot 15 + 1 \cdot 10 \\ 5 \cdot 7 + 3 \cdot 23 & 5 \cdot 15 + 3 \cdot 10 \end{pmatrix}$$
$$\equiv \begin{pmatrix} 4 + 23 & 16 + 10 \\ 9 + 17 & 23 + 4 \end{pmatrix}$$
(mod 26)
$$\equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
(mod 26)

which is the identity matrix since

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

for any values of a, b, c, and d. The matrix inverses and the identity matrix work just like the multiplicative inverses and multiplicative identities we discussed when we learned about affine ciphers and modular arithmetic; just as in that case, not all matrices will have a multiplicative inverse.

**Definition 6.2.11 Determinant.** The *determinant* of a  $2 \times 2$  matrix

$$m = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

is equal to

$$det(m) = ad - bc.$$

 $\diamond$ 

The matrix m will have a multiplicative inverse  $m^{-1}$  if and only if det(m) has a multiplicative inverse, in which case we get

$$m^{-1} = (det(m))^{-1} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Going back to our previous example of

$$m = \begin{pmatrix} 8 & 1 \\ 5 & 3 \end{pmatrix},$$

we calculate the inverse as follows

$$m^{-1} = (8 \cdot 3 - 5 \cdot 1)^{-1} \begin{pmatrix} 3 & -1 \\ -5 & 8 \end{pmatrix}$$
$$\equiv 19^{-1} \begin{pmatrix} 3 & 25 \\ 21 & 8 \end{pmatrix}$$
(mod 26)

$$\equiv 11 \cdot \begin{pmatrix} 3 & 25\\ 21 & 8 \end{pmatrix} \pmod{26}$$

$$\equiv \begin{pmatrix} 11 \cdot 3 & 11 \cdot 25\\ 11 \cdot 21 & 11 \cdot 8 \end{pmatrix} \pmod{26}$$

$$\equiv \begin{pmatrix} 7 & 15\\ 23 & 10 \end{pmatrix} \tag{mod 26}$$

which is what we had before.

Checkpoint 6.2.12 Check your understanding by finding the inverse of

$$m = \begin{pmatrix} 17 & 21\\ 15 & 8 \end{pmatrix}.$$

Hint.

$$det(m) = ad - bc.$$

and

$$m^{-1} = (det(m))^{-1} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Answer.

$$m^{-1} = \begin{pmatrix} 20 & 19\\ 21 & 23 \end{pmatrix}.$$

We are now, finally, ready to decipher a message. Suppose the message NDGUF SJV had been enciphered using Hill's cipher with multiplier

$$m = \begin{pmatrix} 8 & 1\\ 5 & 3 \end{pmatrix},$$

to decipher we split the message into pairs and multiply those pairs by

$$m^{-1} = \begin{pmatrix} 7 & 15\\ 23 & 10 \end{pmatrix}.$$

So for the first two letters we get

$$m^{-1} \begin{pmatrix} N \\ D \end{pmatrix} = \begin{pmatrix} 7 & 15 \\ 23 & 10 \end{pmatrix} \begin{pmatrix} 13 \\ 3 \end{pmatrix} \equiv \begin{pmatrix} 6 \\ 17 \end{pmatrix} \pmod{26} \equiv \begin{pmatrix} g \\ r \end{pmatrix}.$$

And, for the next two letters of cipher text

$$m^{-1}\begin{pmatrix}G\\U\end{pmatrix} = \begin{pmatrix}7 & 15\\23 & 10\end{pmatrix}\begin{pmatrix}6\\20\end{pmatrix} \equiv \begin{pmatrix}4\\0\end{pmatrix} \pmod{26} \equiv \begin{pmatrix}e\\a\end{pmatrix}$$

**Checkpoint 6.2.13** Finish deciphering the message NDGUF SJV. **Hint**.

- 1. Change each pair of consecutive letters into a pair of numbers, this is a vector.
- 2. Multiply the vector by the matrix  $m^{-1}$ .
- 3. Reduce the result modulo 26 and change back to letters, this is your plain text.

#### Answer. great job

 ${\bf Checkpoint}~6.2.14$  Decipher the message <code>TYEMC QDE</code> which was enciphered with the matrix

$$m = \begin{pmatrix} 7 & 4 \\ 0 & 3 \end{pmatrix}.$$

## Hint.

- 1. Find the inverse matrix  $m^{-1}$ .
- 2. Change each pair of consecutive letters into a pair of numbers, this is a vector.
- 3. Multiply the vector by the matrix  $m^{-1}$ .
- 4. Reduce the result modulo 26 and change back to letters, this is your plain text.

### Answer. nice work

**Checkpoint 6.2.15** Decipher the message URYMB FXFLS which was enciphered by multiplying the plain text by

$$m = \begin{pmatrix} 7 & 4 \\ 0 & 3 \end{pmatrix},$$

and then adding the vector

$$s = \begin{pmatrix} 7\\5 \end{pmatrix}.$$

#### Hint.

- 1. Find the inverse matrix  $m^{-1}$ .
- 2. Change each pair of consecutive letters into a pair of numbers, this is a vector.
- 3. Subtract the vector s from this vector.
- 4. Multiply the new vector by the matrix  $m^{-1}$ .
- 5. Reduce the result modulo 26 and change back to letters, this is your plain text.

#### Answer. hello again

In conclusion we can give Hill's cipher the following general definition.

**Definition 6.2.16 Hill's Cipher.** *Hill's cipher* is a cipher with a two part key, a multiplier m which is a square  $n \times n$  matrix and a shift s which is a vector with n entries; typically all the arithmetic is done modulo 26. Characters of the plain text are enciphered n at a time with the formula

$$CIPHER \equiv m(plain) + s \pmod{26},$$

where plain is a vector of n characters. Likewise, characters of the cipher text are deciphered n at a time with the formula

$$plain \equiv m^{-1}(CIPHER - s) \pmod{26}$$

or

$$plain \equiv m^{-1}CIPHER - m^{-1}s \pmod{26}$$

Note that the multiplier m must have a multiplicative inverse.

We end with two important observations. First, modern explanations of Hill's cipher focus on the simplest case when the matrix has dimension  $2 \times 2$  and there is no shift. Second, you should be able to see that Hill's cipher is a variation on a affine cipher which enciphers multiple characters at a time in order to mix up the frequencies, this means it is a type of **polygraphic cipher**.

## 6.2.2.3 Alternate Explanation



YouTube: https://www.youtube.com/watch?v=4RhLNDqcjpA

Figure 6.2.17 Modern look at Hill's Cipher

## 6.3 Up Hill struggle?

- 1. Construct an addition and multiplication table modulo 6. Which pairs of numbers are additive inverses? Which pairs are multiplicative inverses, and how do they relate to 6?
- **2.** Construct an addition and multiplication table modulo 13. Which pairs of numbers are additive inverses? Which pairs are multiplicative inverses, and how do they relate to 13?
- **3.** Encipher zygomorphic using an affine cipher with key m = 7 and s = 12.
- 4. Encipher quaquaversal using an affine cipher with key m = 17 and s = 3.
- 5. Decipher TWWFS NLVBY FVNBX GPNWY XWZVN BKLVM FSHFA FKWT which was enciphered using an affine cipher with key m = 15 and s = 23.
- 6. Decipher YTGXZ GHACB HQGBN XABFU ZAGXN TJJ which was enciphered using an affine cipher with key m = 3 and s = 7.
- 7. Decrypt this message which was enciphered using an affine cipher: EHWQC CIYKH EYKVK PIXKH ZKZKQ THZIP YGPXE KDFYX CZKXT IAOZI TYKCX KAIRK XKFPX IYHZK CHXQE DAQMC KFVGZ ECEYY KDCKK BKXHE IDCED HZKCN XEDUI PHZKW ZITKS MKCHE IDIPH ZKDKH ZKXTQ DFCMY QHXQA IYNQD GQDFI PHZKA ITICC QTCAZ KYKCI PVQXI DYQMN KXHME CQXKH IIXKA KDHED HZKYE DFCIP HZKNM VTEAQ DFQXK HIIED HEYQH KTGAI DAKXD KFWEH ZNITE HEACQ DFPED QDAKH IVKPE HHEDU CMVJK AHCPI XHZEC CKXEK CIPCO KHAZK CHZKG TKFZI WKRKX EDQDE DFEXK AHPQC ZEIDH IQCED UMTQX QDFAI YNTKB NXIVT KYWZE AZUQR KYGPX EKDFQ DINNI XHMDE HGIPF KYIDC HXQHE DUHZK RQTMK IPQPX KCZWK QNIDQ YIDUH ZKYQD GWEHZ WZEAZ ZKWQU KFZEC TEPKT IDUVQ HHTKQ UQEDC HAXEY K
- 8. Decrypt this message which was enciphered using an affine cipher:

 $\Diamond$ 

ZTWFBJOJLTZZZTWXVAJOKOJWZMHOLJKTHWTWKHJFHWMBZZHOHMKIBHOQBOUJTZBLBJVCRJZWHAHWXBOKIBZJLBLJWVSKHKIJKSBOTHQKIBSAJFBRITFIJOJLTZIJQIBAQTWKIBRHOKINXHGBOWHOZBZKTLJKTHWRJZKIJKHMJSOBAJKBRIHLIBOBZSBFKBQJWQJMOTBWQKHRIHLIBHRBQJQBUKHMXOJKTKVQBUVKWRRIBMBAKITLZBAMJWTWMBOTHOJWQKIJKJOJLTZRJZITZLJZKBOIBITLZBAMATXIKBQJAJWKBOWZVLLHWBQJKVOWPBNJWQZJTQOBKVOWTWXKHJOJLTZTJLJKNHVOHOQBOZLHWZBTXWBVOJOJLTZLBOBANWHQQBQITZIBJQJZLVFIJZKHZJNGBONXHHQJWQZTXWBQKHITLRTKIITZJJWQKHABJQKIBRJNNKITZKBATAKENGAN

9. Encipher order more pizza using Hill's cipher and the matrix,

$$m = \begin{pmatrix} 1 & 5\\ 7 & 10 \end{pmatrix},$$

be sure to how your work.

Answer. VIXJZ FVIBW DUZT

10. Encipher we need soda too using Hill's cipher and the matrix,

$$m = \begin{pmatrix} 9 & 6\\ 13 & 7 \end{pmatrix},$$

be sure to how your work.

11. Decipher ZEMUM KJDIK NEWSX XHM which was enciphered using Hill's cipher and the matrix,

$$m = \begin{pmatrix} 13 & 19\\ 1 & 6 \end{pmatrix},$$

be sure to how your work.

Answer. spicy chicken wings

12. Decipher MRUKD QFDWI DHNL which was enciphered using Hill's cipher and the matrix,

$$m = \begin{pmatrix} 7 & 17\\ 2 & 5 \end{pmatrix},$$

be sure to how your work.

13. Decrypt this quote about "Chuck Norris" which was enciphered using Hill's cipher: FYQAB YTAHX XYERO MQMFP NNBXN ARJFK HLSNK QVVME IFDUJ AVOUT ZYLA

**Answer**. "Chuck Norris threw a grenade and killed 50 zombies, then it exploded."

14. Decrypt this quote about "Molly Weasley" which was enciphered using Hill's cipher: CQFUM OEAZH YUMAW MYGCV GEQDD MKCEA BIKCU ZSMGN VUGC

Affine Cipher Cell:

This SAGE cell can help you check your work when you encipher and decipher with a affine cipher, but you should be able to do the basic calculations your self.

```
import re
@interact
def _(me=input_box('sage', label="Enter your message",
```
```
height=3, width=50, type=str),m=[1..25], s=[0..25]):
S = AffineCryptosystem(AlphabeticStrings())
clean_text =
    S.encoding(str(me.encode('ascii','replace').decode()))
try:
    e = S(m,s)
    cipher_text = e(clean_text)
    print("Your affine enciphering text is\n")
    for i in range(0,len(cipher_text),5):
        print(cipher_text[i:i+5])
        if (i+5)%50 == 0: print("\n")
except:
    print("error, likely in your key")
    cipher_text = clean_text
```

Hill's Cipher Cell:

This SAGE cell can help you check your work when you encipher and decipher with a basic Hill's cipher, but you should be able to do the basic calculations your self.

```
@interact
def _(m=input_box('sage', label="Enter your message",
   height=3,
   width=50, type=str),
   key=input_grid(2,2,default=[[7,12],[3,3]],type=int,width=2)):
   S = HillCryptosystem(AlphabeticStrings(),2)
   M = MatrixSpace(IntegerModRing(26),2,2)
    e = S(M(key).transpose())
    clean_text =
       S.encoding(str(m.encode('ascii', 'replace').decode()))
    if len(clean_text)%2 == 1:
        clean_text = S.encoding(str(m)+"Z")
    cipher_text = e(clean_text)
    print(str("Your Hill enciphering text is\n"))
    for i in range(0,len(cipher_text),5):
        print(cipher_text[i:i+5],)
        if (i+5)%50 == 0: print("\n")
```

# Chapter 7

# (\*) The Modern Age

## Objectives

- Symmetric ciphers (DES, AES) and the roll of group theory and linear algebra
- Asymmetric ciphers (Diffie-Hellman, RSA, El Gamal?, etc.) and the roll of number theory
- Absolute secrecy

# 7.1 Going Public

## **Objectives**

- Biographies for Diffie and Hellman
- Public Key Cryptography Overview

## 7.1.1

Enter a little history

#### 7.1.2

Enter some quotes from DH paper

## 7.1.3

Enter comments on general ideas for enciphering vs. key exchange vs. signature schemes  $\ldots$ 

## 7.2 Symmetric Ciphers

#### **Objectives**

- DES and Lucifer
- AES
- PGP

## 7.2.1 Feistel Cipher

General Description Graphic of Cipher Specific Mini-Example Practice Video

## 7.2.2 Data Encryption Standard and Lucifer (DES)

Enter a Little History Enter Quoets from Mini-DES Enter Questions

## 7.2.3 Advanced Encryption Standard (AES)

Enter a Little History Enter Quoets from Mini-AES Enter Questions

Comments about PGP combining DES and RSA

# 7.3 Asymmetric Ciphers

#### **Objectives**

- Biographies for Rivest, Shamir, and Adelman
- A Little More Math
- RSA Encryption
- Diffie-Hellman Key Exchange

#### 7.3.1

Enter a little history ...

#### 7.3.2

Enter mathematical material from RSA  $\ldots$ 

## 7.3.3

Enter some general content on Number Theory...

# 7.3.4

Enter cipher material from RSA  $\ldots$ 

## 7.3.5

Enter material about DH key exchange

# 7.4 Public Inquiry?

**1.** Some questions ...

# Chapter 8

# (\*) The Future Age

## Objectives

- Symmetric ciphers (DES, AES) and the roll of group theory and linear algebra
- Asymmetric ciphers (Diffie-Hellman, RSA, El Gamal?, etc.) and the roll of number theory
- Absolute secrecy

# 8.1 (\*)Quantum Ciphers

#### **Objectives**

- basic definitions
- mini-example

# 8.2 (\*)Lattice Ciphers

#### **Objectives**

- basic definitions
- mini-example

# 8.3 (\*)Elliptic Ciphers

## Objectives

- basic definitions
- mini-example

# 8.4 (\*)Homomorphic Ciphers

# Objectives

- basic definitions
- mini-example

# Appendix A

# Sage Cell Analysis Utilities

Word Count Cell

```
import re
@interact
def word_count_simple(
                    text=input_box('Place the text that you
                        would like to analyze here!!!',
                    label="Enter your message", height=3,
                        width=50, type=str),
                    nWords=input_box(30,height=1,width=4,label="Number
                        of Words", type=int)):
    message = re.sub('[^A-Z]','
        ',str(text.encode('ascii', 'replace').decode()).upper())+"
    count = \{\}
    words = 0
    c=""
    for l in message:
        if l==" ":
            if c in count:
                words += 1
                count[c] += 1
            elif c!="":
                count[c] = 1
                words += 1
            c=""
        else: c=c+l
    print("The total number of words in your message is
       {}".format(words))
    print("The number of distinct words in your message is
       {}".format(len(count)))
    count = sorted([(value, key) for key, value in
        count.items()])
    print("Word\t\tCount\t\tPercent\t\tTotal Percent")
    count.reverse()
    totalPercent=0
    for c in count[0:min(nWords,len(count))]:
        if len(c[1])>6:
            tab="\t"
        else:
            tab="\t\t"
        percent=(c[0]/words*100.0)
```

```
if percent>1:
    tab2="\t\t"
else:
    tab2="\t"
totalPercent+=percent
print(c[1],tab,c[0],"\t\t %.4G" %
    percent,tab2,"%.4G" % totalPercent)
```

String Count Cell

```
import re
@interact
def string_count_simple(
                    text=input_box('Place the text that you
                        would like to analyze here !!! ',
                    label="Enter your message", height=3,
                        width=50, type=str),
                    key=input_box('sage', label="String to
                        search", height=1, width=20,
                        type=str)):
    message = re.sub('[^A-Z]', '
        ',str(text.encode('ascii', 'replace').decode()).upper())+"
    message = re.sub('\\s+',' ',message)
    print("This is what your cleaned up message looks
        like:\n\n",message[0:min(60,len(message))],"...\n\nis
       this what you intended?\n")
    count = 0
    K = re.sub('[^A-Z]','
        ',str(key.encode('ascii', 'replace').decode()).upper())
    N = len(K)
    c=""
    for i in range(len(message)-N+1):
        c = message[i:i+N]
        if c == K:
            count += 1
            print("Instance", str(count), "\t", message[max(i-2,0):min(i+N+2,len(message)))
    print("\nYour string appears ",str(count)," times.")
```

N-Gram Analysis Cell

```
message = re.sub('\\s+','',message)
count = \{\}
for i in range(len(message)-N+1):
    c = message[i:i+N]
    if c in count: count[c] += 1
    else: count[c] = 1
print("The length of your message is {}
   characters".format(len(message)))
count = sorted([(value, key) for key, value in
   count.items()])
print("\tRank\tN-Gram\tCount\tPercent")
count.reverse()
rank = 1
for c in count[0:min(30,len(count))]:
    if len(c[1])>6:
        tab=" "
    else:
        tab="∖t"
    print(str("\t"+str(rank)+"."),tab,c[1],tab,c[0],"\t
       %.4G" % (c[0]/len(message)*100.0))
    rank += 1
if len(count)==26 and N==1 and mode=='Alpha Only':
   print("Pangram!!!")
```

Transposition Tool Cell

```
@interact
def _(message=input_box('EUS HJY TXZ UPE ISE QML COP BEN KVI
   WHO RRG OTD FL NEG
   OA',label="Message",type=str,width=50,height=3),
        key_order_str=input_box('1,2,3,4,5,6,7,8,9,10,11,12,13,14,15',label="Column
           Order",type=str,width=50,height=2)):
    message_blocks = message.split()
    key_order = list(map(int,key_order_str.split(',')))
    print(message_blocks)
    print(key_order)
    for i in range(len(max(message_blocks, key=len))+1):
        row=''+str(i)
        for j in key_order:
            row += ' '
            if i == 0:
                row += str(j)
            else:
                if len(str(j))>1:
                    row += '
                try:
                    row += message_blocks[j-1][i-1]
                except:
                    row += ' '
        print(row)
```

Subset Analysis Cell

```
@interact
def subset_count(
```

```
text=input_box('Place the text that you
                    would like to analyze here !!! ',
                label="Enter your message", height=3,
                    width=50, type=str),
                Subsets=[1..10]):
message =
   AlphabeticStrings().encoding(str(text.encode('ascii', 'replace').decode()))
S = Subsets
sets = [""]*S
for i in range(0,len(message)):
    sets[i%S] += str(message[i])
for i in range(S):
    tmp_message = sets[i]
    count = \{\}
    for i in range(len(tmp_message)):
        c = tmp_message[i:i+1]
        if c in count: count[c] += 1
        else: count[c] = 1
    print("The length of this subset is
       {}".format(len(tmp_message)))
    count = sorted([(value, key) for key, value in
       count.items()])
    print("Character\tCount\t\tPercent")
    count.reverse()
    IC = 0
    for c in count[0:min(30,len(count))]:
        if len(c[1])>6:
            tab="∖t"
        else:
            tab="\t\t"
        print(c[1],tab,c[0],"\t\t %.4G" %
            (c[0]/len(tmp_message)*100.0))
        IC += c[0]*(c[0]-1.0)/len(tmp_message)^2
    print("And, the index of coincidence for this set is
       %.4G\n" % (IC))
```

# Appendix B

# Sage Cell Cipher Utilities

Substitution Cipher Cell

```
import textwrap
@interact
def _(p =
    input_box('abcdefghijklmnopqrstuvwxyz',label='Plain',
    type=str,width=50,height=1),
            с =
                input_box('ZYXWVUTSRQPONMLKJIHGFEDCBA',label='Cipher',
                type=str,width=50,height=1),
            shift=[0..25],
            mode=selector(['encipher', 'decipher'],
                buttons=True),
            spaces = selector(['yes', 'no'], buttons=True),
            m=input_box('sage', label="Message", height=5,
                width=50, type=str)):
    P = str(p.encode('ascii', 'replace').decode()).upper()
    C = str(c.encode('ascii', 'replace').decode()).upper()
    C = C[shift:]+C[:shift]
    Message =
        str(m.encode('ascii', 'replace').decode()).upper()
    print("\nPlain alphabet: \t", str(P).lower())
    print("Cipher alphabet:\t", str(C))
    if len(C)!=len(P):
        print("Key lengths do not match.")
    else:
        if mode == 'encipher':
            inText = P
            outText = C
        else:
            inText = C.lower()
            outText = P.lower()
            Message = Message.lower()
        output = ""
        for char in Message:
            try:
                position = inText.index(char)
                output += outText[position]
            except:
                if spaces == 'yes': output += char
                else: pass
        print("\nHere is your output:\n")
```

```
if spaces == 'yes':
    print(textwrap.fill(output, 50))
else:
    for i in range(0,len(output),5):
        print(output[i:i+5],)
        if (i+5)%50 == 0: print("\n")
```

Affine Cipher Cell

```
import re
@interact
def _(me=input_box('sage', label="Enter your message",
   height=3, width=50, type=str),m=[1..25], s=[0..25]):
    S = AffineCryptosystem(AlphabeticStrings())
    clean_text =
        S.encoding(str(me.encode('ascii', 'replace').decode()))
    try:
        e = S(m,s)
        cipher_text = e(clean_text)
        print("Your affine enciphering text is\n")
        for i in range(0, len(cipher_text), 5):
            print(cipher_text[i:i+5])
            if (i+5)%50 == 0: print("\n")
    except:
        print("error, likely in your key")
        cipher_text = clean_text
```

Alberti Cipher Formula

```
# constants:
plain_alphabet = "ABCDEFGILMNOPQRSTVXZ1234"
cipher_alphabet = "acegklnprtvz&xysomqihfdb"
U = [plain_alphabet[i] for i in range(len(plain_alphabet))]
L = [cipher_alphabet[i] for i in range(len(cipher_alphabet))]
# interactive:
@interact
def _(Key=("Key Letter",slider(U)),
      index_letter=("Index Letter",slider(L))):
    alpha_len = min(len(plain_alphabet), len(cipher_alphabet))
    angle = 2*pi/alpha_len
    #print Key, index_letter
    text_plot_sum = text("",(0,0))
    radius=1
    index1 = cipher_alphabet.index(index_letter)
    index2 = plain_alphabet.index(Key)
    index = index1-index2
    #print index
    for i in range(alpha_len):
        if cipher_alphabet[i]==index_letter:
            text_plot_sum +=
                text(cipher_alphabet[i],(radius*cos(-angle*i),radius*sin(-angle*i)),rota
            text_plot_sum +=
                text(plain_alphabet[(i-index)%alpha_len],(1.25*radius*cos(-angle*i),1.25
        else:
            text_plot_sum +=
```

```
text(cipher_alphabet[i],(radius*cos(-angle*i), radius*sin(-angle*i)),rota
text_plot_sum +=
    text(plain_alphabet[(i-index)%alpha_len],(1.25*radius*cos(-angle*i),1.25
text_plot_sum.show(axes=False,aspect_ratio=1,figsize=[4,4])
```

Vigenere Cipher Cell

```
import re
import textwrap
@interact
def _(m=input_box('sage', label="Enter your message",
   height=3, width=50, type=str),
        key=input_box('sage', label="Enter your key",
            height=1, width=20, type=str),
        mode = selector(['encipher', 'decipher'],
           buttons=True),
        spaces = selector(['yes', 'no'], buttons=True)):
    plain_alpha = "ABCDEFGHIJKLMNOPQRSTUVWXYZ"
    clean_message =
        str(m.encode('ascii', 'replace').decode()).upper()
    if spaces == 'no':
        clean_message = re.sub('[^A-Z]', '', clean_message)
    cipher_key =
       re.sub('[^A-Z]','', str(key.encode('ascii', 'replace').decode()).upper())
    key_list = [plain_alpha.index(ch) for ch in cipher_key]
    if mode == 'decipher':
        key_list = [-1*k for k in key_list]
    cipher_text = ""
    key_counter = 0
    for ch in clean_message:
        try:
            tmp_pos = plain_alpha.index(ch)
            cipher_pos = (tmp_pos+key_list[key_counter])%26
            cipher_text += plain_alpha[cipher_pos]
            key_counter = (key_counter+1)%len(key_list)
        except:
            if spaces == 'yes':
                cipher_text += ch
    print("\nHere is your output:\n")
    if spaces == 'yes':
        print(textwrap.fill(cipher_text, 42))
    else:
        for i in range(0,len(cipher_text),6):
            print(cipher_text[i:i+6],)
            if (i+6)%42 == 0: print("\n")
```

Falconer Cipher Cell

```
keys=input_grid(1,6,default=["CBA", "CAB",
             "ACB", "BCA", "BAC", ""],
                label="Keys:", to_value=list, type=str),
        chars=[3..5]):
text =
   re.sub('[^A-Z]','', str(message.encode('ascii', 'replace').decode()).upper())
columns = "ABCDE"
key = keys[0]
while "" in key: key.remove("")
message_table = [["" for x in range(chars)] for y in
   range(len(key))]
for i in range(0,len(text),chars):
    row = (i/chars)%len(key)
    for j in range(chars):
        try:
            col = columns.index(key[row][j])
        except:
            col = chars-1 #pass
        trv:
            message_table[row][col] += str(text[i+j])
        except:
            pass
out_message = ""
print("Chracters in text: ",len(text))
print("Cipher Table:")
for k in range(len(key)):
    print("\t",str(key[k][0:chars]),":\t","\t".join(message_table[k]))
    for i in range(chars):
        out_message += str(message_table[k][i])+" "
print("Completed Message:")
#for i in xrange(0,len(out_message),50):
#
    print("\t",out_message[i:min(i+50,len(out_message))].strip())
print(textwrap.fill(out_message, 50))
```

Jefferson's Wheel Cipher

```
# wheels:
disk_0 = list("A0LOVTZSMHNJRUIEWCDQPKBYGXF")
disk_1 = list("AXZTKYMHPESDURFLWIVOGNBQ1JC")
disk_2 = list("AJDWQIYCFXMUSGLBZTH2RVENKPO")
disk_3 = list("AEVDQULIBSJ3HNPFWMYKGXCZTRO")
disk_4 = list("AHUNFRILTG4MD0QPEXSYJKVWBCZ")
disk_5 = list("AEUMDGZRJBQTXC5SLYFPNIVOHKW")
disk_6 = list("A6LOVTZSMHNJRUIEWCDQPKBYGXF")
disk_7 = list("AXZTKYMHPESDURFLWIVOGNBQ7JC")
disk_8 = list("AJDWQIYCFXMUSGLBZTH8RVENKPO")
disk_9 = list("AEVDQULIBSJ9HNPFWMYKGXCZTRO")
disk_A = list("AHUNFRILTGdMDOQPEXSYJKVWBCZ")
disk_B = list("AEGZRJBQTXCoSLYFPNIVOHKUMDW")
# cylinder:
cylinder =
   [disk_0,disk_1,disk_2,disk_3,disk_4,disk_5,disk_6,disk_7,disk_8,disk_9,disk_A,disk_B
# interactive:
@interact(layout={'top': [['wheel0',
    'wheel1', 'wheel2', 'wheel3'],['wheel4',
```

```
'wheel5', 'wheel6', 'wheel7'], ['wheel8', 'wheel9', 'wheelA', 'wheelB']]})
def _(
        wheel0=selector(disk_0,label="0"),
        wheel1=selector(disk_1,label="1"),
        wheel2=selector(disk_2,label="2"),
        wheel3=selector(disk_3,label="3"),
        wheel4=selector(disk_4,label="4"),
        wheel5=selector(disk_5,label="5"),
        wheel6=selector(disk_6,label="6"),
        wheel7=selector(disk_7,label="7"),
        wheel8=selector(disk_8,label="8"),
        wheel9=selector(disk_9,label="9"),
        wheelA=selector(disk_A,label="10"),
        wheelB=selector(disk_B,label="11")
    ):
    args=locals()
    text_plot_sum = text("",(0,0))
    column_index = 3
    for alphabet, wheel in zip(cylinder, sorted(args)):
        character = args[wheel]
        temp_wheel =
            alphabet[alphabet.index(character):]+alphabet[:alphabet.index(character)]
        row_index=0
        for i in range(-3,4):
            column_head = (column_index-3)/2
            text_plot_sum +=
                text(str(column_head),(column_index,1.5),fontsize=16,color="blue")
            if temp_wheel[i]==character:
                text_plot_sum +=
                    text(temp_wheel[i],(column_index,-row_index),fontsize=16,color="red"
            else:
                text_plot_sum +=
                    text(temp_wheel[i],(column_index,-row_inde*),fontsize=16,color="gree
            row_index+=2
        column_index+=2
    text_plot_sum.show(axes=False,aspect_ratio=1,figsize=[5,5])
```

Hill Cipher Cell

```
@interact
def _(m=input_box('sage', label="Enter your message",
    height=3,
    width=50, type=str),
    key=input_grid(2,2,default=[[7,12],[3,3]],type=int,width=2)):
    S = HillCryptosystem(AlphabeticStrings(),2)
    M = MatrixSpace(IntegerModRing(26),2,2)
    e = S(M(key).transpose())
    clean_text =
        S.encoding(str(m.encode('ascii', 'replace').decode()))
    if len(clean_text)%2 == 1:
        clean_text = S.encoding(str(m)+"Z")
    cipher_text = e(clean_text)
    print(str("Your Hill enciphering text is\n"))
    for i in range(0,len(cipher_text),5):
        print(cipher_text[i:i+5],)
        if (i+5)%50 == 0: print("\n")
```

# Appendix C

# Blank Tables, Charts, and Images

Table C.0.1	Monoalphabetic	Substitution	Table
-------------	----------------	--------------	-------

plain	a	b	с	d	e	f	g	h	i
CIPHER									
plain	j	k	1	m	n	о	р	q	r
CIPHER									
plain	s	t	u	v	w	x	У	z	
CIPHER									

Table C.0.2 A Numerical Alphabet

1	2	3	4	5	6	7	8	9
a	b	с	d	е	f	g	h	i
10	20	30	40	50	60	70	80	90
j	k	1	m	n	0	р	q	r
100	200	300	400	500	600	700	800	900
s	t	u	v	W	х	у	z	



Figure C.0.3 Axes for Mapping Letter Frequencies



Figure C.0.4 Blank Tables for Frequency Counts



Figure C.0.5 Alberti's Cipher Disk or "Formula"



Figure C.0.6 Modern Cipher Disk or "Formula"  $\,$ 

	11		0	P	Q	R	SI	T	V	X	A	B	C	D	E	F	G	H	I	L	M	N
			E	F	G	H		L	M	N	0	2	2	R	5	T	V	X	11	B	C	D
0	1	E	a  1	6	c	d	e	f	g	h	<b> i</b>	1	m	n	0	P	q	r	<b> </b> ſ	t	v	x
P		F	6	c	d	e	f	g	[h	i	11	m	n	0	P	9	r	1	t.	v	x	a
Q		G	c	d	c	f	g	h	i	11	m	n	0	P	9	r	16	1	V	x	a	b
R		H	d	c	f	19	h	1	1	m	n	0	(P	9	r	1	t	V	X	a	6	C
S		L	e	t	g	h	1	11	m	n	0	P	9	r	1	1	V	X	a	6	C	d
T		L	<u>t</u>	g	Ih I:	1	1	m	n	0	P	9	r lc	1	C	IV Iv	X	a IL	D	C	d	C
V		14	0.0	-	12	1-	10	10	10	1P	IT.		1	IV.	v	IX In	1a Ib	10	Id	la	IF	IT In
X		N		1 .	Im	In		0	IP IQ	14	1	t	IV.	IV.	12	1a Ib		Id	le	IC.		15 h
A		0		m	In	10		In	Ir Ir	1	t	v	x	12	lb		Id	le	If	10	15 1h	11
	-   -	P	m	n	10	lp	la	r	16	t	V	łx	la	16	C	Id	c	If	10	h	li	11
	i.	Q	n	0	p	q	r	11	t	V	x	a	6	C	d	le	f	g	h	i	110	m
E	15	S	0	p	9	r	1	t	V	x	a .	b	Ic.	d	c.	İf	g	.h	i	1	m	n
F	-	T	PI	q	r	11	t	V	x	a	Ь	C	d	e	f	g	h	i	11	m	In	0
G		V	9	r	1	t	V	x	la	6	С	d	e	f	g	h	i	1	m	n	0	P
H	-	X	r	ſ	lt	v	x	a	b	c	d	e	f	lg	h	i	11	m	n	0	1p	9
I		1	<u>f</u>	t	V	·X	a	ļþ	C	d	e	f_	g	h	li	1	m	n	0	P	9	r
L		B	T	V	X	a	6	C	d	e	It	g	h	1	1	m	n	0	P	9	r	{
M		C	V	X	a	10	C.	d	e	11	g	h	1	1	m	In	0	P	q	r	1 1	t
IN	1	D		a	D	C	d	IC.	11	g	In	1	11	Im	n n	0	IP	19	r	1	E .	V.

Figure C.0.7 Vigenère's Tableau

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z A A B C D E F G H I J K L M N O P Q R S T U V W X Y Z B B C D E F G H I J K L M N O P Q R S T U V W X Y Z A C C D E F G H I J K L M N O P Q R S T U V W X Y Z A B D D E F G H I J K L M N O P Q R S T U V W X Y Z A B C E F G H I J K L M N O P Q R S T U V W X Y Z A B C D F F G H I J K L M N O P Q R S T U V W X Y Z A B C D E G G H I J K L M N O P Q R S T U V W X Y Z A B C D E F H H I J K L M N O P Q R S T U V W X Y Z A B C D E F G I J K L M N O P Q R S T U V W X Y Z A B C D E F G H Ι J J K L M N O P Q R S T U V W X Y Z A B C D E F G H I K K L M N O P Q R S T U V W X Y Z A B C D E F G H I J L L M N O P Q R S T U V W X Y Z A B C D E F G H I J K M M N O P Q R S T U V W X Y Z A B C D E F G H I J K L N N O P Q R S T U V W X Y Z A B C D E F G H I J K L M O O P Q R S T U V W X Y Z A B C D E F G H I J K L M N P P Q R S T U V W X Y Z A B C D E F G H I J K L M N O Q Q R S T U V W X Y Z A B C D E F G H I J K L M N O P R R S T U V W X Y Z A B C D E F G H I J K L M N O P Q S T U V W X Y Z A B C D E F G H I J K L M N O P Q R ST T U V W X Y Z A B C D E F G H I J K L M N O P Q R S U U V W X Y Z A B C D E F G H I J K L M N O P Q R S T V V W X Y Z A B C D E F G H I J K L M N O P Q R S T U WWXYZABCDEFGHIJKLMNOPQRSTUV X X Y Z A B C D E F G H I J K L M N O P Q R S T U V W YYZABCDEFGHIJKLMNOPQRSTUVWX Z Z A B C D E F G H I J K L M N O P Q R S T U V W X Y

Figure C.0.8 Updated Vigenère Tableau

1	a	b	c	d	e	f	g	h	i	1
$\overline{B} \parallel$	m	n	0	p	q	r	S	t	u	x
	a	b	C	d	e	f	g	h	i	1
D	x	m	n	0	P	q	r	S	t	u
E	a	b	c	d	e	f	g	h	i	1
$F \parallel$	u	x	m	n	0	P	<u>q</u>	r	S	t
G	a	b	h.c	d	c	f	g	h	i	1
$H \parallel$	t	u	X	m	n	0	P	<b>P</b>	r	S
I	a	b	C	d	e	f	g	h	i	1
	S	tio	u	x	m	n	0	P	<b>q</b>	r
M	a	b	C	d	e	f	g	h	i)	1
$N \mid$	r	S	t	u	x	m	n	0	P	<u>q</u>
0	a	b	C	d	e	f	g	h	i	1
$P \parallel$	q	r	S	t	u	x	m	n	0,	P
2	a	b	c	d	e	f	g	h	i	1
$R \parallel$	р	q	r	S	t	u	x	m	n	0
S	a	b	C	d	e	f	g	h	i	1.
$T \parallel$	0	P	q	r	S	t	u	x	m	n
V	a	b	С	d	e	f	g.	h	i	1
X	n	0	<b>p</b>	q	r	s	t	u	x	m

Figure C.0.9 Vigenère's Autokey Tableau



Figure C.0.10 Pigpen Cipher Key

	А	В	С
1			
2			
3			
4			
5			
6			

 Table C.0.11 Falconer's Transposition Table

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 12.22% eW X Y Z A B C D E F G H I J K L M N O P Q R S T U V 9.28% t H I J K L M N O P Q R S T U V W X Y Z A B C D E F G 8.06% a A B C D E F G H I J K L M N O P Q R S T U V W X Y Z 7.62% o M N O P Q R S T U V W X Y Z A B C D E F G H I J K L 7.10% i STUVWXYZABCDEFGHIJKLMNOPQR 6.82% n N O P Q R S T U V W X Y Z A B C D E F G H I J K L M 6 45% I J K L M N O P Q R S T U V W X Y Z A B C D E F G H 5.91% r J K L M N O P Q R S T U V W X Y Z A B C D E F G H I 5.76%  $h \mid T \cup V \cup X Y Z A B C D E F G H I J K L M N O P Q R S$ 4.19% *l P Q R S T U V W X Y Z A B C D E F G H I J K L M N O* d X Y Z A B C D E F G H I J K L M N O P Q R S T U V W 3.93%

Figure C.0.12 Kasiski Key Table

*B* 1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 C2 D3 0 3 6 9 12 15 18 21 24 1 4 7 10 13 16 19 22 25 2 5 8 11 14 17 20 23 E4 0 4 8 12 16 20 24 2 6 10 14 18 22 0 4 8 12 16 20 24 2 6 10 14 18 22  $F \quad 5 \quad 0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \quad 4 \quad 9 \quad 14 \quad 19 \quad 24 \quad 3 \quad 8 \quad 13 \quad 18 \quad 23 \quad 2 \quad 7 \quad 12 \quad 17 \quad 22 \quad 1 \quad 6 \quad 11 \quad 16 \quad 21$ G6 0 6 12 18 24 4 10 16 22 2 8 14 20 0 6 12 18 24 4 10 16 22 2 8 14 20 H 7 0 7 14 21 2 9 16 23 4 11 18 25 6 13 20 1 8 15 22 3 10 17 24 5 12 19 T 0 8 16 24 6 14 22 4 12 20 2 10 18 0 8 16 24 6 14 22 4 12 20 2 10 18 8 J9 0 9 18 1 10 19 2 11 20 3 12 21 4 13 22 5 14 23 6 15 24 7 16 25 8 17  $M \hspace{.1in} 12 \hspace{.1in} 0 \hspace{.1in} 12 \hspace{.1in} 24 \hspace{.1in} 10 \hspace{.1in} 22 \hspace{.1in} 8 \hspace{.1in} 20 \hspace{.1in} 6 \hspace{.1in} 18 \hspace{.1in} 4 \hspace{.1in} 16 \hspace{.1in} 2 \hspace{.1in} 14 \hspace{.1in} 0 \hspace{.1in} 12 \hspace{.1in} 24 \hspace{.1in} 10 \hspace{.1in} 22 \hspace{.1in} 8 \hspace{.1in} 20 \hspace{.1in} 6 \hspace{.1in} 18 \hspace{.1in} 4 \hspace{.1in} 16 \hspace{.1in} 2 \hspace{.1in} 14 \hspace{.1in} 14 \hspace{.1in} 12 \hspace{.1in$  $N \quad 13 \quad 0  14 0 14 2 16 4 18 6 20 8 22 10 24 12 0 14 2 16 4 18 6 20 8 22 10 24 12 0 15 0 15 4 19 8 23 12 1 16 5 20 9 24 13 2 17 6 21 10 25 14 3 18 7 22 11 P $Q \hspace{.1cm} 16 \hspace{.1cm} 0 \hspace{.1cm} 16 \hspace{.1cm} 6 \hspace{.1cm} 22 \hspace{.1cm} 12 \hspace{.1cm} 2 \hspace{.1cm} 18 \hspace{.1cm} 8 \hspace{.1cm} 24 \hspace{.1cm} 14 \hspace{.1cm} 4 \hspace{.1cm} 20 \hspace{.1cm} 10 \hspace{.1cm} 0 \hspace{.1cm} 16 \hspace{.1cm} 6 \hspace{.1cm} 22 \hspace{.1cm} 12 \hspace{.1cm} 2 \hspace{.1cm} 18 \hspace{.1cm} 8 \hspace{.1cm} 24 \hspace{.1cm} 14 \hspace{.1cm} 4 \hspace{.1cm} 20 \hspace{.1cm} 10 \hspace{.1cm$  $R \quad 17 \quad 0 \quad 17 \quad 8 \quad 25 \quad 16 \quad 7 \quad 24 \quad 15 \quad 6 \quad 23 \quad 14 \quad 5 \quad 22 \quad 13 \quad 4 \quad 21 \quad 12 \quad 3 \quad 20 \quad 11 \quad 2 \quad 19 \quad 10 \quad 1 \quad 18 \quad 9 \\$ S 18 0 18 10 2 20 12 4 22 14 6 24 16 8 0 18 10 2 20 12 4 22 14 6 24 16 8  $T \quad 19 \quad 0 \quad 19 \quad 12 \quad 5 \quad 24 \quad 17 \quad 10 \quad 3 \quad 22 \quad 15 \quad 8 \quad 1 \quad 20 \quad 13 \quad 6 \quad 25 \quad 18 \quad 11 \quad 4 \quad 23 \quad 16 \quad 9 \quad 2 \quad 21 \quad 14 \quad 7$  $U \quad 20 \quad 0 \quad 20 \quad 14 \quad 8 \quad 2 \quad 22 \quad 16 \quad 10 \quad 4 \quad 24 \quad 18 \quad 12 \quad 6 \quad 0 \quad 20 \quad 14 \quad 8 \quad 2 \quad 22 \quad 16 \quad 10 \quad 4 \quad 24 \quad 18 \quad 12 \quad 6$ V21 0 21 16 11 6 1 22 17 12 7 2 23 18 13 8 3 24 19 14 9 4 25 20 15 10 5 W22 0 22 18 14 10 6 2 24 20 16 12 8 4 0 22 18 14 10 6 2 24 20 16 12 8 4 X 23 0 23 20 17 14 11 8 5 2 25 22 19 16 13 10 7 4 1 24 21 18 15 12 9 6 3 24 0 24 22 20 18 16 14 12 10 8 6 4 2 0 24 22 20 18 16 14 12 10 8 6 4 2 Z 25 0 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Figure C.0.13 Multiplication table modulo 26

Two Round rSDES (really Simple DES)



Figure C.0.14 A Highly Simplified two step DES

CHARACTER	MORSE CODE	TELEPHONY	PHONIC (PRONUNCIATION)
A	•-	Alfa	(AL-FAH)
В	-•••	Bravo	(BRAH-VOH)
С	-•-•	Charlie	(CHAR-LEE) or (SHAR-LEE)
D	-••	Delta	(DELL-TAH)
Е	•	Echo	(ECK-OH)
F	••-•	Foxtrot	(FOKS-TROT)
G	•	Golf	(GOLF)
Н	••••	Hotel	(HOH-TEL)
I	••	India	(IN-DEE-AH)
J	•	Juliett	(JEW-LEE-ETT)
K	-•-	Kilo	(KEY-LOH)
L	•-••	Lima	(LEE-MAH)
М		Mike	(MIKE)
Ν	-•	November	(NO-VEM-BER)
0		Oscar	(OSS-CAH)
Р	••	Papa	(PAH-PAH)
Q	•-	Quebec	(KEH-BECK)
R	•-•	Romeo	(ROW-ME-OH)
S	•••	Sierra	(SEE-AIR-RAH)
Т	-	Tango	(TANG-GO)
U	••-	Uniform	(YOU-NEE-FORM) or (OO-NEE-FORM)
V	•••-	Victor	(VIK-TAH)
W	•	Whiskey	(WISS-KEY)
Х	-••-	Xray	(ECKS-RAY)
Y	- •	Yankee	(YANG-KEY)
Z	••	Zulu	(ZOO-LOO)
1	•	One	(WUN)
2	••	Two	(TOO)
3	•••	Three	(TREE)
4	•••-	Four	(FOW-ER)
5	• • • • •	Five	(FIFE)
6	- • • • •	Six	(SIX)
7	••	Seven	(SEV-EN)
8	•	Eight	(AIT)
9		Nine	(NIN-ER)
0		Zero	(ZEE-RO)

 ${\bf Figure~C.0.15~NATO~Phonetic~Alphabet~(from~Wikimedia~Commons)}$ 

# Appendix D

# **Quotations** Appendix

# D.1 Chapter 1 Quotes:

Julius Caesar Shift: Augustus Caesar Shift:

"When he had occasion to write in cypher he put b for a, c for b [and] so forth and instead of z, aa." [15, p. 134]

Falconer on 26!:

Schottus demonstrates, (though the calculation in his book be not exact) that a thousand million of men in as many years could not write down all those different transpositions of the alphabet, granting every one should complete forty pages a day, and every page contain forty several positions: For if one writer in one day write forty pages, everyone containing forty combinations, 40 multiplied by 40, gives 1,600, the number he completes in one day, which multiplied by 366, the number (and more) of days in a year; a writer in one year shall compass 585,600 distinct rows. Therefore in a thousand million of years he could write

585,600,000,000,000,

which being multiplied by 1,000,000,000, the number of writers supposed, the product will be

585,600,000,000,000,000,000,000,

which wants of the number of combinations no less than

348, 484, 017, 332, 394, 393, 600, 000.

[4, pages 5-6]

Ibn ad-Durayhim on Numerical Ciphers:

#### "5. On the replacement of letters using the decimallyweighted numerical alphabet:

- By substituting decimal numerical alphabet for letters in four different ways: by writing the numbers in words as pronounced; or by finger-bending, using the fingers to communicate the message visually to a recipient; or by writing the numbers as numerals such as writing (mhmd: forty, eight, forty, four); or by giving the cryptogram a semblance of a page of a financial register.
- By recovering the cryptogram numeral into a number of letters - a method of encipherment which involves more sophistication. There are many combinations that can be used in this method; for example in (mhmd: jl, fb, jl, ca) or (kk, ga, kk, bb). One can even form delusive words such as (mhmd: lead, cad, deal, baa), or substitute two words for a letter, e.g. (ali: dig fad, cab ab), in which case a line is to be drawn over two words to denote that they represent one letter.
- By multiplying the number representing the letter by two, and so write (mhmd: q, jf, q, h) and (ali: ob, jh), etc; or multiply it by three, thus writing (mhmd: sk, kd, sk, jb) and (ali: rc, kg). Numbers can also be multiplied by four or five." <sup>1</sup> [12, vol. 3, pp. 69-70]

## D.2 Chapter 2 Quotes:

Al-Kindi on Frequency Analysis:

#### "Algorithms of Cryptanalysis

So we say, the enciphered letters are either in numerical proportions, that is poetry -because poetic meter, ipso facto, sets measures to the number of letters in each line-, or they are not. Non- poetry can be cryptanalyzed using either quantitative or qualitative expedients.

The *quantitative* expedients include determining the most frequently occurring letters in the language in which cryptograms are to be cryptanalyzed. If vowels functioned as the material from which any language is made, and non-vowels functioned as the shape of any language, and since many shapes can be made from the same material, then the number of vowels in any language would be greater than non-vowels. For instance, gold is the material of many shapes of finery and vessels; it may cover crowns, bangles, cups, etc.. The gold in these realizations is more than the shapes made

 $<sup>^{1}</sup>$ The examples here are very loosely based on the Arabic examples in the translation. The "mhmd" is Mohamed since Arabic is written without vowels, and for "ali" the a and l together are treated as a single letter.

of it. Similarly, the vowels which are the material of any kind of text are more than the non- vowels in any language. I mean by vowels the letters: (a), (y or i or e) and (o or u). Therefore the vowels in any language, inevitably, exceed in number the non-vowels in a text of that language. It happens that in certain languages some vowels are greater in number than some other vowels, while non-vowels may be frequent or scarce according to their usage in each language, such as the letter (s), of which frequency of occurrence is high in Latin.

Among the expedients we use in cryptanalyzing a cryptogram if the language is already known, is to acquire a fairly long plaintext in that language, and count the number of each of its letters. We mark the most frequent letter "first", the second most frequent "second", and the following one "third", and so forth until we have covered all its letters. Then we go back to the message we want to cryptanalyze, and classify the different symbols, searching for the most frequent symbol of the cryptogram and we regard it as being the same letter we have marked "first" -in the plaintext-; then we go to the second frequent letter and consider it as being the same letter we have termed "second", and the following one "third", and so on until we exhaust all the symbols used in this cryptogram sought for cryptanalysis.

It could happen sometimes that short cryptograms are encountered, too short to contain all the symbols of the alphabet, and where the order of letter frequency cannot be applied. Indeed the order of letter frequency can normally be applied in long texts, where the scarcity of letters in one part of the text is compensated for by their abundance in another part.

Consequently, if the cryptogram was short, then the correlation between the order of letter frequency in it and in that of the language would no longer be reliable, and thereupon you should use another, *qualitative* expedient in cryptanalyzing the letters. It is to detect in the language in which cryptograms are enciphered the associable letters and the dissociable ones. When you discern two of them using the letter order of frequency, you see whether they are associable in that language. If so, you seek each of them elsewhere in the cryptogram, comparing it with the preceding and following dissociable letters by educing from the order of frequency of letters, so as to see whether they are combinable or non-combinable. If you find that all these letters are combinable with that letter, you look for letters combinable with the second letter. If found really combinable, so they are the expected letters suggested by the combination and non-combination of letters, and also by their order of frequency. Those expected letters are correlated with words that make sense. The same procedure is repeated elsewhere in the ciphertext until the whole message is cryptanalyzed."

#### [12, vol. 1, pp. 121-123]

#### Falconer's Analysis Step 1-4:

1.	And first, the vowels generally discover themselves by their frequency; for because they are but few in number and no word made up without some of them, they must frequently be used in any writing.
2.	Where you find any character or letter standing by it self, it must be a vowel.
3.	If you find any character doubled in the beginning of a word, in any language it is a vowel, as <i>Aaron, Eel, Jilt</i> , <i>Oogala, Vulture, etc.</i> , except for some English proper names, as Llandaff or Lloyd.
4.	In monosyllables of two letters you may distinguish it from the consonant joined with it by its frequency.
5.	In a word of three letters beginning and ending in the same letter the vowel is probably included.
6.	When you find a character doubled in the middle of a word of four letters, `tis probably the vowel $e$ or $o$ .
7.	In Polysyllables, where a character is double in the middle of the word, it is for the most part a consonant; and if so, the precedent letter is always a vowel, and very often the following.[4, pp. 8-9]

# 1. Compare their frequency, and *e*, as we observed before, is generally the most used in the English tongue, next *o*, then *a* and *i*; but *u* and *y* are not so frequently used as some of the consonants.

- 2. It is remarkable that amongst the vowels, e and o are often doubled, the rest seldom or never.
- 3. *e* is very often a terminal letter, and *y* terminates words, but they are distinguishable, because there is no proportion to their frequency: *o* is not often in the end of words, except in monosyllables.
- 4. *e* is the only vowel that can be doubled in the end of an English word, except *o* in *too*, etc.
- 5. You may consider which of the vowels, in any language, can stand alone, as *a*, *i*, and sometimes *o* in English, *a*, *e*, *o*, in Latin or *i* the imperative of *eo*.[4, pp. 9-10]

#### Distinguish the Consonants Amongst Themselves..

- 1. As before observe their frequency. Those of most use in English are d, h, n, r, s, t, and next to those may be reckoned c, f, g, l, m, w, in third rank may be placed b, k, p, and lastly q, x, z...
- 2. You may consider which consonants may be doubled in the middle or end of words.
- 3. What are terminal letters, etc.
- 4. The number and nature of consonants and vowels that fall together, or do usually fall together. [4, p. 10]

#### $Additional \ Observations.$

- 1. A word of three letters, beginning and ending with the same, may be supposed *did*
- 2. A word consisting of four characters, with the same letter in the beginning and end, is probably *that* or *hath*
- 3. A word consisting of five letters, when the second and last are the same, is commonly *which*, though it may be otherways, as in *known*, *serve*, etc. And you may judge of the truth of such suppositions by the frequency of the letters in the word supposed.

Next you may compare words one with another, as *on* and *no*, each being the other reversed; so *of* and *for*, the last being the first reversed with the addition of a letter; for and from will discover each other, etc.

You may also likewise observe some of the usual propositions and terminations of words, such as *com*, *con*, *ing*, *ed*, etc. Note that t and h are often joined in the beginning and end of English words, and sometimes in the middle.[4, pp. 11-12]

## D.3 Chapter 3 Quotes:

#### Alberti's Polyalphabetic Cipher:

"Say for example we mutually establish k as the index of the mobile circle. Writing, the formulae are positioned at will, say such k lies under the uppercase B [Figure 3.1.2] and the next letter corresponds to the letter that comes next. In writing to you, I will first put the uppercase B under which lies the index k in the formula; this is to signal you far away, wanting to read what I have written, that you should set up the twin formula in your keeping, positioning the mobile circle so that the B sits over the index k. Then all of the rest of the lowercase letters present in the coded text will take their meaning and sound from those of the fixed circle above them."

"After I have written three or four words I will mutate the position of the index in our formula, rotating the disk let's say, so that the index k falls below the upper case R. Then in this missive I write an uppercase R to indicate that k no longer refers to B, but to R, and the letters that will follow will assume new meanings." - Alberti [17, p. 181]

Vigenère's Polyalphabetic Cipher:

"... set the capitals which run across the top [of the table] for the message to be conveyed & those that run perpendicularly down the left for the keys. I have put two rows of capitals here, one black and the other red, to show that alphabets of the text as, the keys, may be transposed and changed in as many ways as you want to keep knowledge of them from all others except ones correspondents. Therefore, to encipher the saying we used previously "au nom de l'eternel," with the key "le jour obscur" proceed in this way; from a in the alphabet across the top in red, come to the row with l and the box with b: u from e will be q: n from i, n: o from o, s: m from u, a: d from r, m: e from o, i: l from b, c: e from s, o: t from c, n: e from u, q: r from r, c: n from l, o: e from e, a: l from i, l. Altogether we get bgnsamiconcoal & so on for the remainder." [16, pp. 49b, 50, 50b (100-102)], [11, p. 110]<sup>1</sup>

 $<sup>^{1}</sup>$ Most of this translation is due to Mendelsohn [11], I have only filled in with my own at the end which he had not translated.

1=	1	•	OPQRSTVXABCDEFGHIILMN
11			EFGHILMNOPQRSTVXABCD
	ō	Ë	a  b  c  d  c  f  g  h  i  1  m  n  o  p  q  r  f  t  v  x
	P	F	$\frac{ \mathbf{b} \mathbf{c} \mathbf{d} \mathbf{e} \mathbf{f} \mathbf{g} \mathbf{h} \mathbf{i} \mathbf{l} \mathbf{m} \mathbf{n} \mathbf{o} \mathbf{p} \mathbf{q} \mathbf{r} \mathbf{l} \mathbf{t} \mathbf{v} \mathbf{x} \mathbf{a} $
1	2	G	c d c f g h i l m n o p q t f t v x a b
	R	H	d   c   f   g   h   i   l   m   n   o   p   q   r   l   t   v   x   a   b   c
	S	L	$\begin{bmatrix} e & f & g & h & i \end{bmatrix} \begin{bmatrix} m & n & o & p & q & r \end{bmatrix} \begin{bmatrix} t & v & x & a & b & c & d \end{bmatrix}$
-	Γ	L	f   g   h   1   l   m   n   o   p   q   r   1   t   v   x   a   b   c   d   c
	V	A	
-	X	N	
	A	0	
-	B	P	
		2	$\frac{ \mathbf{n}  \mathbf{n}  0  \mathbf{p}  \mathbf{q}  1  1  \mathbf{v}  \mathbf{x}  \mathbf{a}  0  \mathbf{c}  \mathbf{q}  \mathbf{c}  1  \mathbf{g}  1  1  \mathbf{n}  \mathbf$
		R	$   \mathbf{p}    \mathbf{p}    \mathbf{q}    \mathbf{r}    \mathbf$
	F	S	$\frac{ \mathbf{r} }{ \mathbf{r}  } = \frac{ \mathbf{r}$
	G	T	q r f t v x a b c d c f g h i  m n o p
-	H	V	$  \mathbf{r}  (\mathbf{t}   \mathbf{v}   \mathbf{x}   \mathbf{a}   \mathbf{b}   \mathbf{c}   \mathbf{d}   \mathbf{e}   \mathbf{f}   \mathbf{g}   \mathbf{h}   \mathbf{i}   \mathbf{l}   \mathbf{m}   \mathbf{n}   \mathbf{o}   \mathbf{p}   \mathbf{q} $
	I		f   t   v   x   a   b   c   d   c   f   g   h   i   l   m   n   o   p   q   r
-	L	B	r  v  x  a  b  c  d  e  f  g  h  i (l  m  n  o  p  q  r  f
	M	C	$ \mathbf{v}  \mathbf{x}  \mathbf{a}  \mathbf{b}  \mathbf{c}  \mathbf{d}  \mathbf{c}  \mathbf{f}  \mathbf{g}  \mathbf{h}  \mathbf{i}  \mathbf{l}  \mathbf{m}  \mathbf{n}  \mathbf{o}  \mathbf{p}  \mathbf{q}  \mathbf{r}^*  \mathbf{f}  \mathbf{r}$
	N	D	$\ x \ _{a} \ b \ c \ d \ c \ f \ g \ h \ i \ l \ m n  o  p  q r  f  r  v $

Figure D.3.1 Vigenère's Tableau

Agripa's Pigpen Cipher





Vigenère's Autokey Cipher

"Each letter may be enciphered by the preceding letter, thus: with the . . . text. . . . "Au nom de l'eternel" and the key D, we say, a [the first letter of the text] from D [the key letter] gives x; u [the second letter of the text] from A [the first letter of the text, which now becomes the key] gives i; n from U, a; o from N, h; m from O, g; d from M, u; e from D, p; l from E, t; e from L; m, t from E, l; e from T, s; r from E, h; n from R, i; e from N, x; l from E, t. By which method, taking D for the key, we would arrive at DXIAHGUPTMLSHIXT. The other method, which is more secret, is to encipher each letter of the clear text not by the letter [of that text] which precedes it, but by the letter by which the preceding letter is enciphered. Thus, a from D, which is the key, gives x; u from X [the first letter of the cipher text] h; n from H, e; o from E, e; m from E, c; d from C, o; e from O, u; l from U, m; ... etc." [16, pp. 49-49b] [11, p. 128]

<sup>&</sup>lt;sup>2</sup>The translation is based, for the most part, on [2] with reference to the original text. The example was changed some in order to make the work more accessible.`

1	a	b	c	d	e	f	g	h	i	1
$B \parallel$	m	n	0	p	q	r	S	t	u	x
C	a	Ь	c	d	e	f	g	h	i	1
D	x	m	n	0	P	q	r	S	t	u
E	a	b	C	d	e	f	g	h	i	1
$F \parallel$	u	x	m	n	0	P	<u>q</u>	r	S	t
G	a	b	₩.C	d	c	f	g	h	i	1
$H \parallel$	t	u	X	m	n	0	P	<b>P</b>	r	S
I	a	b	C	d	e	f	g	h	i	1
	S	1	u	x	m	n	0	P	q	r
M	a	Ь	c	d	e	f	g	h	i)	1
$N \parallel$	r	S	t	u	x	m	n	0	P_	<u>P</u>
	State Street 1			1 1	1	If	la	1 h	1 ;	11
0	a	Ь	C	d	e	1.1	18	+ 11	1	<u>                                     </u>
$\begin{array}{c c} O \\ \hline P \\ \hline \end{array}$	a 	r b	C S	d t	u e		<u>m</u>	n		<u>p</u>
	a <b>q</b> a	b r b	C   S   C	d t d	e   u   e	1   x   f	<u>g</u>	n   h	1   0   i	
0             	a 9 a 9	b r b q	C S C I	d t d s	c   u   c   t	I   x   f   u	g   g   x	n   n   h	1   0   i   n	p   1   1   0
0 P 2 R S	a q a p a	b   r   b   b	C S C I C	d t d s d	e   u   e t   e	f f f	m   g   x	n   n   h   m   h	1   0   i   1   1	P   1   0   1
0 P    Q    R    S    T	a 9 a 9 a 0	b r b q b p	c       s       c       r       c       q	d t d s d r	e   u   e t   e s	I     x     f     u     f     t	g   g   g   g   g   u	n   h   m   h x	1   0   1   1   1   1   1	P   1   0   1   n
0 P 2 R S T V	a q a p a · o a	b r b q b p b	c       s       c       r       c       q       c	d t d s d r	e u e t s e	I     x     f     u     f     t     f	m   g   x   g   u   g.	n   h   m   h x h	1   0   i   1   1   1   1	p       l       o       l       n       l

Figure D.3.2 Table used for Vigenère's Autokey Cipher

# D.4 Chapter 6 Quotes:

Hill's	Cryptograph	iy in d	an Algebraic	Alphabet:
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#### Cryptography in an Algebraic Alphabet.

By Lester S. Hill, Hunter College

1. The Bi-Operational Alphabet

Let  $a_0, a_1, \ldots, a_{25}$  denote any permutation of the letters of the English alphabet; and let us associate the letter  $a_i$  with the integer *i*. We define operations of *modular addition* and *multiplication* (modulo 26) over the alphabet as follows:

$$a_i + a_j = a_r$$

 $a_i a_j = a_t,$ 

where r is the remainder obtained upon dividing the integer i+j by the integer 26 and t is the reaminder obtained on dividing ij by 26. The integers i and j may be the same or different.

It is easy to verify the following salient propositions concerning the bi-operational alphabet thus set up:

(1) If  $\alpha$ ,  $\beta$ ,  $\gamma$  are letters of the alphabet,

•  $\alpha + \beta = \beta + \alpha$  and  $\alpha \beta = \beta \alpha$  [commutative law]

•  $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$  and  $\alpha(\beta\gamma) = (\alpha\beta)\gamma$  [associative law]

•  $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$  [distributive law]

(2) There is exactly one "zero" letter, namely  $a_0$ , characterized by the fact that the equation  $\alpha + a_0 = \alpha$  is satisfied whatever the letter denoted by *alpha*.

(3) Given any letter  $\alpha$ , we can find exactly one letter  $\beta$ , dependent on  $\alpha$ , such that  $\alpha + \beta = a_0$ . We call  $\beta$  the "negative" of  $\alpha$ , and we write:  $\beta = -\alpha$ .

(4) Given any letters  $\alpha$ ,  $\beta$  we can find exactly on letter  $\gamma$  such that  $\alpha + \gamma = \beta$  [i.e.  $\gamma = \beta - \alpha$  is unique].

(5) Distinguishing the twelve letters,

 $a_1, a_3, a_5, a_7, a_9, a_{11}, a_{15}, a_{17}, a_{19}, a_{21}, a_{23}, a_{25},$ 

with subscripts prime to 26, as "primary" letters, we make the assertion, easily proved: If  $\alpha$  is any primary letter and  $\beta$  is any letter, there is exactly one letter  $\gamma$  for which  $\alpha \gamma = \beta$ .

(6) In any algebraic sum of terms, we may clearly omit terms of which the letter  $a_0$  is a factor; and we need not write the letter  $a_1$  explicitly as a factor in any product.

2. An Illustration

Let the letters of the alphabet be associated with the integers as follows:or, in another convenient formulation:

0	1	2	3	4	5	6	7	8	9	10	11	12
k	р	с	0	h	a	r	n	g	$\mathbf{z}$	е	у	S
13	14	15	16	17	18	19	20	21	22	23	24	25
m	w	f	1	v	i	q	d	u	x	b	$\mathbf{t}$	j

It will be seen that

The so t + v = f.

$$c + x = t, \ j + w = m, \ f + y = k, \ -f = y, \ -y = f, \ etc.$$
  
 $an = z, \ hm = k, \ cr = s, \ etc.$ 

The zero letter is k, and the unit letter is p. The primary letters are:  $a \ b \ f \ j \ n \ o \ p \ q \ u \ v \ y \ z$ .

Since this particular alphabet will be used several times, in illustration of further developments, we append the following table of negatives and reciprocals:

	Letter	:	a	b	$\mathbf{c}$	d	е	f	g	h	i		k	1	m	n	0	р	$\mathbf{q}$
	Neg.	:	u	0	$\mathbf{t}$	r	1	у	i	х	g	þ	k	е	$\mathbf{m}$	$\mathbf{q}$	b	j	n
	Rec.	:	u	v				n								f	$\mathbf{Z}$	р	у
lution to the equation $z + \alpha = t$ is $\alpha = t - z$ or $\alpha = t + (-z) =$																			
stem of linear	tem of linear equations: $o \alpha + u \beta = x$ , $n \alpha + i \beta = q$ has																		

The system of linear equations:  $o \alpha + u \beta = x$ ,  $n \alpha + i \beta = q$  has solution  $\alpha = u$ ,  $\beta = o$ , which may be obtained by the familiar method of elimination or by formula. [5, pp.306-308]

# Appendix E

# Sample Analysis Appendix

## E.1 Monoalphabetic Analysis

Let us see if we can determine what is happening in this message:

VGHTHTCMYHOGCRCBWTHMFAJCUZNRBYHOGCRVGHTYHOGCRZNRJTSWUDHRTVZRHVHMFNWVAJCUZNRBBCKCVHMFAMUBWOKHYAVCKCVVCRTAMBVGCMZRHVHMFBNZMVGCRCTVNDVGCAKOGASCVHMNRBCRVGHTARRAMFCLCMVNDVGCAKOGASCVVGCMDNRLTUNWRYHOGCRAKOGASCVVGHTYHOGCRAMBVGCNVGCRLNMNAKOGASCVHYHOGCRTZCZHKKTVWBUGATATVRWYWRCVGAVZCYAMCQOKNHVVNPRAYJHVAMBDHFWRCNWVVGCLCTTAFCGNZCXCRVGHTHTMNVHMFCMCRAKVRWCVGCRCARCVCYGMHYAKKUVZCMVUTHQDAYVNRAABBCOCMBHMFNMGNZKCZKTTAFCGNZCXCRVGHTHTMNVHMFCMCRAKVRWCVGCRCARCVCYGMHYAKKUVZCMVUTHQDAYVNRAABBCOCMBHMFNMGNZ<td

We begin with a basic frequency analysis (see Figure E.1.1 and Figure E.1.2), counting how many times each character appears, and find that "C", "V", and "H" are most common. This leads us to guess that C=e, V=t, and H=a since "e", "t", and "a" are the three most common letters in English.

Next we look for repeated strings of characters. We find, among others, that "VGC" appeared very frequently. Based on what we did before this would be "t\_e" and so it appears that G=h.

The next step is a bigram analysis, that is we look at two letter combinations. We find that "CR"="e\_" and "RC"="\_e" are very common and comparing this to a table of common bigrams we guess that R=r.

We also find that "VH" which is "ta" was more common than it should be and thus one of our previous letter assignments is perhaps wrong. Going back to our repeated strings we find "VGHT" in a couple of places and when we look at common words starting with "th" that could match "VGHT" the only one that really works is "this." Therefore  $H \neq a$ , H=i, and T=s.

Finally, the digraph "CM"="e\_" is very frequent so should represent either "er" or "en." Since, "er"="CR" we conclude that M=n

Going back to repeated strings we find that "HMF"="in\_" appears in a number of places and so it is likely that F=g. Finally, we know "a" should be represented by a common letter and the only really common cipher letter that is not used is "A" so A=a.

At this point we have nine possible substitutions, C=e, V=t, G=h, R=r, H=i, T=s, M=n, F=g, and A=a, which we have some confidence in. Now we need to write out lines of the message filling in what we have and looking for words we can recognize. As a result we find:

- 1. ZRHVHMG = \_\_riting so we get Z=w
- 2.  $AMB = an_{and}$  is a common trigraph so B=d
- 3. WTHMF = \_sing and BWOKHYAVC = d\_pli\_ate so we get W=u and Y=c
- 4. KCVVCR = \_\_etter so K=l
- 5. AKOGASCV = al\_ha\_et and YHOGCR = ci\_her so O=p and S=b
- 6.  $ZNRB = w_rd \text{ so } N=0$
- 7. ARRAMFCLCMV = arrange\_ent so L=m
- 8. ND = o\_ and DNRLT = \_orms so D=f
- 9. AMU = an\_ and UNWR = \_our and TVWBU = stud\_ so U=y
- 10. JCU = \_ey and YRAYJ = crac\_ and ZNRJ = wor\_ so J=k
- 11. CQOKNHV =  $e_{\text{ploit so }}Q=x$

Now the only letters left to figure out are "P" and "X." Looking for "P's" we see that there are only two and they are both followed by "W"="u," so it looks like P=q and this is confirmed by DRCPWCMYU = frequency. Then scanning through what we have for a message we see XCRU = \_\_ery and thus X=v fills in our last unknown letter very nicely.

Final Deciphered Message:

"This is enciphered using a keyword cipher. This cipher works buy first writing out a key word, deleting any duplicate letters, and then writing down the rest of the alphabet in order. This arrangement of the alphabet then forms your cipher alphabet. This cipher and the other monoalphabetic ciphers we will study has a structure that we can exploit to crack it and figure out the message. However this is not in general true. There are technically twenty six factorial different monoalphabetic substitution ciphers and depending on how they are used they could be made unbreakable. In general however monoalphabetic ciphers can be cracked with some simple and very old techniques, in particular with the use of frequency analysis is our first tool."



Figure E.1.1 Sample Keyword Analy sis Page 1:

Figure E.1.2 Sample Keyword Analysis Page 2:

## E.2 Transposition Analysis

Consider this exercise from Exercises 4.4:

Checkpoint E.2.1 Decrypt this transposition cipher with crib WEATHER:HIPYEN TAUTCI WSDWPF ETTEEA ARONRN ENPMNC RTEPTHRERHR EMTCA PPHHI OREOL FAIEL TETCA RNRNF OSIANMYWRR RTNIU CEDNN ASIAO RLSFA IITHU TDENA HASODHYAEQ RNTUA FETRE TTENR RITDO YHNDT IMOER

#### Answer.

weather report for march thirty first rain temps nearly steady in the mid to upper thirties winds e at ten to twenty mph chance of rain one hundred percent rain fall around a quarter of an inch

To crack the cipher you need to first transpose each block of characters into a vertical column.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	Н	Т	W	Е	А	Е	R	R	Е	Ρ	0	F	Т	R	0	Μ	R	С	Α	R	Ι	Т	Н	Н	R	F	Т	R	Y	Ι
2	Ι	А	S	Т	R	Ν	Т	Е	М	Ρ	R	А	Е	Ν	S	Y	Т	Е	S	L	Ι	D	Α	Y	Ν	Е	Т	Ι	Н	Μ
3	Ρ	U	D	Т	0	Ρ	Е	R	Т	Н	Е	Ι	Т	R	Ι	W	Ν	D	Ι	S	Т	Е	S	А	Т	Т	Е	Т	Ν	0
4	Y	Т	W	Е	Ν	М	Ρ	Н	С	Н	0	Е	С	Ν	А	R	Ι	Ν	Α	F	Н	Ν	0	Е	U	R	Ν	D	D	Е
5	Е	С	Ρ	Е	R	Ν	Т	R	А	Ι	L	L	Α	F	Ν	R	U	Ν	0	А	U	А	D	Q	А	Е	R	0	Т	R
6	Ν	Ι	F	А	Ν	С	Н																							

Next we need to identify words in the text which are jumbled up like an anagram. Since we were given the crib WEATHER our task will be easier. In the first row columns 3, 4, 5, 2, 1, 6, 7 can spell out weather.
0 3 4 5 2 1 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 А 1 WEATHERRE P 0 F Т R 0 М R С R Ι Т Н Н R F Т R Ι Υ S 2 S T R A I N T E M Ρ R Е Ν S Y Т Е L D Е Т Ι А Ι А Υ Ν Н Μ 3 D T O U P P E R T Н Е Ι Т R Ι W Ν D Ι S Т Е S А Т Т Е Т Ν 0 R Ι F Е U D 4 W E N T Y M P H C H 0 Е С Ν А Ν А н Ν 0 R Ν D F 5 PERCENTRA I L L F Ν R U 0 U OAER Α Ν А А D 0 Т R 6 F A N I N C H

Then if we take columns 8, 9, 10, 11, 14, 13 we get the word report. And, after making that change we can see than 12, 15, 20 spells for.

0 3 4 5 2 1 6 7 8 9 10 11 14 13 12 15 17 16 18 19 20 21 22 23 24 25 26 27 28 29 30 1 W E A T H E R R E Ρ Т 0 R С R Ι Т Н R F Т R 0 R F М Α Н Υ Ι 2 S T R A I N T E M Р R Ν Е А S Т Υ Е S L Ι D А Υ Ν Е Т Ι Н Μ 3 D T O U P P E R T Н Е R Т Ι Ι Ν W D Ι S Т Е S А Т Т Е Т Ν 0 4 W E N T Y M P H C H 0 Ν С Е Α Ι R Ν А F Н Ν 0 Е U R Ν D D F 0 5 P E R C E N T R A I L F А L Ν U R Ν А U А D 0 Α Е R 0 Т R 6 F A N I N C H

It would make since for the next few words to be a date and if we arrange the columns as 16, 19, 20, 18, 23, 22, 24, 21, 25, 27, 29, 26, 30, 28 then we get march thirty first.

0 3 4 5 2 1 6 7 8 9 10 11 14 13 12 15 17 16 19 20 18 23 22 24 21 25 27 29 26 30 28 1 W E A T H E R R E Ρ 0 R Т F 0 R Μ А R С Н Т Н Ι R Т Υ F Ι R 2 S T R A I N T E M Ρ R Ν Е А S Т Y S L Е А D Y Ι Ν Т Н Е М Ι 3 D T O U P P E R T Н Е R Т Ι Ι Ν W Ι S D S Е A Т Т Е Ν Т 0 Т 4 WENTYMPHC H С Е Ι R F Е U E D 0 Ν Α А Ν 0 Ν Н Ν D R 5 PERCENTRA I L F Ν U 0 UΑ Α L R Α Ν D А 0 RTE R 0 6 F A N I N C H

Now the first row looks good, but the others are a little off. After some inspection you should see that swapping columns 11 and 15, and columns 17 and 20 should fix that.

0 3 4 5 2 1 6 7 8 9 10 15 14 13 12 11 20 16 19 17 18 23 22 24 21 25 27 29 26 30 28 1 WEATHERRE P 0 R Т F 0 R R С Т н Т R Т Υ F R М Δ н Т 2 S T R A I N T E M Ρ S Ν Е А R L Y S Т Е А D Υ Ι Ν Т Н Е М Ι 3 D T O U P P E R T ΗI R Т Ι Е S W D S Е Т Т Е Т Т Ι Ν А Ν 0 4 W E N T Y M P H C H А Ν С Е 0 F R Ι Ν 0 Ν Е U Ν D Е Α Н R D 5 PERCENTRA I N F ALLAROU Ν D А 0 U A R Е Т R 0 6 F A N I N C H

Which, with a little work, gives:

weather report for march thirty first rain temps nearly steady in the mid to upper thirties winds e at ten to twenty mph chance of rain one hundred percent rain fall around a quarter of an inch

Along the way this is the sort of scrap work you will produce.



Figure E.2.2 Sample Transposition Analysis:

#### E.3 Vigenere Analysis

We start by looking through this cipher text for repeated strings and counting the spaces between them:

PRMOULFMJOZXJOLDLNKRHMFBQIEQOEWDKIJCKOGXRMZXDLCILNTRDRXORFYSVBIYWHVBLNCKZIKMRUCNEEUYQESOFALCHTYOUENKVVVBBLZDWLVLXSZXHSJKWAEIWIDODNUZUATDLCRVOYEYQERDDLCLHFFBHTYOHVVXLNXWUVVBOOTMDRVNEUKVLTKVHASYXTYSVOJDHNJSELVLXSZXHSJKQDDYUEFFHRYSVWZPHWRCLNTRDRXORFYSVBIYWHVBLNCKZTYOVHFZZAJCPACVDNUCRWRCWHVRRUJOLTNKVOEORFKRRSVQUIDIERZMNHFEVEJGKITRHXZCWEUSQLRBJEHEDNKSWIVCEEWYUEKRHEIKRFIOFOECWRLMWIFXGANXHDLZRNCYQDFXWHVCKOGGDSRCTURBHBFHRFRZOATOZIKRWHVPUOEDJLRJHDZXVMRVOPRXHSZXWHVNDYKSPEKRHDFYURWUDIEOGCCYVEUSQTYOHVXLNXSWSKY

There are a lot of different repeated strings at different spacings, but the most common divisor for the spacings is 4. Thus, 4 is likely the length of the key.

- LN: 32, 20, 96, 72, 20
- TYO: 64, 104, 228
- VB: 6, 36, 68, 78, 6
- JO: 4
- VO: 100
- YQE: 60
- LC: 45, 46, 13
- DLC: 13, 91, 104
- LVLXSZXHSJK: 96
- FX: 16
- LNTRDRXORFYSVBIYWHVBLNCKZ: 188

Dividing the cipher text into four subsets and finding frequencies for each we get:

The length of th	is subset	is 134	The length of this subset is 134				
Character	Count	Percent	Character	Count	Percent		
Η	20	14.93	Ε	14	10.45		
W	14	10.45	Ν	12	8.955		
V	12	8.955	Ι	11	8.209		
R	12	8.955	$\mathbf{S}$	9	6.716		
D	11	8.209	О	9	6.716		
$\mathbf{L}$	10	7.463	L	9	6.716		

And, the index of coincidence for thisAnd, the index of coincidence for this set is 0.06973 set is 0.05558

The length of th	is subset	is 134	The length	The length of this subset is 133				
Character	Count	Percent	Chara	acter	Count	Percent		
V	16	11.94	О		15	11.28		
Κ	12	8.955	Х		13	9.774		
R	11	8.209	$\mathbf{S}$		12	9.023		
$\mathbf{F}$	11	8.209	$\mathbf{C}$		12	9.023		
$\mathbf{Z}$	10	7.463	Υ		11	8.271		
$\mathbf{C}$	9	6.716	R		9	6.767		

And, the index of coincidence for thisAnd, the index of coincidence for this set is 0.05814 set is 0.0597

Now looking in Kasiski's Key Table Figure 4.3.2 for the columns labeled with the most common characters in each subset we get a likely key of "DARK." Testing this we get this deciphered message:

MRVERL OCGOIN GOUTIN THEMOR NINGLE FTHISS HOPNOM INALLY INCHAR GEOFHI SBROTH ERINLA WITCOU LDBEDO NEBECA USETHE REWASV ERYLIT TLEBUS INESSA TANYTI MEANDP RACTIC ALLYNO NEATAL LBEFOR ETHEEV ENINGM RVERLO CCARED BUTLIT TLEABO UTHISO STENSI BLEBUS INESSA NDMORE OVERHI SWIFEW ASINCH ARGEOF HISBRO THERIN LAWTHE SHOPWA SSMALL ANDSOW ASTHEH OUSEIT WASONE OFTHOS EGRIMY BRICKH OUSESW HICHEX ISTEDI NLARGE QUANTI TIESBE FORETH EERAOF RECONS TRUCTI ONDAWN EDUPON LONDON THESHO PWASAS QUAREB OXOFAP LACEWI THTHEF RONTGL AZEDIN SMALLP ANESIN THEDAY TIMETH EDOORR EMAINE DCLOSE DINTHE EVENIN GITSTO

Once we clean this up and do a quick search for the text online we find this passage:

"Mr Verloc, going out in the morning, left his shop nominally in charge of his brother-in-law. It could be done, because there was

very little business at any time, and practically none at all before the evening. Mr Verloc cared but little about his ostensible business. And, moreover, his wife was in charge of his brother-in-law."

"The shop was small, and so was the house. It was one of those grimy brick houses which existed in large quantities before the era of reconstruction dawned upon London. The shop was a square box of a place, with the front glazed in small panes. In the daytime the door remained closed; in the evening it stood discreetly but suspiciously ajar." - *The Secret Agent: A Simple Tale* by Joseph Conrad

Finally, here is the sort of hand scrap work you would expect to do.

	1								
		2	3	4	5	6	7		Subset 11
t	PRMOUL	FMJOEX)	JOLD (N	KRHMFB	QIEQOE	WDKIJC	KOGXRM	Note	H: DOHTZUPQAWE
z	zx6rdi	LNTRDR	XORFYS	VBIYWH	<u>ve(</u> ln)ck	<b>Z</b> EKMRÚ	CNEEU	7 blocks at 6 Chernoters por line = 42 Changeter 5.	W:SOWIOJEEPLT V:RCVHNIGEOKS
3	QEBOFA	LCHTYO	UENKVV	VBBLZD	WEVEXS	(zx) HSJK	WAEIWI		Subset 2:
ષ	DODNUZ	uat <u>(dlc</u> )	RVOYEY	QERDDL	Днғғв	HEYOHV	vxLnjkw	LVLXSZXHSJK Hx6+12 = 15.6	E: ALEOWRMNXTB N: JUNZFAVWGZK
5	07 <u>78</u> 00	TMDRVN	EUKVLT	KVHASY	XTYSVO	JDHNJS	ELVIXS	= 32.3	I ! EPIVA VARBXI
6	ZXHSJK	QDDYUE	FFHRYS	VWZPHW	R	DRXORF	YŚ <u>VB</u> I¥	= 2 . 5	Subset 3:
7	wh <u>ve</u> (ln)	CKZTYO	VHFZZA	JCPACV	DNUCRW 1	RCWHVR	RUJOLT	UNTRORXORF	VI (RCVH NIVEDICH KI FRKW CXSTDZH
8	NK <u>VO</u> EO I	RFKRRS	VQUIDI	ERZMNH	FEVEJG I	KITRHX	ZCWEUS	(42+5) XXVBIYW # VOLIN CKZ"; YX42+20 = 4(42+5)	R: NYBD JEZAN
9	QLRBJE I	IEDNKS	VIVCEE	WYUEKR	HEIKRF	IOFOEC	WRLMWI	=4(47)	Subset Y!
10	(exiganx f	HDLZRN (	CYQD(FX)	WHVCKO (	GDSRC 1	URBHB	FHRFRZ		O QUOAF BWX X TEXJPR FG R MU
0	OATOZI K	RWHVP (	JOEDJL I	RJHE(ZX)	MRVOP F	XHSZX	WHVNDY		S: OZSE(SFAS LIF
IZ	KSPEKR H	DFYUR V	WDIEO (	SCCYVE (	ISQTYO H	IVVX(LN)	XSWSKY		DARK
	RDUSVC I	OHTCIE 32=2 <sup>5</sup>	UKCXS (	SFIFE V 4.5 ,(	/LPKMA I 5-6+6 =	96=8.1	L=16.31	11.6+6=12.6=8.9,20=4.5 4(0=4	

most common factor.

**Figure E.3.2** Sample Vigenere Analysis Page 2:

Figure E.3.1 Sample Vigenere Analysis Page 1:

TYO:  $64 = 2^6$ ,  $16x6 + 8 = 10^4 = 8 \cdot 13$ ,  $38 \cdot 6 = 4 \cdot 3 \cdot 19$  6cd = 4

VB : 6, 36 = 4.9, 68 = 4.17, 78 = 2.39 = 2.3.15, 6

LC : 45=5.9 , 46=2.23 , 13 P.CC : 13 , 15:6+1 = 91 = 7.13 , 14:6+2 = 104 = 8.13=2.4.13

50 : 4

VO ; (6.6+ 4= 4(25)

YQE: 8.6+12 = 4(15)

## Appendix F

# Solutions to Selected Exercises

# 1 · Caesar's Shifty Idea 1.1 · Simple Early Ciphers (and a little math)

#### Checkpoint 1.1.3

Answer. KHOOR ZRUOG

Checkpoint 1.1.4

Answer. this is a test

Checkpoint 1.1.5

Answer. zombies like zebras

#### Checkpoint 1.1.6

**Answer**. Here is a longer message for you to practice on. The shift for this message was by a factor of seven, so the a goes to the h. This doesn't really offer much more security, especially since there are only twenty six possible shift ciphers and one of those doesn't change anything. While you were deciphering this what patterns did you notice? What words stood out? How could we make this a more secure (harder to crack) cipher? These are all questions that we will investigate in more detail throughout the text.

#### Checkpoint 1.1.7

#### Answer.

"Be copy now to men of grosser blood, And teach them how to war. And you, good yeoman, Whose limbs were made in England, show us here The mettle of your pasture; let us swear That you are worth your breeding; which I doubt not; For there is none of you so mean and base, That hath not noble lustre in your eyes. I see you stand like greyhounds in the slips, Straining upon the start. The game's afoot: Follow your spirit, and upon this charge Cry 'God for Harry, England, and Saint George!" - Henry V, William Shakespeare

#### Checkpoint 1.1.8

#### Answer.

"We few, we happy few, we band of brothers; For he to-day that sheds his blood with me Shall be my brother; be he ne'er so vile, This day shall gentle his condition: And gentlemen in England now a-bed Shall think themselves accursed they were not here, And hold their manhoods cheap whiles any speaks That fought with us upon Saint Crispin's day." - Henry V, by William Shakespeare

#### Checkpoint 1.1.9

**Answer**. 40,329,146,112,660,563,558,400,000 seconds, 11,202,540,586,850,156,544,000 hours, or 1,275,334,766,262,540,590 and 10/61 years

#### Checkpoint 1.1.11

Answer.

- 4!=24
- 7!=5040
- 10!=3628800

#### Checkpoint 1.1.12

**Answer**. "Before you begin a thing, remind yourself that difficulties and delays quite impossible to foresee are ahead. If you could see them clearly, naturally you could do a great deal to get rid of them but you can't. You can only see one thing clearly and that is your goal. Form a mental vision of that and cling to it through thick and thin." - Kathleen Norris"

#### **1.2** • Arabic Numerical Ciphers

Checkpoint 1.2.2

#### Answer.

- 1. NINE TWO FIVE SIXHUNDRED
- 2. HA AA CB VT

3. KG DB JE ZXV

#### Checkpoint 1.2.3

Answer. There are multiple possible encipherments, a couple are  $V\ B\ S$  and another is QUK AA JJOK.

#### Checkpoint 1.2.4

**Answer**. "When I want to understand what is happening today or try to decide what will happen tomorrow, I look back." - Omar Khayyam

#### Checkpoint 1.2.5

**Answer**. One possible solution is LEAD A SOM CAD.

#### $1.3 \cdot \text{How Shifty are You?}$

#### 1.3.1.

#### Answer.

``FN'EN JUU QNJAM CQJC J VRUURXW VXWTNHB KJWPRWP XW J VRUURXW CHYNFARCNAB FRUU NENWCDJUUH ANYAXMDLN CQN NWCRAN FXATB XO BQJTNBYNJAN. WXF, CQJWTB CX CQN RWCNAWNC, FN TWXF CQRB RB WXC CADN.'' -YAXONBBXA AXKNAC BRUNWBTH

#### 1.3.2.

**Answer**. There are in fact multiple answers to this, one of them would be:

STUVVWXYZ TRJ ST SN U LE ME VS VX

#### 1.3.3.

Answer. BAA LEEK BAD

#### 1.3.4.

Answer.

NRHGZPVH ZIV Z KZIG LU YVRMT SFNZM. ZKKIVXRZGV BLFI NRHGZPVH ULI DSZG GSVB ZIV: KIVXRLFH ORUV OVHHLMH GSZG XZM LMOB YV OVZIMVW GSV SZIW DZB. FMOVHH RG?H Z UZGZO NRHGZPV, DSRXS, ZG OVZHG, LGSVIH XZM OVZIM UILN. - ZO UIZMPVM, LS, GSV GSRMTH R PMLD, 2002

#### 1.3.5.

#### Answer.

"Now they show you how detergents take out bloodstains, a pretty violent image there. I think if you've got a T-shirt with a bloodstain all over it, maybe laundry isn't your biggest problem. Maybe you should get rid of the body before you do the wash." - Jerry Seinfeld

#### 1.3.6.

#### Answer.

"One should guard against preaching to young people success in the customary form as the main aim in life. The most important motive for work in school and in life is pleasure in work, pleasure in its result, and the knowledge of the value of the result to the community." - Albert Einstein

#### 1.3.7.

Answer.

"In a completely rational society, the best of us would be teachers and the rest of us would have to settle for something less, because passing civilization along from one generation to the next ought to be the highest honor and the highest responsibility anyone could have." - Lee Iacocca

#### 1.3.8.

#### Answer.

"The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that mathematicians have made a covenant with the devil to darken the spirit and confine man in the bonds of Hell." - St. Augustine

#### 1.3.9.

#### Answer.

MPKRGOPL AHRM GHP ZHFR NKHF TDGGDGO. VHQK MPKQOOERM ARSREHI VHQK MPKRGOPLM. TLRG VHQ OH PLKHQOL LXKAMLDIM XGA ARZDAR GHP PH MQKKRGARK, PLXP DM MPKRGOPL. - XKGHEA MZLTXKWRGROORK

#### 2 · Attacking the Alphabet

2.4 · Bringing it all Together

#### 2.4.5.

**Answer**. "of all that is good, sublimity is supreme. succeeding is the coming together of all that is beautiful. furtherance is the agreement of all that is just. perseverance is the foundation of all actions." - lao tzu

**Solution**. When you look at the frequencies you see that G and V are most common, next most common are R and Z. So we know that one of the first two is probably e while the other is t, and one of the latter two is likely a, assuming the frequencies are fairly normal.

Then we see that GSV and GSZG appear frequently, and GS is the most common bi-gram (so that it is likely th). Putting this together we see that e is V, t is G, a is Z, and h is S. So that GSV and GSZG represent the and that.

We can also see that ZOO appears four times with Z replaced by a this looks like aOO, the likely candidate is that ZOO is all, so that O is l.

At this point we can start writing down the letters we have in the monoalphabetic substitution table (Table C.0.1). When you do that you will hopefully notice that the cipher letters are in reverse alphabetic order, so this was likely enciphered with atabash which we learned about in Exercise 1.3.4.

#### **2.4.6**.

**Answer**. "far better is it to dare mighty things, to win glorious triumphs, even though checkered by failure ... than to rank with those poor spirits who neither enjoy nor suffer much, because they live in a gray twilight that knows not victory nor defeat." - theodore roosevelt

**Solution**. Looking at a basic frequency analysis we see that the most common single letters are G, R, B, E, V, U, A, F, N, H, the most common bi-gram is GU, and the most common tri-gram is GUR; from this we may conclude that G is t, U is h, and R is e.

Since the word spacing is preserved we also have the two letter words VF, VG, GB, and VA, and the one letter word N, these allow us to deduce the ciphertext-plaintext pairs N - a, B - o, V - i, and F - s.

When we start writing down the ciphertext we have worked out under the plaintext alphabet (use Table C.0.1) we can see that the letters we have discovered are in the correct order and with the correct spacing so that we appear to have a shift, trying this as a possible solution we see that we are correct and that a was shifted to N.

#### 2.4.7.

**Answer**. "... what is out of the common is usually a guide rather than a hindrance. In solving a problem of this sort, the grand thing is to be able to reason backwards. That is a very useful accomplishment, and a very easy one, but people do not practice it much. In the every-day affairs of life it is more useful to reason forwards, and so the other comes to be neglected. There are fifty who can reason synthetically for one who can reason analytically." - Sherlock Holmes in Study in Scarlet by Sir Arthur Conan Doyle

**Solution**. As always you need to start with a basic frequency analysis. From this you immediately get that the ten most common letters are L, T, J, S, Q, P, I, K, F, D, the most common bi-gram and tri-gram are TK and TKL, L appears frequently before and after other letters (in particular P), and finally TKLPL and TKLDP appear multiple times; from all of this we get the ciphertext-plaintext pairs L - e, T - t, H - k, P - r, and D - i.

Then we can start filling in bits and pieces of the message and looking for more clues, for example LVLPY becomes eVerY which we reasonably assume is every. Also, with the hint that it is a keyword cipher we can line up the ciphertext letters we have worked out underneath a copy of the plaintext alphabet (Table C.0.1) and look for patterns. Continuing in this way we can finally arrive at the solution and the key is KEYWORD: SHERLOCKED, key letter: a.

#### $3 \cdot \text{Mixing Things Up}$

3.1 · Alberti's Great Idea

#### 3.1.2 · Alberti's Polyalphabetic Cipher

Checkpoint 3.1.3

Answer. G vbb ke V xsdba m&bbry&

Checkpoint 3.1.4

Answer. cats and dogs

#### 3.1.3 · Vigenère's Cipher

Checkpoint 3.1.8

Answer. Ciphertext: QOTEASLQACVI

Checkpoint 3.1.9

Answer. Ciphertext: IIREHXSL

Checkpoint 3.1.10

Answer. Plaintext: tour de france

Checkpoint 3.1.11

Answer. Plaintext: baguette

### 3.2 · Variation on a Theme

#### 3.2.2 · Pigpen Cipher

#### Checkpoint 3.2.3

**Answer**. Here are the first few words of the message:



Figure F.0.1 Cipher Text:

#### Checkpoint 3.2.5

Answer. Plaintext: "Pork 'n Beans"

#### Checkpoint 3.2.8

**Answer**. "I am fond of pigs. Dogs look up to us. Cats look down on us. Pigs treat us as equals." Winston S. Churchill

Checkpoint 3.2.10

Answer.



Figure F.0.2 Cipher Text

3.3 · An Automatic Hit3.3.2 · The Autokey Cipher

#### Checkpoint 3.3.2

**Answer**. Method 1:

Plaintext:a u t o m a t i c c o n f u s i o nKey Letter:Q A U T O M A T I C C O N F U S I OCiphertext:Q P I G A G R H M U N D H M A F M G H

Method 2:

Plaintext: a u t o m a t i c c o n f u s i o n Key Letter: Q P E L G D X G R N T A B R F I Q L Ciphertext: Q P E L G D X G R R R L F P E I Q L F

#### Checkpoint 3.3.3

Answer.

Ciphertext:C H F A B H C X N X M G F M R R M D D M RKey Letter:C S T O R M T H E B A S T I L L E N U LPlaintext:s t o r m t h e b a s t i l l e n u l l

#### Checkpoint 3.3.4

Answer.

Ciphertext: Q M P T M G P O H G S O O H L Q X U G C Key Letter: Q M P T M G P O H G S O O H L Q X U G Plaintext: h i d i n g i n p l a i n s i g h t x

### 3.4 · Stirring Things Up 3.4.2 · Transposition

#### Checkpoint 3.4.4

Answer. Now I'm all turned around.

**Solution**. First put the blocks of text into the table row by row in alphabetical order:

#### Table F.0.3 Falconer's Transposition Practice

		А	В	С
1	CBA	WU	00	NR
2	CAB	MD	Α	IN
3	ACB	L	Т	L
4	BCA	Ν	U	R
5	BAC	D	Е	Α

Now read the message out character by character according to the combinations at the left:

nowimall turned around

Finally, we get the original message.

Now I'm all turned around.

#### Checkpoint 3.4.6

**Answer**.  $\triangle$ LSNDNE SINENM ADIBOS NUNZMB GRGZAR OTIIAL ANWYKO TNRNMK SIEAEO WNOET HGEDIS EAGWS RSNEK ODNRA LPAGE

**Solution**. First we break up the message into blocks of three to make it easier to work with.

ASL ONG AST HEW ORL DIS TUR NIN GAN DSP

INN ING WER EGO NNA BED IZZ YAN DWE REG ONN AMA KEM IST AKE SME LBR OOK S

Then copy each character into the table using the combination keys at the side to tell us which column to put each character in.

		A	В	С
1	CBA	LSNDNE	SINENM	ADIBOS
2	CAB	NUNZMB	GRGZAR	OTIIAL
3	ACB	ANWYKO	TNRNMK	SIEAEO
4	BCA	WNOET	HGEDIS	EAGWS
5	BAC	RSNEK	ODNRA	LPAGE

Table F.0.4 Falconer's Transposition Practice

Finally, we read the message out of the table from left to right and from top to bottom.

 $\triangle {\rm LSNDNE}$  SINENM ADIBOS NUNZMB GRGZAR OTIIAL ANWYKO TNRNMK SIEAEO WNOET HGEDIS EAGWS RSNEK ODNRA LPAGE

#### Checkpoint 3.4.8

**Answer**.  $\triangle$ AVADAI ERENB PEKALN SAETR ODVHT RMNYE WARNI NEEIE HMTGN OIRNS NSIET ETEWE

**Solution**. First we break up the message into blocks of three to make it easier to work with.

APERSONWHONEVERMADEAMISTAKENEVERTRIEDANYTHINGNEWALBERTEINSTEIN

Then copy each character into the table using the combination keys at the side to tell us which column to put each character in.

 Table F.0.5 Falconer's Transposition Practice

		А	В	С
1	ACB	AVADAI	ERENB	PEKALN
2	CAB	SAETR	ODVHT	RMNYE
3	BAC	WARNI	NEEIE	HMTGN
4	ABC	OIRNS	NSIET	ETEWE

Finally, we read the message out of the table from left to right and from top to bottom.

 $\bigtriangleup AVADAI ERENB PEKALN SAETR ODVHT RMNYE WARNI NEEIE HMTGN OIRNS NSIET ETEWE$ 

#### 3.5 · Presidential Secrets

#### 3.5.2 · Thomas Jefferson and Vigenère's Cipher

#### Checkpoint 3.5.1

Answer.

UVYVQB &MGTSF RCSSSN JEMCUQ ISLQHK AJTWXG NS&&UP

XQQHBH MYBXJF UJBBXI RIAWSK BUSENE TSKNCT FJ&PZX

AIWIFF HERLGC & IFPEW JCHLJA SQEJBC VXYJSK LUPRID

PHXQYG WYMCPH & CWSSG SGUVU

**Solution**. Start by writing out the plaintext with the key text above it, just as Jefferson did:

antipod esan tipodes anti podesa ... pursues some greatly good intent ...

Next, using the key letter to tell you the column and the plaintext to tell you the row, look up the cipher letters inside the table:

antipod esan tipodes anti podesa ... pursues some greatly good intent ... QHKAJTW XG ...

Checkpoint 3.5.2

Answer.

IQGYJ IDQWB K

**Solution**. As before write out the message with the key lined up above it. Then you can look up the key letter along the top of Jefferson's table and the message letters or numbers along the sides.

happy happy h april 13174 3 IQGYJ IDQWB K

#### 3.5.3 • Thomas Jefferson's Wheel Cipher

#### Checkpoint 3.5.3

**Answer**. Because after lining up the letters you want you can choose to write down any other row, there are multiple different as nwers. One posibility is: MMIHODVWR9RN

Checkpoint 3.5.4

Answer.

attack at dawn

#### 3.6 · How Mixed Up Do You Feel?

#### 3.6.1.

Answer.



Figure F.0.6 "You can put wings on a pig, ..."

#### 3.6.2.

Answer.

That's All Folks

#### 3.6.3.

Answer.

Cdmb sxb G&h tmm Xxkpg& ke Clm nnhff

**Solution**. Start by setting the index to s and the key letter to C. Then you can start translating the plaintext to ciphertext. Remember that you need to write the key letter down each time you change it, the plaintext for Alberti are the uppercase letters around the outside of his formula, and sometimes you need to leave letters out or make substitutions since not all the letters are available. In the end you should have something like: Cdmb sxb G&h tmm Xxkpg& ke Clm nnhff.

#### **3.6.4.**

Answer.

MFPRKW FXOBFF WVAXHE XKBHVM YZSNDE IJUCRI W

**Solution**. Write down the plaintext with the key above it and then, using the key letter to indicate the row and plain letter the column, look up the ciphertext in the table.

france france france france france f hopeis agoodb reakfa stbuti tisaba dsuppe r MFPRKW FXOBFF WVAXHE XKBHVM YZSNDE IJUCRI W

#### 3.6.5.

**Answer**. "There are no secrets to success, it is the result of prparation, hardwork, and learning from failure." - Colin Powell

**Solution**. Write out the ciphertext with the key word written above it as before. Now look use each key letter to tell which row to look in and slide your finger along to the cipher letter; the column header is your plaintext letter.

```
state state state state state state state ...
LAEKI SKEGS KXCKI LLTHW MVCXW KBTBW LAEKI KNLMS ...
there are ...
```

**3.6.6.** 

Answer.

ERGFO OUMIO FTLXF CSPHN OQATA M ...

**Solution**. write out the message with the key above, like Vigenere's polyalphabetic cipher, but now the key is essentially the message its self. Above the first letter is our key letter, then after that you write down the message. Thus lok up the key letter on the side of the table and find the plain letter in the corresponding row. This last step is a little tricky because the letters are in pairs, so if the key letter is S and we need a plain t the cipher letter will be F.

```
FOBST ACLES ARETH OSEFR IGHTF U ... obsta clesa retho sefri ghtfu l ... ERGFO OUMIO FTLXF CSPHN OQATA M ...
```

#### 3.6.7.

Answer.

KMWEG WKAAF BHYNV NLQFI TFLVM AFBHY FTTLT BBALA LGQIL ...

Solution. As before write out the message, place the key letter above the

first plaintext letter, and write out the rest of your message after that as a key. Then, using the modern Vigenere table, the key letters in the top row tell you which row to look in and the plain letters gives you the column.

SSUCC ESSIS NOTFI NALFA ILURE ISNOT FATAL ITIST HECOU ... succe ssisn otfin alfai lurei snotf atali tisth ecour ... KMWEG WKAAF BHYNV NLQFI TFLVM AFBHY FTTLT BBALA LGQIL ...

#### 3.6.8.

Answer. "Donuts, is there anything they can't do?" - Matt Groening

**Solution**. To decipher this you need to decipher the first character with the X, look in row X for the A and then there is a d at the top of that column. Now repeate the process with the D you just found and the R, this is how you get the o. Keep repeating this process until you find the message.

Xdonu tsist herea nythi ngthe ycant domat tgroe ning ARBHN LAALA LVVEN LRAPV TZALC ACNGW RAMTM ZXFSR VVT donut sisth erean ythin gthey cantd omatt groen ing

3.6.9.

Answer.

WGI RWE EIV NLI EOR IUS TBY AEM ODT KAA IUN NN

**Solution**. Create a table with three three columns and four rows. Label the rows with the keys and the columns with A, B, and C. Write down the message character by character with the keys at the side telling you which columns to write the characters in. Finally, write down the blocks of text from left to right and top to bottom.

Cipher Table:

		А	В	С								
ACB	:	WGI	RWE	EIV								
BCA	:	NLI	EOR	IUS								
CAB	:	TBY	AEM	ODT								
ABC	:	KAA	IUN	NN								
Fina	al cip	oher	text:									
WGI	RWE	EIV	NLI	EOR	IUS	TBY	AEM	ODT	KAA	IUN	NN	
<b>3.6</b> .	<b>10.</b>											
Ans	swer	•										
AHF	HDE	INR	VAO	ETR	LEE	ENI	ODI	VID	JQI	UUN	DIE	SIE
IYI	ттх	EHE	CTL	AE								
Sol	utio	n. (	Ciphe	er Ta	ble:							
		А	В	С								
CBA	:	AHF	HDE	INR								
ABC	:	VAO	ETR	LEE								
BCA	:	ENI	ODI	VID								
CAB	:	JQI	UUN	DIE								

ACB : SIE IYI TTX

BAC : EHE CTL AE

Completed Message:

AHF HDE INR VAO ETR LEE ENI ODI VID JQI UUN DIE SIE IYI TTX EHE CTL AE

#### **3.6.11.**

**Answer**. "It is not in the stars to hold our destiny, but in ourselves." - William Shakespeare

**Solution**. Set up a table with columns labeled A, B, C, and rows labeled with the keys. Write each block of text into the table from right to left and top to bottom. Finally, read the characters out of the table one at a time based in the order given by the keys.

A B C ABC : ISUIIP TTRNLE IADOLA BCA : OTTSM SREUIR NSSRAE BCA : NOYVA TOIES IHNLH ACB : TLBEK EOTWS HDUSE

Completed Message:

"It is not in the stars to hold our destiny, but in ourselves." - William Shakespeare

**3.6.12.** 

Answer.

WHIQCGXXWTH

3.6.13.

Answer.

EBWVEUJUNPZG

**3.6.14.** 

Answer.

moving on up to

#### 4 · Triumphs of Logic and Statistics

#### 4.1 · A Simple Solution

### 4.1.1 · The Power of Prime Factors

Checkpoint 4.1.1

Answer.

Table F.0.7 Prime Factorization Table F.0.8 All Possible Factors

210	105	35	7	1		2	3	5	7
2	3	5	7				6	10	14
								15	21
								30	42
									35
									70
									105
									210

**Solution**. First make a table of the primes factors by writing down the 210 and under that a 2 (since it is even), then next to the 210 write 105=210/2. Next, with a little work, we see that 105 is divisible by 3, so write 3 under the 105 and 35 next to 105, since 35=105/3. Continue this process to fill in Table F.0.9 with the rest of the prime factors.

Now make a table with the prime factors written across the top. Starting with the second column, multiply each number in the table by all the numbers to its left by which it has not already been multiplied. So, we multiply 3 by 2 to get 6, then 5 by 2 to get 10 and so on. Try to finish filling in Table F.0.10.

#### Table F.0.9 Prime Factorization Table F.0.10 All Possible Factors

210 2	$\frac{105}{3}$	35			2	$\frac{3}{6}$	5 10	$7 \\ 14 \\ 21$
								210
								210

#### Checkpoint 4.1.6

Answer.

Table F.0.11 Prime Factorization

980	490	245	49	7	1
2	2	5	7	7	

 Table F.0.12 All Possible Factors

2

2	5	7	7
4	10	14	49
	20	28	98
		35	196
		70	245
		140	490
			980

#### Checkpoint 4.1.9

**Answer**. There are thirty blocks of text. Factoring that, we get the prime factors 2,3, and 5. Putting these together, the complete list of factors of 30 is 2, 3, 5, 6, 10, 15, and 30. Since Falconer's cipher requires writing letters into a table, we can conclude that the table's dimensions were likely 3 by 10, 5 by 6, 6 by 5, or 10 by 3. (It is unlikely that the table was 1 by 30 or 2 by 15.) Using this information we can conclude that there were likely 3, 5, or 6 key letters and we could try to pick apart the message.

# 4.1.2 · But Really There's an Easier Way (sort of)

#### Checkpoint 4.1.12

**Answer**. "If winter comes, can spring be far behind" By Percy Bysshe Shelly **Solution**. Begin by writing each block of text vertically and looking for scrambled words. From the hint we know to look for the word WINTER. Spotting the letters to spell WINTER in the top row we rearrange the columns and then start to look for other words.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 I W F I N R O T C E M S E C A R P S N I B F N E G A B R E H P D N I E Y Y R B C S H A E S L L E H E Y

You should notice that columns 3 and 4 can spell IF in the first row and if we

put them at the beginning we also get the word SPRING in the second row.

2 1 5 8 10 6 3 4 7 9 11 12 13 14 15 W I N T E R F I O C M E S C A P R I N G B S N F E A B R E H ....

Reassuringly we see that this also gives us the name PERCY in the third row and SHELLY wrapping around from row 3 to 4. Finally, you may notice that swapping columns 9 and 7, and columns 12 and 13 will give us the word COMES in the first row.

4 3 2 1 5 8 10 6 7 9 11 12 13 14 15 I F W I N T E R O C M S E C A N S P R I N G B F E A B R E H I N D P E R C Y Y B S H S E S H E L L E Y

After make those last two swaps we can now read off an entire message.

4 3 2 1 5 8 10 6 9 7 11 13 12 14 15 I F W I N T E R C O M E S C A N S P R I N G B E F A R B E H I N D P E R C Y B Y S S H E S H E L L E Y

"If winter comes, can spring be far behind" By Percy Bysshe Shelly

#### Checkpoint 4.1.13

**Answer**. "Laughter is the sun that drives winter from the human face." Victor Hugo

#### Checkpoint 4.1.14

**Answer**. "It was one of those March days, when the sun shines hot and the wind blows cold. When it is summer in the light and winter in the shade." Charles Dickens

#### 4.3 · Nineteenth Century Revelations

# 4.3.1 · Kasiski's Attack4.3.1.2 · Determining the Key

#### Checkpoint 4.3.3

#### Answer.

"Since Aramis's singular transformation into a confessor of the order, Baisemeaux was no longer the same man. Up to that period, the place which Aramis had held in the worthy governor's estimation was that of a prelate whom he respected and a friend to whom he owed a debt of gratitude; but now he felt himself an inferior, and that Aramis was his master. He himself lighted a lantern, summoned a turnkey, and said, returning to Aramis, "I am at your orders, monseigneur." Aramis merely nodded his head, as much as to say, "Very good"; and signed to him with his hand to lead the way." The Man in the Iron Mask, Alexandre Dumas

#### Checkpoint 4.3.4

**Answer**. The keyword is "ironmask" The message is:

"And from that moment, D'Artagnan, accommodating his action to the pace of the horse, like a true centaur, gave up his thoughts to nothing - that is to say, to everything. He asked himself why the king had sent for him back; why the Iron Mask had thrown the silver plate at the feet of Raoul. As to the first subject, the reply was negative; he knew right well that the king's calling him was from necessity. He still further knew that Louis XIV. must experience an imperious desire for a private conversation with one whom the possession of such a secret placed on a level with the highest powers of the kingdom. But as to saying exactly what the king's wish was, D'Artagnan found himself completely at a loss. The musketeer had no doubts, either, upon the reason which had urged the unfortunate Philippe to reveal his character and birth. Philippe, buried forever beneath a mask of steel, exiled to a country where the men seemed little more than slaves of the elements; Philippe, deprived even of the society of D'Artagnan, who had loaded him with honors ..." The Man in the Iron Mask, Alexandre Dumas

#### 4.4 · Do You Feel Logical?

4.4.1.

Answer.

ANWZS UOLS QTIE KHEG IFLI CTJX VNAB OEPN MEOU MMDA EERO EYIT

**Solution**. Set up a cipher table with the rows labled by the set of keys and the columns by the letters A, B, and C. Then place the characters from the message into the table one at a time following the key for each row.

Cipher Table:

		А	В	С
ACB	:	ANWZS	UOLS	QTIE
BCA	:	KHEG	IFLI	СТЈХ
CBA	:	VNAB	OEPN	MEOU
BAC	:	MMDA	EERO	EYIT

Completed Message:

ANWZS UOLS QTIE KHEG IFLI CTJX VNAB OEPN MEOU MMDA EERO EYIT

#### **4.4.2.**

Answer.

weather report for march thirty first rain temps nearly steady in the mid to upper thirties winds e at ten to twenty mph chance of rain one hundred percent rain fall around a quarter of an inch

#### **4.4.3**.

Answer.

how are you he said cordially gripping my hand with a strength for which I should hardly have given him credit you have been in afghanistan i perceive

#### 4.4.4.

Answer.

painful zombies quickly watch a jinxed graveyard

4.4.5.

Answer.

WBRCMN NUXDAM QXOSWO RLRGNU KKBBPY FHITBU YGNIMI DJBXMN RG

#### **4.4.6**.

Answer. Keyword "lock".

we met next day as he had arranged and inspected the rooms at no b baker street of which he had spoken at our meeting they consisted of a couple of comfortable bedrooms and a single large airy sitting room cheerfully furnished and illuminated by two broad windows so desirable in every way were the apartments and so moderate did the terms seem when divided between us that the bargain was concluded upon the spot and we at once entered into possession that very evening i moved my things round from the hotel and on the following morning sherlock holmes followed me with several boxes and portmanteaus for a day or two we were busily employed in unpacking and laying out our property to the best advantage that done we gradually began to settle down and to accommodate ourselves to our new surroundings

#### 4.4.7.

Answer. Keyword "bug".

many years ago i contracted an intimacy with a mr william legrand he was of an ancient huguenot family and had once been wealthy but a series of misfortunes had reduced him to want to avoid the mortification consequent upon his disasters he left new orleans the city of his forefathers and took up his residence at sullivans island near charleston south carolina this island is a very singular one it consists of little else than the sea sand and is about three miles long its breadth at no point exceeds a guarter of a mile it is separated from the main land by a scarcely perceptible creek oozing its way through a wilderness of reeds and slime a favorite resort of the marsh hen the vegetation as might be supposed is scant or at least dwarfish no trees of any magnitude are to be seen near the western extremity where fort moultrie stands and where are some miserable frame buildings tenanted during summer by the fugitives from charleston dust and fever may be found indeed the bristly palmetto but the whole island with the exception of this western point and a line of hard white beach on the seacoast is covered with a dense undergrowth of the sweet myrtle so much prized by the horticulturists of england the shrub here often attains the height of fifteen or twenty feet and forms an almost impenetrable coppice burthening the air with its fragrance

#### 4.4.8.

Answer. Keyword "vampire".

i went straight to my own room and commenced to work afresh on the biliteral cipher more than ever had i the conviction upon me that

the reading of the secret writing would be the first step to the attainment of my wishes regarding marjory it would have been strange therefore if i had not first attempted the method which she had herself suggested the reducing the baconian cipher to its lowest elements for many hours i laboured at this work and finally when i had reduced the baconian five symbols to three i felt that i had accomplished all that was possible in that way when i had arrived at this result and had tested its accuracy in working i felt in a position to experiment with my new knowledge on the old number cipher first i wrote out my method of reduction as a sort of addendum to the paper which i had prepared for marjory then i made a key to cipher and one to decipher by this time the night was well on and the grey of early morning was beginning to steal in by the edges of the blinds i was not sleepy however i was too much excited to think of sleep for the solving of the problem seemed almost within my grasp excited to a state which almost frightened me by its intensity i got ready my copy of the number cipher and my newly prepared key with an effort which took me all my resolution i went on steadily writing its proper letter under each combination without once looking back for i knew that even should some of the letters be misplaced in the key the chance of recognising the right ones would be largely increased by seeing a considerable number of letters together then i glanced over the whole and found that many of the symbols made up letters with such a basis to work on the rest was only labour a few tentative efforts and i had corrected the key to agreement with some of the combinations in the cipher

# 6 · Mathematics to the Rescue6.1 · Affine Ciphers

#### 6.1.3 · Affine Cipher

#### Checkpoint 6.1.18

Answer. ZKTTI WIXTF

#### Checkpoint 6.1.19

Answer. cryptology rocks

#### Checkpoint 6.1.20

Answer. MTSZC MTTSZ CUSHB CP

Checkpoint 6.1.21

Answer. more than a shift

#### Checkpoint 6.1.22

**Answer**. The message begins with "One summer night, a few months after my ..."

#### 6.2 · Hill's Cipher

#### 6.2.1 · Matrices

Checkpoint 6.2.2

Answer.

$$\begin{pmatrix} 4\\14 \end{pmatrix} \pmod{26}$$

#### Checkpoint 6.2.3

Answer.

$$\begin{pmatrix} 12 & 10 \\ 18 & 9 \end{pmatrix} \pmod{26}$$

#### Checkpoint 6.2.4

Answer.

$$\binom{22}{9} \pmod{26}$$

Checkpoint 6.2.5

Answer.

$$\binom{15}{3} \pmod{26}$$

Checkpoint 6.2.6

Answer.

# 6.2.2 · Hill's Cipher6.2.2.1 · Enciphering with Matrices

#### Checkpoint 6.2.7

Answer. (ph) $\equiv$ HO and (er) $\equiv$ YL

#### Checkpoint 6.2.8

 $\mathbf{Answer.}$  Your enciphered message should be something like <code>JJNPA PYWTM IMKN</code>.

#### Checkpoint 6.2.9

Answer. CKEADO

#### 6.2.2.2 · Matrix Inverses and Deciphering

#### Checkpoint 6.2.12

Answer.

$$m^{-1} = \begin{pmatrix} 20 & 19\\ 21 & 23 \end{pmatrix}.$$

- Checkpoint 6.2.13
- Answer. great job

Answer. nice work

Checkpoint 6.2.15

Answer. hello again

#### 6.3 · Up Hill struggle?

6.3.9. Answer. VIXJZ FVIBW DUZT

#### 6.3.11.

Answer. spicy chicken wings

**6.3.13.** 

**Answer**. "Chuck Norris threw a grenade and killed 50 zombies, then it exploded."

## Appendix G

# Glossary

- **Block Cipher** A *block cipher* repeatedly uses a single key in order to encipher successive pieces of plaintext.
- **Cipher** A *cipher* is a system by which a letter or block of letters in a message is replaced by another letter or block of letters in a systematic way.
- **Ciphertext** The *ciphertext* is the message you wish to transmit after it has been rewritten in order to hide its meaning.
- **Code** A *code* is a system by which words or phrses in a message are replaced by other words, phrases, or symbols in a systematic way.
- **Crib** In cryptology a *crib* is a known piece of plaintext which may be compared to as sample ciphertext in order to aid cryptanalysis.
- **Cryptoanalysis** *Cryptoanalysis* is an attempt to compute the key or determine the meaning of a message from the ciphertext.
- Cryptography Cryptography focuses on concealing the meaning of a message.
- **Cryptology** *Cryptology* is the science of maintaining secrets by either hiding the meaning or the existence of messages.
- **Decipher** *Deciphering* is the process by which blocks of ciphertext are replaced by plaintext.
- **Decrypt** *Decrypting* is the process by we discover the meaning of a ciphertext without the aid of a cipher key.
- **Encipher** *Enciphering* is the process by which blocks of plaintext are replaced by ciphertext.
- **Factorial** Given a counting number n we define n-factorial as the number of ways in which we can arrange n objects and we calculate it by taking the product of all the consecutive positive integers from n down to 1, and we write:

$$n! = n(n-1)(n-2)\cdots 2\cdot 1$$

Note that for a variety of reasons 0! = 1.

**Frequency Analysis** *Frequency analysis* is the process of counting the characters, blocks of characters, or words in a text in order to determine how many of each there are relative to the entire length of the text.

- **Greatest Common Divisor** The greatest common divisor of two integers is the largest positive integer which divides both. For a variety of reasons it is important to know that it is also defined as the least positive linear combination of the two integers, i.e. 5 is the gcd(15, 35) because not larger positive integer divides both 15 and 35 and because  $5 = 1 \cdot (35) - 2 \cdot (15)$ is the smallest possible positive combination of 15 and 35.
- **Homophonic Cipher** A *homophonic cipher* is a cipher in which a single plaintext letter may be replaced by multiple different ciphertext letters throughout a message.
- **Index of Coincidence** The *index of coincidence* for a block of text is the probability that two characters chosen at random are the same.
- **Modular Equivalence** Given a positive integer n two integers are *equivalent* modulo n if they have the same remainder when you divide them by n. Equivalently, two integers a and b are equivalent modulo n if a b is divisible by n. When this is the case we write  $a \equiv b \pmod{n}$
- Monoalphabetic Substitution Cipher A monoalphabetic substitution cipher is a cipher in which there is a one-to-one correspondence between plaintext letters and the cipher text so that a single plaintext letter is always replaced by the same letter, group of letters, or symbol and every letter, group of letters, or symbol in the ciphertext represents the same plaintext letter.
- **Multiplicative Identity** A number a is the *multiplicative identity* if for all numbers b,

$$a \cdot b = b \cdot a = b.$$

#### Multiplicative Inverse See Reciprocal

**N-Gram** An n-gram is a block of n consecutive characters.

- **Nomenclator** A *nomenclator* is a system for disguising the meaning of a message which uses both cipher alphabets and a set of code words.
- **Null** A *null* is an extra character added to a cipher text to increase confusion. It may consist of extra characters added throughout the ciphertext designed to alter frequencies or characters added to the end of message to disguise the length of the message.
- **Pangrams** *Pangrams* are words or sentences containing every letter of the alphabet at least once.
- **Plaintext** The *plaintext* is the message you wish to transmit written in a readable form.
- **Polyalphabetic Substitution Cipher** A *polyalphabetic substitution cipher* is a cipher in which a single plaintext letter maybe replaced by several differnt ciphertext letters, groups of letters, or symbols and every letter, group of letters, or symbol in the ciphertext may represent more than one plain text letter.
- **Polygraphic Cipher** A *polygraphic cipher* is a cipher in which multiple plaintext characters are enciphered at a time so that how a character is enciphered depends on which other letters it is beside.

- **Polyphonic Cipher** A *polyphonic cipher* is a cipher in which a single ciphertext letter may be represent multiple different plaintext letters throughout a message.
- **Prime Number** A positive integer is *prime* if it has precisely two divisors, one and its self.
- **Reciprocal** The *reciprocal* (or *multiplicative inverse*) of a number x is some number y such that  $x \cdot y = 1$ .
- **Relatively Prime Numbers** Two integers are *relatively prime* if there greatest common divisor is one. In this case we may say that one of the integers is *prime* to the other.
- **Shift cipher** Shift Cipher generally refers to a monoalphabetic cipher in which the order of letters is not changed in the ciphertext each letter is just replaced with a letter a fixed distance further along in the alphabet.
- **Steganography** *Steganography* focuses on concealing the existence of a message.
- **Stream Cipher** A *stream cipher* uses a sequence (or stream) of different keys in order to encipher successive pieces of plaintext.
- **Substitution Cipher** A *substitution cipher* changes individual characters or groups of characters, but does not change their position.
- **Transposition Cipher** A *transposition cipher* rearranges the positions of characters, but does not change the characters themselves.

## Appendix H

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